Organization: TRIUMF

Supervisor’s Name: Dr. Stanley Yen

Student’s Job Title: Undergraduate Co-op Student

Report Title: M11 Beamline Tuning Progress and Study

<table>
<thead>
<tr>
<th>Student’s Name:</th>
<th>Adrian Pikor</th>
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<tbody>
<tr>
<td>Degree (BSc, BA, MSc, etc.):</td>
<td>BSc</td>
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<td>Major:</td>
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<td>Work Term # (1, 2, 3, etc.):</td>
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<td>Semester (e.g., Fall 2016):</td>
<td>Summer 2018</td>
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Type of Report:

- ☑ Technical
- ☐ Lit. Review
- ☐ Interview
- ☐ Presentation
- ☐ Poster
- ☐ Reflective
- ☐ Article/Blog Post
- ☐ Other
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1 Abstract

The M11 beamline at TRIUMF is one of the facility’s originals, intended to produce a beam of pions in order to study nuclear structure. However, the beamline’s septum magnet had developed a water leak which rendered it un-useable. As such, this summer’s project was to improve the particle flux given these circumstances using a variety of beamline optimization programs to tune the magnet strengths, as well as to improve understanding of it to avoid treating it as a ”black box” with a nominal momentum as much as possible. A particle flux of around 65 kHz was achieved with a new tune, which was a significant improvement over the previous flux of 4 kHz. A 3D model in G4Beamline of M11 was also produced, which can be used to run simulations with and to ultimately aid in the understanding and further optimization of the beamline in the future.
2 Introduction

M11 is a beamline located near the south of the Meson Hall in TRIUMF (fig.2). It is designed to deliver pions, muons, and positrons of momenta varying between approximately 20 and 450 MeV/c, and consists of a total of six quadrupole magnets (Q1 to Q6), three dipole bending magnets (a septum magnet, B1, and B2), and four sextupole magnets (SX1, SX2, SX2.5, and SX3). However, the septum magnet, designed to bend particles by a further 12° between the BL1A quadrupole 1AQ9 and M11’s first quadrupole Q1, developed a water leak over 20 years ago, rendering it unusable. As a result, the particle flux is decreased by about five orders of magnitude (down from $\sim 10^8$ to $\sim 10^3$ or less), and the beam is very ill-defined. The resulting beam is speculated to be mostly tertiary particles, which are particles created from collisions of the secondary particles or protons scattered off of the target T1 with the beampipe/septum. The four sextupole magnets are also no longer in use, since their purpose is to refine aberrations created by the first-order behaviour of the dipole and quadrupole magnets, which is futile to attempt with such a poor beam.

Figure 1: Diagram of the M11 beamline circa 1987 [1], when B2 was bending particles west by 60°. Now, it is bending particles 30° to the east, and the sextupole SX4 has been removed.
Despite the state of the beamline, there is still interest in using it to perform some particle detector tests, which can still be done with particle fluxes as low as a few hundred or so per second. The project is to tune the beamline to maximize the particle flux, using programs such as TRANSOPTR and G4Beamline, under the supervision of Dr. Stanley Yen, as well as Dr. Rick Baartman.

3 Theory

In charged particle beam optics [2], the coordinate system used is such that the z-axis coincides with the trajectory of a reference particle, i.e. one with a specific momentum and position with respect to all of the beamline elements. \( x \) and \( y \) are the transverse distances from the reference trajectory along the horizontal and vertical axes, respectively. Positive \( y \) is defined to be what is normally considered as "upwards", and positive \( x \) has a varying convention, but is often taken to be "beam left", i.e. positive towards the left if following the reference particle down the beamline, with positive \( z \) being in this direction (fig. 1). Along with these positional coordinates, there are also directional coordinates, \( \theta \) and \( \phi \). Assuming paraxial rays, the small-angle approximation is used, such that \( \theta = dx/dz \) and \( \phi = dy/dz \). Two more coordinates are also often used, and are usually some combination of time and energy or momentum coordinates. For example, a commonly-used set of 5th and 6th coordinates are the path-length difference, \( \Delta l = (l - l_0)/l_0 \), and the percent deviation in the momentum, \( \delta = (p - p_0)/p_0 \).

Figure 2: Example diagram of an arbitrary particle travelling through a beam element with the transfer matrix \( R \). The beam element converts the coordinates \((x_2, \theta_2, y_2, \phi_2)\) into \((x_3, \theta_3, y_3, \phi_3)\). The black line with arrows is the trajectory of the particle, whereas that without arrows is the reference trajectory.
The equation that governs the motion of a charged particle in an electromagnetic field is the Lorentz equation,

$$\frac{dp}{dt} = \frac{q}{m} \left( E + v \times B \right),$$

(1)

and, assuming there is no electric field ($E = 0$), one obtains the following equations of motion:

$$m\ddot{x} = \frac{q}{m} \left( y B_z - z B_y \right)$$

$$m\ddot{y} = \frac{q}{m} \left( z B_x - x B_z \right)$$

$$m\ddot{z} = \frac{q}{m} \left( x B_y - y B_x \right),$$

(2)

where a dot over a variable denotes differentiation with respect to time ($\dot{x} = dx/dt$). Using the chain rule to convert these from equations of functions of time to functions of position ($z$),

$$\frac{d}{dt} \to \frac{dz}{dz},$$

(3)

one obtains the following equations of motion for $x(z)$ and $y(z)$,

$$\frac{d}{dz} = \frac{dz}{dz} \frac{d}{dt} = \dot{z} \frac{d}{dz} \to \frac{d^2}{dz^2} = \ddot{z} \frac{d}{dz} + \dot{z} \frac{d}{dz},$$

(3)

$$x''(z) = \left( \frac{q}{p} \right) \sqrt{1 + x'^2 + y'^2} \left( x' y' B_z - (1 + x'^2) B_y + y' B_z \right)$$

$$y''(z) = \left( \frac{q}{p} \right) \sqrt{1 + x'^2 + y'^2} \left( (1 + y'^2) B_x - x' y' B_y - x' B_z \right),$$

(4)

(5)

where a prime denotes differentiation with respect to $z$ ($x' = dx/dz$). If we assume paraxial rays once more,

$$v^2 = v_x^2 + v_y^2 + v_z^2 \approx v_z^2.$$  

(6)

With this approximation, it also suggests that terms higher than first order in $x'$ and $y'$ can be neglected, resulting in,

$$x''(z) = \left( \frac{q}{p} \right) \left( y' B_z - B_y \right)$$

$$y''(z) = \left( \frac{q}{p} \right) \left( B_x - x' B_z \right).$$

(7)

(8)

If we solve eq.1 in cylindrical coordinates for the special case where $\vec{B}$ and $\vec{p}$ are mutually perpendicular, we get a very simple relation $B = pqr$, or $Br = p/q$, where the quantity $Br$ is known as the magnetic rigidity of the particle.

Using these equations of motion, one can then use the expressions for the magnetic field of a certain beam element to determine the transfer matrix $R$, that relates the coordinate vector before entering the element, $\vec{x}_0$, with the coordinate vector after passing through the element, $\vec{x}_1$, i.e.,

$$\vec{r}_1 = R \vec{r}_0,$$

(9)

where,
\[
\vec{r}_0 = \begin{bmatrix}
x_0 \\
\theta_0 \\
y_0 \\
\phi_0 \\
\delta_0
\end{bmatrix}.
\]

As an example, we will derive the transfer matrix for a simple quadrupole magnet, assuming hyperbolic pole-tips. For such a quadrupole, the magnetic field equations are,

\begin{align*}
B_x &= gy \\
B_y &= gx \\
B_z &= 0,
\end{align*}

where,

\[
g = \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \frac{B_0}{a},
\]

with \(g\) known as the gradient of the quadrupole, and where \(B_0\) is the magnetic field at the pole-tips, and \(a\) is the aperture radius. Substituting equations (11) into equation (7) gives the following equation of motion in the \(x\)-direction:

\[
x'' = -(q/p)B_y = -(q/p)gx = -gx/(BR_0) = -k^2 x.
\]

Solving this for \(x(z)\) gives,

\[
x(z) = A \cos(kz) + B \sin(kz),
\]

and using the boundary conditions \(z_0 = 0, \ z_1 = L,\)

\[
x_1 = \begin{bmatrix} x_0 \\ \theta_0 \\ y_0 \\ \phi_0 \end{bmatrix} = \begin{bmatrix} \cos(kL) & \frac{\sin(kL)}{k} \\ -k \sin(kL) & \cos(kL) \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \end{bmatrix} = R_{Qx}(kL)x_0,
\]

where \(k^2 = g/(BR_0) = (B_0/a)/(BR_0)\). Similarly, in the \(y\)-direction we have,

\[
y'' = (q/p)B_x = (q/p)gy = gx/(BR_0) = k^2 y.
\]

which gives a similar matrix,

\[
y_1 = \begin{bmatrix} y_0 \\ \phi_0 \end{bmatrix} = \begin{bmatrix} \cosh(kL) & \frac{\sinh(kL)}{k} \\ k \sinh(kL) & \cosh(kL) \end{bmatrix} \begin{bmatrix} y_0 \\ \phi_0 \end{bmatrix} = R_{Qy}(kL)y_0.
\]

Putting it all together, we have the full matrix equation \(\vec{r}_1 = R_{Q}(kL)\vec{r}_0,\)

\[
\begin{bmatrix} x_1 \\ \theta_1 \\ y_1 \\ \phi_1 \\ \delta_1 \end{bmatrix} = \begin{bmatrix} \cos(kL) & \frac{\sin(kL)}{k} & 0 & 0 & 0 \\ -k \sin(kL) & \cos(kL) & 0 & 0 & 0 \\ 0 & 0 & \cosh(kL) & \frac{\sinh(kL)}{k} & 0 \\ 0 & 0 & k \sinh(kL) & \cosh(kL) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ y_0 \\ \phi_0 \\ \delta_0 \end{bmatrix}.
\]
This transfer matrix $R_Q(kL)$ is that of a horizontally-focussing quadrupole magnet; that of a vertically-focussing quadrupole would have the two submatrices $R_{Qx}(kL)$ and $R_{Qy}(kL)$ interchanged.

Refer to Stinson’s *A Brief Course in Beam Optics* [2] (or any other comprehensive beam optics text) for a full derivation of the transfer matrices of a variety of beamline elements, such as drift lengths, dipole magnets, solenoids, and more. It also includes a discussion of phase-space concepts, which explains the idea behind the $\sigma$-matrix.

## 4 The Project

The first two weeks was spent learning how to use TRANSPORT and TRANSOPTR, the charged particle beam optimization programs, as well as studying beam optics itself. Consequently, a familiarity with Linux needed to be developed first, as the programs required the operating system. TRANSPORT was looked at first, as that program was easier to use, largely because it had good documentation. As soon as some experience with TRANSPORT had been gained, the focus moved towards TRANSOPTR, which is the main program that has been used this whole summer. TRANSOPTR is very similar, but it is instead programmed in FORTRAN, with each element being its own separate subroutine in a .f file, and as such they can be modified as needed or new elements created entirely. TRANSOPTR was also easier to use, having a more intuitive input format. However, there was considerable difficulty due to the documentation being limited, as it had been heavily modified, requiring the consulting of Dr.Baartman and a fellow co-op student with a similar project, Dylan Bassi.

After having gained the ability to use TRANSOPTR in a basic sense, it was played with to attempt to get a tune with the initial beam parameters given to me, first assuming the septum magnet to still be operational, as it turns out TRANSOPTR cannot actually handle the case of a beam like M11 (see fig.3 for an example envelope). During this time, the beam area was shown, as well as how to locate and use the power supplies for each magnet, and how they are controlled remotely. Since the beamline had to be properly recommissioned with a radiation test (which was not done until the the beginning of July), whatever tunes had been developed could not actually be tested. On top of this, it was realized by Dr.Baartman and I that it was most likely not possible to create an accurate tune assuming that the beam originates near the beginning of the septum magnet, when in reality particles, most likely protons, were hitting the inside of the septum magnet and the beampipe upstream it, meaning that there was most likely a wide area of pions originating off of the inside of the beamline wall (fig.3), resulting in a widely misaligned beam. TRANSOPTR cannot handle a situation like this, so Dr.Baartman proposed an alternate procedure that could be used to try to find a tune.
Figure 3: Diagram of the beginning of the first half of M11, showing the approximate area where particles could be hitting to create pions.
The procedure involved essentially tracking individual particles and trying to minimize the amount lost. The first step was to figure out where the particles were generally originating from inside the beampipe (fig.4) in the coordinates \((x, \theta, y, \phi)\). This resulted in two phase-space distributions which define the spread of tertiary particles. Unfortunately, TRANSOPTR can only handle elliptical phase-space distributions, and so the best-fitting ellipse for each of the \(x-\theta\) and \(y-\phi\) planes had to be found. Since what TRANSOPTR would now calculate the beam with didn’t exactly represent the real distribution but contained it, the idea was to then perform a multitude of single-particle tracking runs (TRANSOPTR’s mode 2), where each point falls within the phase-space distributions. In order to facilitate this, however, a script needed to be written that could feed in each phase-space point and perform a mode 2 TRANSOPTR run. At first, this was attempted with bash shell scripting, but it proved very impractical when it came to the calculation of points. Therefore, most of the month of June was spent learning Python from scratch, at the recommendation of Dylan. Here is a basic outline of what the script was meant to do:

1) Perform an envelope fit (mode 3, fitting \(R_{12}, R_{34} = 0\); often had to fit \(R_{11}, R_{33} = 0\) instead) using the previously determined elliptical phase-space parameters

2) Extract the relevant sigma matrix parameters \((\sigma_{11}, \sigma_{12}, \sigma_{33}, \sigma_{34})\) and the fitted magnet strengths from the output into a separate file
3) Input a previously defined file of the phase-space distribution’s vertices to calculate points along and within the edges

4) Perform particle tracking runs (mode 2) with the previously fitted magnet strengths for each point in phase-space

5) Extract the coordinates of each point at each element into files, one for each element

Once the coordinates of each point are recorded to a file, these can then be plotted all at once to determine the resulting phase-space distribution after it passes each element. Knowing the acceptance of each element in phase-space, the fraction of particles lost can be determined and thus minimized.

However, once the script had been developed to the point where it could create files containing the full distributions at each element, a bug in TRANSOPTR was discovered. When trying to perform a particle tracking run, the initial angular coordinates did not match what was input, resulting in the calculated trajectories being off. Because this was something that didn’t seem to be fixable anytime soon, this small project had to be abandoned in the meantime. However, both the script and the general idea behind the procedure could be useful later on in the use of M11, as it’s most likely going to remain in its current state and in theory it is a very precise and accurate form of tuning that works with any type of beamline.

After this turn of events, it was decided by Dr. Yen and I that the best course of action was to design M11 in a program called G4Beamline (G4BL for short), which is a program that uses the Geant4 libraries (C++ particle physics libraries) to create an environment which makes it easy to build a beam of many possible elements in 3D and to then simulate a particle beam travelling through it, accounting for its interactions with the surrounding elements and its decay processes as well. G4BL could be used in tandem with TRANSOPTR to try to find a tune that minimizes particle loss. At the same time as the program was being learned and developing the model of M11, two visitors from other universities as well as Isabel Trigger and Aleksey Sher of TRIUMF were helping to get the particle detectors and the corresponding data acquisition devices set up (fig.5,6). The particle detectors consisted of a plastic scintillator mounted on a photomultiplier tube (BC1) right in front of the opening of M11, and then approximately 3.4 m away was a NaI crystal detector (called "MINA", which stands for Montreal INa) that was equipped with six photomultiplier tubes along the direction of the beam and two more equipped with scintillators (BC2 and BC3) oriented transverse to the beam, just like BC1.
Figure 5: Picture of M11’s counting setup, consisting of a multitude of modules such as a linear fan-in/fan-out, discriminators, a coincidence unit, gate and delay units, as well as an ADC and a TDC for recording to a computer.

Figure 6: Picture of BC1 (left) in front of the opening of M11 and MINA (right) equipped with BC2 and BC3. BC1 and BC2/3 are located approximately 3.4 m away from each other. Pictures taken by Aleksey Sher [3], pulled from the GRIDS M11 experiment manual.
After all of the equipment had been set up, some time was spent by Dr. Yen and I near the end of July testing and manually tuning M11. The tuning involved adjusting pairs and triplets of quadrupoles together by a proportional increase and decrease in the current supplied (and thus magnetic field), as well as flipping the polarities of different combinations of quadrupoles, since it was realized that the polarities of the magnets were not actually known beyond an assumption that the previous setting (all set to \( \pi^+ \)) as labelled above the power supplies) matched that of the previous tune discovered around the early 2000's, a 120 MeV/c tune listed in the manual. It was found that, using a rough tune developed in TRANSOPT with the elliptical phase-space parameters and a system starting just before Q1, keeping the original polarities, the particle flux coincident with BC1 and BC2 was around 39.6 kHz, and after flipping Q6 and adjusting it on its own, a rate of about 54.8 kHz was achieved. With further adjustment of Q1 and Q2, a maximum rate of 65.4 kHz was achieved. This was a significant improvement of over an order of magnitude over the particle flux using the old 120 MeV/c tune listed in the manual, which was the general goal of this project. Of course, the search for an even better tune and/or tuning schemes continued so that they could be used by Dr. Yen and other co-op students in the future.

![Time Difference between BC1 and BC2](image)

**Figure 7:** Time histogram at 100 MeV/c nominal. Note the poor resolution in comparison with the old plot in figure 9.
Figure 8: Charge deposited vs. time difference plot at 100 MeV/c nominal. The three distinct peaks represent three different particles: positrons, positive muons, and positive pions.

Figure 9: Old histogram done in the early 2000’s for 108 MeV/c nominal (and actual momentum based on time-of-flight measurements). The septum magnet was already broken at this time. Notice the time resolution of the peaks is considerably better than now (fig.7).
Settling on this tune, time of flight measurements were then taken by Dr. Yen and I. The ADC and TDC software had been setup to plot a histogram of time differences as well as a 2D plot of charge deposited in the scintillators vs. the time difference between them. Figure 7,8 is an example pair of plots at 100 MeV/c, showing three distinct peaks representing the time difference of positrons, positive muons, and positive pions, respectively. However, the time differences between the means of the peaks do not match those with the older histograms; a difference of approximately 4.2 ns is seen between the first (from the left) and second peak in the new plot vs. a difference of approximately 6.2 ns in the old plot, even when the older plot is at a higher momentum (the first peak from the left is that of positrons which are already travelling at very close to the speed of light, so the other peaks represent much more massive muons and pions that can still gain considerable amounts of speed, thus a smaller time difference means a higher momentum). This means that we were not at the desired momentum, and a quick calculation shows that the momentum was closer to 110 MeV/c; this shift in momentum was most likely due to the fact that the majority of particles were originating maximally off-axis on the beampipe wall, and so were entering the bending magnets at the largest angle away from normal as possible, resulting in an incorrect momentum selection. Another possible cause of this momentum shift is that the BL1A quadrupole magnet 1AQ9 might have been altered since, which would cause the steering of the secondary protons (and other particles) to change. Also, the time resolution of the TDC wasn’t as good as before. As shown in figure 9, the time resolution previously was around 1 ns as opposed to about 2 - 2.5 ns this time around, which was possibly caused at least partially by the same reasons as just given, but is mostly not understood. Attempts were made to improve the resolution, but it was not possible to do so given the time constraints.

Afterwards, development of the G4BL model of M11 continued, with the more-or-less finished model in figure 10 (the only thing not accounted for was the pole-face rotations of B1). At this stage, it was used to determine where the protons were hitting the beampipe more accurately (fig.11). With this knowledge, the idea was to then create a source of pions where the protons hit that could make it down the beampipe, but this would have to be done somewhat crudely with a few planes of particle sources approximating the actual partial-cylindrical area they would be originating from. However, it was noticed that G4BL has a command called "beamlossntuple", which creates something called an "ntuple" (a piece of data containing particle coordinates, momenta, and other information for each individual particle) for a particle that is lost, e.g. through being killed by a pre-defined killzone. Using this instead, it could be made so that the entire pipes and quadrupoles where it is known protons will be hitting to be killzones, and to have this "beamlossntuple" record each particle that is lost, past some value of z that is desired. This data could then be used, particularly with the initial $x$ and $y$ momenta $p_x$ and $p_y$ replaced with some uniform distribution representative of angles between 0 and the maximum angle possible before a particle were to hit the opposite corner of the farthest down pipe, which obviously depends on the distance down the pipe.
Unfortunately, it was found that the "beam" command used with an input file (the edited "beamlossntuple" file) does not correctly handle the conversion between global and centerline coordinates (coordinates along the beam-axis). In order to rotate centerline coordinates, one needs to use either the "corner" or "cornerarc" command, both of which rotate the centerline coordinate system to a new angle as desired. What "beam" ends up doing, though, is seeing the first corner and thus recognizing the centerline coordinates of that "branch" but then placing all of the particles defined in the input file along that branch, resulting in excursions outside of the beam elements that are placed along an area with the same angle as that of the branch. It appeared that this is an issue with the fundamental programming of the global-centerline conversion and not just a small bug, so an alternate idea was to either create a few approximating planes from which the particles originate from, or approximate it even further by creating a line source of particles down the centre.

At the point in time this was being worked on, the work term was nearing an end, and as such documentation of all files used as well as this report had to be started, so unfortunately not much progress was made beyond this. However, the model created in G4BL, along with the ideas on how to use it to improve upon the existing tunes, will prove to be very useful, as they allow another method on top of that developed by Dr. Baartman and I.
5 Conclusion

In working through this project, multiple pitfalls were encountered; the Python script becoming useless because of a bug in TRANSOPTR, G4BL’s global to centerline coordinate conversion bug with the “beam” command, the exact polarities of M11’s quadrupoles not even being known other than assuming the “pi +” label corresponds with the old tune, and so on. Yet, over an order of magnitude increase of the particle flux was achieved, albeit with a rough tune using just an envelope fit in TRANSOPTR. The procedure developed by Dr. Baartman and I, as well as the G4BL model of M11, will hopefully prove to be at least a useful aid, and possibly a new method of beamline tuning, for those who will continue working on the beamline in the future.
6 Bibliography


7 References


8 Appendix

8.1 Table of DAC and current values for old and new tune (100 MeV/c)

Tune A - Old tune for 100 MeV/c

<table>
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Tune B - New optimized tune for 100 MeV/c

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8.2 Time histograms and 2D charge vs. time difference plots for momenta of 60, 70, 80, 100, 125, 160, and 200 MeV/c

![Time Difference between BC1 and BC2](image1.png)

**Figure 12:** Time histogram at 60 MeV/c.

![BC2 Charge vs BC1 - BC2 time](image2.png)

**Figure 13:** Charge deposited vs. time difference plot at 60 MeV/c.
Figure 14: Time histogram at 70 MeV/c.

Figure 15: Charge deposited vs. time difference plot at 70 MeV/c.
Figure 16: Time histogram at 80 MeV/c.

Figure 17: Charge deposited vs. time difference plot at 80 MeV/c.
Figure 18: Time histogram at 100 MeV/c.

Figure 19: Charge deposited vs. time difference plot at 100 MeV/c.
Figure 20: Time histogram at 125 MeV/c.

Figure 21: Charge deposited vs. time difference plot at 125 MeV/c.
Figure 22: Time histogram at 160 MeV/c.

Figure 23: Charge deposited vs. time difference plot at 160 MeV/c.
Figure 24: Time histogram at 200 MeV/c.

Figure 25: Charge deposited vs. time difference plot at 200 MeV/c.