

Shortcut to finding β

$$\beta\gamma = \sqrt{\gamma^2 - 1} = \sqrt{(\gamma - 1)(\gamma + 1)}$$

Commonly, one knows $\gamma - 1$, not γ , through what is usually called the “energy”, but is in fact the energy E_k that’s been added to the rest energy. $\frac{E_k}{mc^2} = \gamma - 1$; let us call this η . So

$$\beta\gamma = \sqrt{\eta(\eta + 2)} \text{ and finally } \beta = \frac{\sqrt{\eta(\eta + 2)}}{\eta + 1}$$

This gives β for any added energy E_k , without loss of precision. But for non-relativistic, this is simply

$$\beta_{\text{nonrel}} = \sqrt{2\eta}.$$