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Automatic Tuning Algorithm for the CANREB HRS Multipole

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Abstract: The CANREB HRS multipole is responsible for correcting higher order aberrations in the HRS beamline. A tuning algorithm designed to determine the appropriate potential configuration for a given beam has been developed and is described in this report. The algorithm determines these potentials based on given phase space outcomes using the method of least squares and is able to achieve runtimes ranging between 10^{-3} and 10^{-2} seconds.

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2 Introduction

The CANREB HRS multipole is used to correct for nonlinear optical effects in the HRS beamline [1]. An algorithm has been developed to determine the required potential configuration of the multipole that would correct these higher order aberrations based on given emittance data. The ultimate goal of the algorithm is to improve the separation of isotopes in a given beam so that each isotope exists in its own independent horizontal region. This is necessary if the desired isotope is to be correctly selected using the exit slit.

3 The Minimization Algorithm

3.1 Initial Test Case

The first example that was used to test and develop the algorithm was generating using Zgoubi. Using a particle tracking code such as Zgoubi was useful in verifying any corrections and assumptions that were made. The phase space plot for this particular example is shown in Figure 1 and represents the center of the beam. The optimization algorithm was written with the goal of minimizing the function f where f returns the total horizontal deviation from the center of the beam for a given set of electrode potentials. These potentials are denoted ϕ_i where i denotes the i^{th} electrode pair. The Python script used to minimize f also generates the resulting phase space plot with the multipole correction and returns the total runtime.

3.2 Functional Approach

It was discovered and verified that the HRS multipole possesses a superposition relationship that exists between the potentials of the electrode pairs and the change in position that each electrode pair induces on a particle located at a given x'. If $x_I(x')$ is the initial position of a particle located at a given x' and $\Delta x_i(x')$ is the change in position at a given x' caused by applying a 1V potential to the i^{th} electrode pair then

$$x_F(x') = x_I(x') + \sum_{i=1}^{23} \phi_i \Delta x_i(x')$$

where $x_F(x')$ is the new position. In other words, it is assumed that the $\Delta x(x')$ contribution due to each electrode pair is linear in its voltage and is independent of the voltages of the other electrode pairs. It follows then that the function returning the total deviation can be formulated mathematically as follows:

$$f = \sum_{x'} \left| x_I(x') + \sum_{i=1}^{23} \phi_i \Delta x_i(x') \right|$$

Defining a mathematical expression for the function that we are minimizing is important if we are to avoid having to run any simulations. The function can then be minimized using many different optimization methods from SciPy. One method that worked particularly well for this case was the Sequential Least Squares Quadratic Programming (SLSQP) method. Using the SLSQP method, the total deviation was reduced from ~ 23.7 mm to ~ 1.63 mm in a total runtime of about 3.5 s. Figure 1 shows this correction in blue along with the original distribution in red.

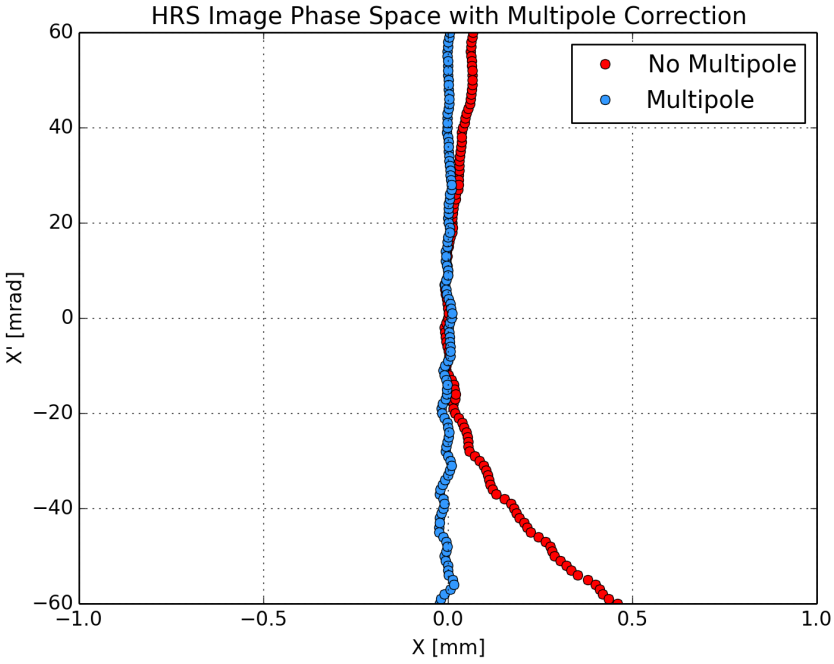

Figure 1: Result of the applied multipole correction based on the potential configuration obtained from the functional approach.

3.3 Matrix Approach

An alternative method to solving this problem would be to reformulate the problem as a matrix equation. Let us denote the upper and lower limits on x' for a given example as x'_U and x'_L respectively. The equations representing $x_F(x')$ at each x' can then be grouped together to form a matrix vector equation as follows.

$$\begin{bmatrix} x_F(x'_U) \\ \vdots \\ x_F(x'_L) \end{bmatrix} = \begin{bmatrix} x_I(x'_U) \\ \vdots \\ x_I(x'_L) \end{bmatrix} + \begin{bmatrix} \Delta x_1(x'_U) & \cdots & \Delta x_{23}(x'_U) \\ \vdots & \ddots & \vdots \\ \Delta x_1(x'_L) & \cdots & \Delta x_{23}(x'_L) \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_{23} \end{bmatrix}$$

Assigning matrix and vector symbols to each of the constructs above we have:

$$\vec{x}_F(x') = \vec{x}_I(x') + A\vec{\phi}$$

This equation can then be solved using the method of least squares. In essence the method will try to determine the best solution $\vec{\phi}$ that minimizes the difference between our desired position vector $\vec{x}_D$ and our resulting position vector $\vec{x}_F$. Of course $\vec{x}_D$ would represent the center of the beam.

$$\min_{\vec{\phi}} \left\| \vec{x}_D(x') - \vec{x}_I(x') - A\vec{\phi} \right\|$$

Since this is an overdetermined system with 23 unknowns and several more equations, the solution that this method produces may not meet our requirements. Indeed if we try to solve the entire system using this method, the resulting solution is very poor. In knowledge

of this, binary weights were assigned to each equation to turn the equations on and off. This allowed the Python script to loop through the possible sets of equations and determine the optimal set of weights. Using this approach, we are able find several sets of equations for which the algorithm was able to provide a strong correction. Using the optimal set, the algorithm was able to generate an improved total deviation of ~ 1.11 mm in a greatly improved runtime of $\sim 10^{-3}$ s. Figure 2 shows this correction graphically and Figure 3 shows the resulting distribution.

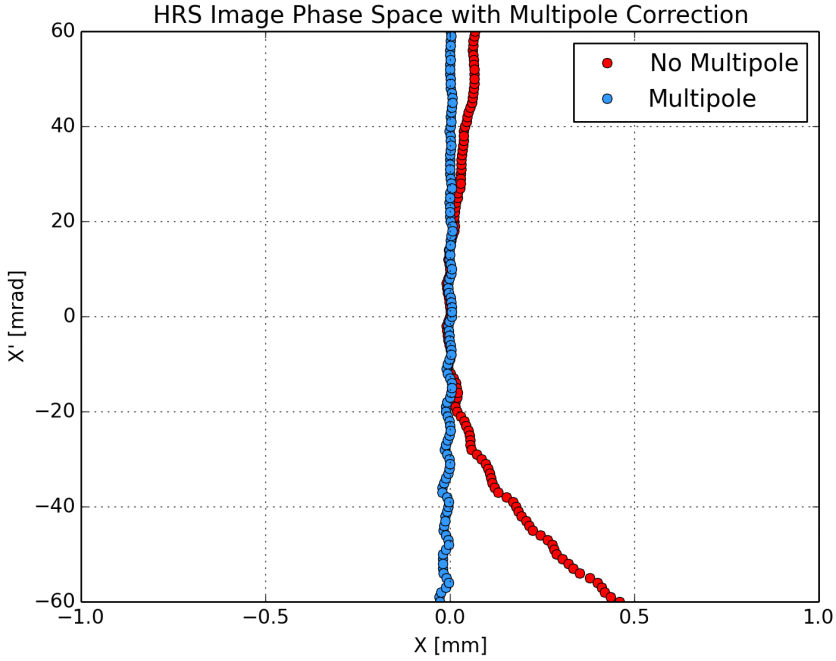

Figure 2: Result of the applied multipole correction based on the potential configuration obtained from the matrix approach.

3.4 Quadratic and Cubic Cases

Following the initial test case, the algorithm was tested on quadratic and cubic cases to mimic S and C shaped phase space diagrams. The range in x was also increased by a factor of 6 in the quadratic case and a factor of 12 in the cubic case compared to the distribution in the initial test case. This was done in order to evaluate how well the algorithm works in terms of obeying the parameter constraints. The algorithm performed almost a perfect correction for both cases while satisfying the parameter constraints. The runtime varied between 10^{-3} and 10^{-2} s for these two cases and several other tested cases. The corrections for the quadratic and cubic cases are shown in Figures 4 and 5 respectively.

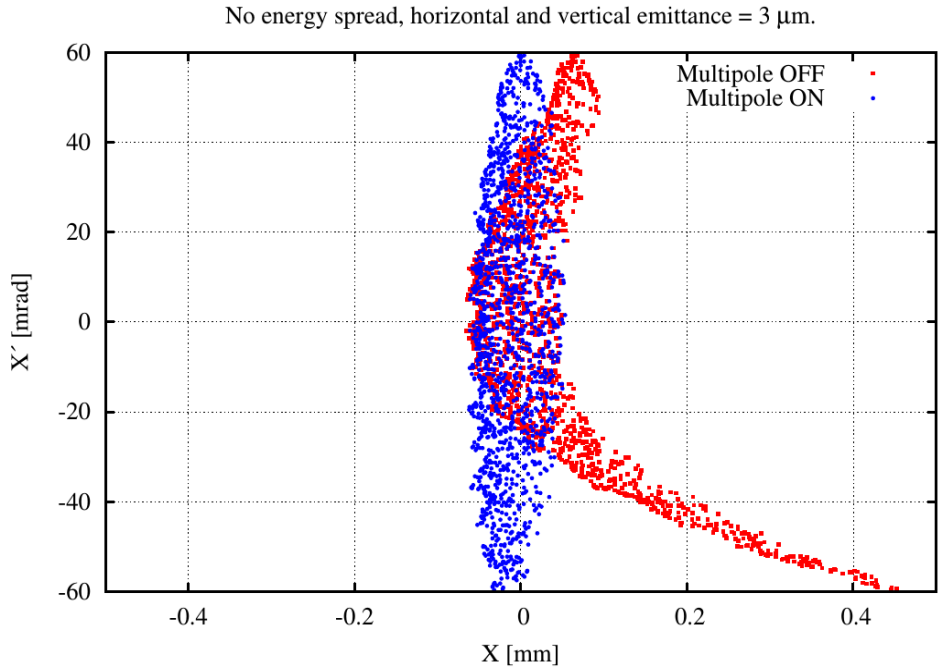

Figure 3: Resulting full particle distribution obtaining from applying the correction obtained using the matrix approach.

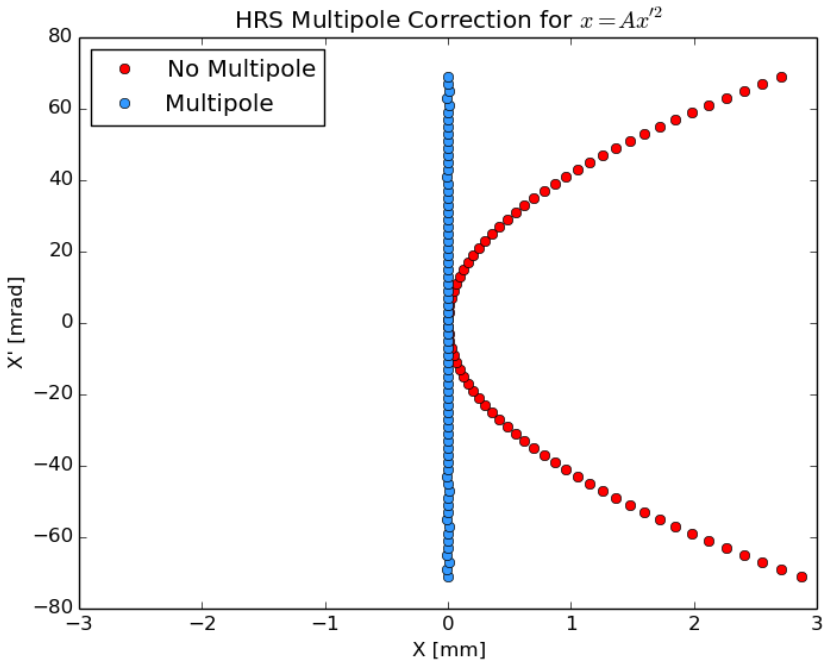

Figure 4: Multipole correction for quadratic relation between x and x'

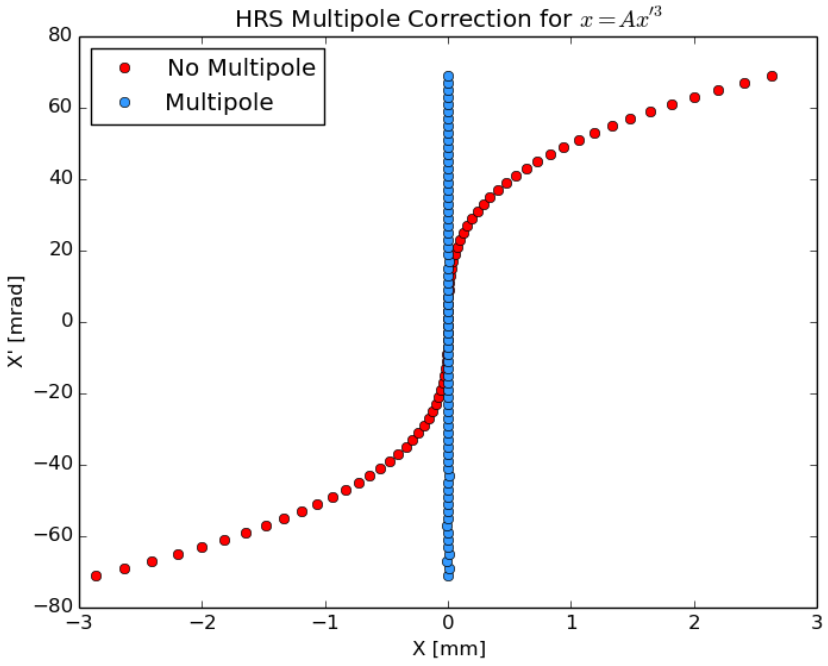

Figure 5: Multipole correction for cubic relation between x and x'

4 Running in Real Time

In order for the algorithm to apply a correction based on given emittance data, it was established that an effective strategy would be to calculate and correct the horizontal centroid of the uncorrected phase space plot along each value of x' using the formula

$$C(x') = \frac{\sum_x xI(x,x')}{\sum_x I(x,x')}$$

where $I(x,x')$ represents the particle intensity at a given point. This would of course be more efficient and more effective then attempting to correct the deviation of every particle in the distribution. The algorithm would then focus on minimizing the total deviation of the centroid from the center of the beam. It should be noted that the horizontal center of the beam is determined by finding the x value for which the maximum value of $I(x,x')$ occurs. Figure 6 shows the horizontal centroid of an S shaped phase space and Figure 7 shows the resulting phase space after applying the multipole correction to the centroid of the beam. This new contour was obtained using the 2D interpolation method from SciPy known as griddata.

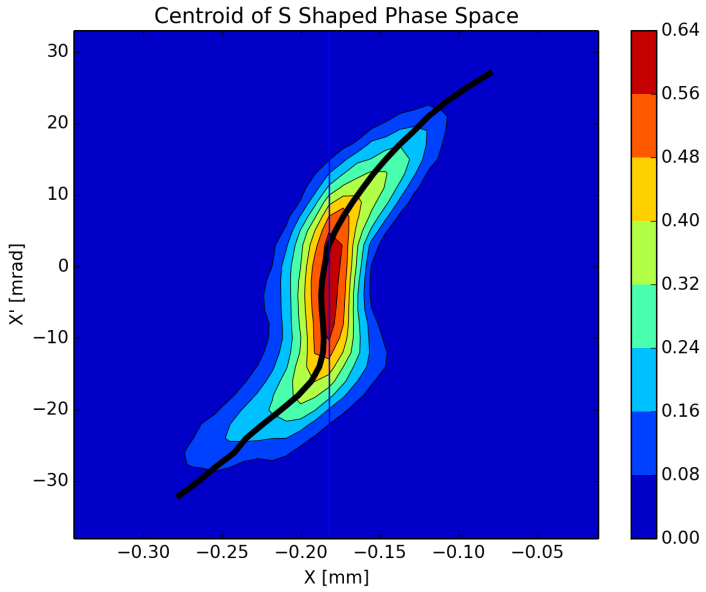

Figure 6: An S shaped phase space and the corresponding centroid. The blue line represents what is taken to be the center of the beam and the black line represents the centroid.

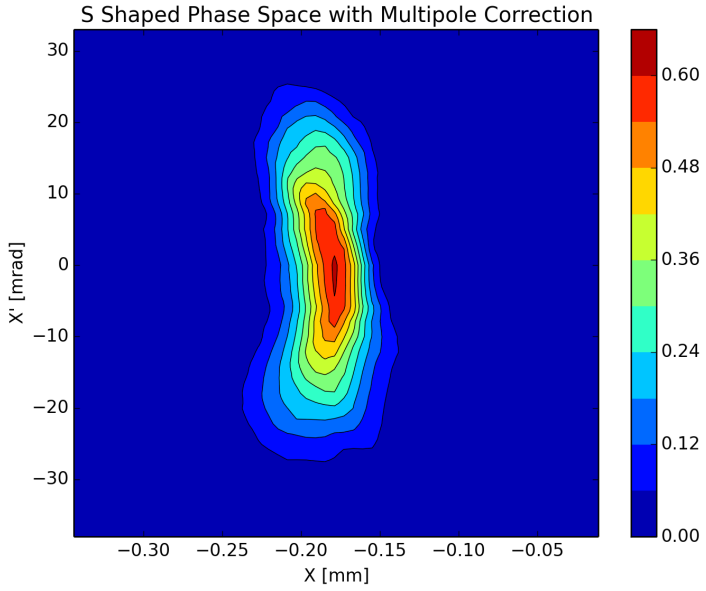

Figure 7: The resulting phase space after applying the multipole correction. The griddata method from SciPy was used for the interpolation.

5 Conclusion

After testing several cases, it is clear the the matrix formalism to solving the optimization problem of tuning the multipole is the more effective method both in terms of reducing the total deviation from the center of the beam and in terms of reducing runtime. While the average runtime when applying the functional approach ranges between 2 and 5 seconds, the average runtime when applying the matrix approach ranges between 10^{-3} and 10^{-2} seconds. Overall, the algorithm proved to be effective at correcting several different types of phase space outcomes while obeying the parameters constraints. In terms of testing on real cases, it seems clear that correcting the centroid results in the best outcomes when compared to other methods such selecting x values corresponding to maximum intensities.

References

[1] J. Maloney, "CANREB HRS Multipole Corrector," Tech. Rep. TRI-DN-16-09, TRI-UMF, 2016.