

$$\begin{aligned}
s &= \rho * \phi; \\
\theta &= \text{ArcSin}[(\rho / R) \text{Sin}[\phi]]; \\
f_1 &= \rho / \text{Tan}[\phi - \theta + \xi]; \\
f_2 &= \rho / \text{Tan}[\phi - \theta - \xi]; \\
d &= 2 R * \text{Sin}[\phi - \theta];
\end{aligned}$$

$$R = \rho \left(1 + \frac{2}{F}\right);$$

$$k = \kappa * \rho / R;$$

$$k_x = \text{Sqrt}[1 + k] / \rho;$$

$$k_y = \text{Sqrt}[k] / \rho;$$

$$M_z := \left(\begin{pmatrix} \text{Cosh}[k_y s] & \text{Sinh}[k_y s] / k_y \\ k_y \text{Sinh}[k_y s] & \text{Cosh}[k_y s] \end{pmatrix} \right).$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}.$$

$$\begin{pmatrix} \text{Cosh}[k_y s] & \text{Sinh}[k_y s] / k_y \\ k_y \text{Sinh}[k_y s] & \text{Cosh}[k_y s] \end{pmatrix}$$

$$\begin{aligned}
v_z^2 &= \text{FullSimplify}[\\
&\quad \text{ArcCos}[\text{Series}[\\
&\quad\quad (M_z[[1, 1]] + M_z[[2, 2]]) / 2, \\
&\quad\quad \{\phi, 0, 3\}]]] / (2 \phi)]^2
\end{aligned}$$

$$\left(-\kappa + \frac{2}{F} (1 + 2 \text{Tan}[\xi]^2)\right) + O[\phi]^2$$

$$M_r := \left(\begin{pmatrix} \text{Cos}[k_x s] & \text{Sin}[k_x s] / k_x \\ -k_x \text{Sin}[k_x s] & \text{Cos}[k_x s] \end{pmatrix} \right).$$

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{f_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1}{f_1} & 1 \end{pmatrix}.$$

$$\begin{pmatrix} \text{Cos}[k_x s] & \text{Sin}[k_x s] / k_x \\ -k_x \text{Sin}[k_x s] & \text{Cos}[k_x s] \end{pmatrix}$$

$$\begin{aligned}
v_r^2 &= \text{FullSimplify}[\\
&\quad \text{ArcCos}[\text{Series}[\\
&\quad\quad (M_r[[1, 1]] + M_r[[2, 2]]) / 2, \\
&\quad\quad \{\phi, 0, 3\}]]] / (2 \phi)]^2
\end{aligned}$$

$$(1 + \kappa) + O[\phi]^2$$