Energy deposition and Temperature Time Constants

Rick Baartman, TRIUMF

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Abstract

I derive the two time constants that apply to the case of an energetic electron beam stopping in a material. The thermal time constant (τ) determines the rate of temperature change from heat conduction, and the stopping time constant (Δt) is the time to melt (or otherwise damage) assuming no conduction.

Typically, $\tau \sim 3$ ms and $\Delta t \sim 0.3$ ms. Pulse lengths must be less than Δt, and duty factor (pulse length, rep rate product) must be less than $\Delta t/\tau$.

Thermal Diffusion

Depends upon quotient of thermal conductivity (k) and volume heat capacity (C_{pV}). For example, Cu: $k = 400$ Watt/metre/Kelvin, and $C_{pV} = 0.40$ Joule/gram/Kelvin $\times\, 9$ gram/cm^3 $= 3.6$ Joule/Kelvin/cm^3, giving $\alpha = 111$ mm^2/s. (Imagine a heat "bump"; this is the rate of spreading of area of the bump.)

This is the diffusion equation $\frac{\partial T}{\partial t} = \alpha\, \nabla^2 T$, and we assume there is a beam of particles hitting the surface and it has a Gaussian distribution, so

$$\Delta T = \Delta T_{\mathrm{c}} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right]$$

ΔT is the temperature relative to the ambient temperature.

We only care about the peak central temperature $\Delta T_{\rm c}$ and it evolves as $\frac{d\Delta T_{\rm c}}{dt} = -\alpha \Delta T_{\rm c} \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right)$, giving a time constant τ

$$\frac{1}{\tau} = \alpha \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} \right).$$

Example: For Cu and $\sigma_x = \sigma_y = 0.5$ mm, $\tau = 1.13$ ms. So if we deposit a bunch that thermally stresses the metal, we must wait more than a millisecond before depositing the next one.

Rate of Heat Deposition

With a beam distribution of $f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right]$, and a material stopping power of $S(E)$ (energy per mass per area), the rate of deposition of energy per unit volume at the surface is highest and its peak is

$$P = \frac{\rho S(E) I/e}{2\pi\sigma_x\sigma_y}$$

where I is the beam intantaneous current[1], not usually more than 10 milliamperes.

To get the rate of temperature increase, we ignore conduction (legitimately as it turns out that the times are short compared with the thermal time constant derived above) and divide this by the heat capacity C_{pV}.

[1] By "instantaneous" we actually mean over time scales longer than a few nanoseconds: the bunch structure is not seen in this physics as we shall see, the shortest time constants are about 100 microseconds.

Example: In Cu, at 10 MeV, $S(E) = 2.0$ MeV-cm^2/gram. We find $\frac{dT}{dt} = 3200$ Kelvin per millisecond. To reach the melting point of 1358 Kelvin requires $\Delta t = 0.33$ ms.

Here are some other materials and other energies. All times are in milliseconds, and $\sigma_x = \sigma_y = 0.63$ mm. For other beam sizes, scale as $\sigma_x \sigma_y$.

Material	Thermal (τ)	$\Delta t_{0.3\mathrm{MeV}}$	$\Delta t_{10\mathrm{MeV}}$	$\Delta t_{50\mathrm{MeV}}$
Aluminum	3.1	0.79	0.75	0.39
Copper	1.8	0.66	0.53	0.21
Tungsten	2.9	0.86	0.49	0.14
Al_2O_3	20.5	2.08	2.08	1.20
YAG	38.1	0.55	0.44	

How does this apply to Modes?

Two conditions:

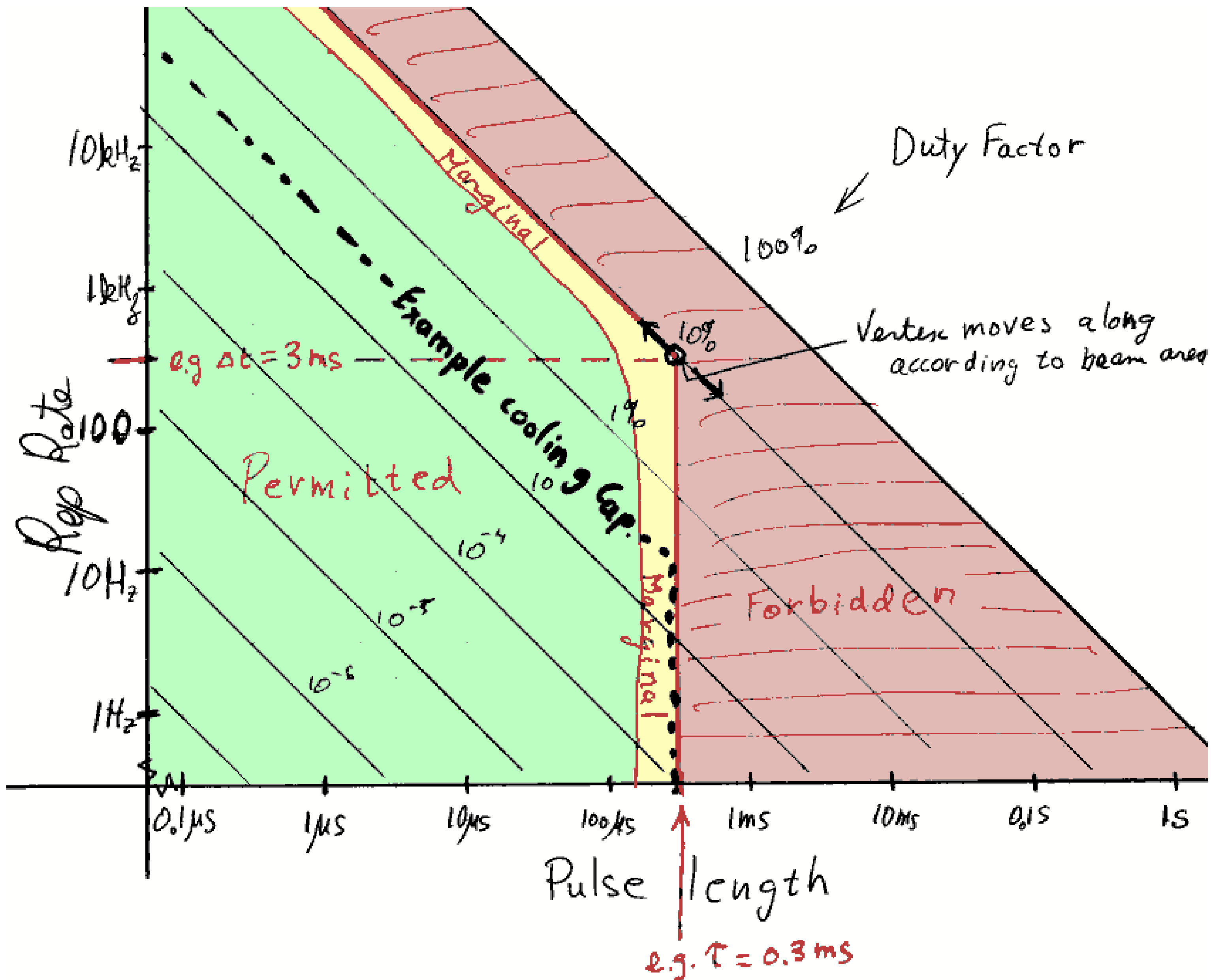
$$\text{Pulse length} < \Delta t$$

$$\text{Duty factor} < \frac{\Delta t}{\tau}$$

For different beam spot area, both τ and Δt change, but the ratio is constant. Thus, maximum duty factor not changed by beam size. In plot below, the vertex moves along a diagonal line.

Orthogonally, changing the peak beam moves the vertex across the diagonal lines.

Thirdly of course is the cooling power capacity of the material. This is also along a diagonal line below.


Duty Factor
100%
10%
1%
Vertex moves along according to beam area
e.g Δt = 3 ms
Example cooling cap.
Marginal
Permitted
Forbidden
Marginal
10kHz
1kHz
100
10Hz
1Hz
Rep Rate
0.1μs
1μs
10μs
100μs
1ms
10ms
0.1s
1s
Pulse length
e.g. τ = 0.3 ms

Model Limitations

The heat flow is 2-dimensional, i.e., implicit assumption that range $>$ beam size.

Here are the ranges in mm:

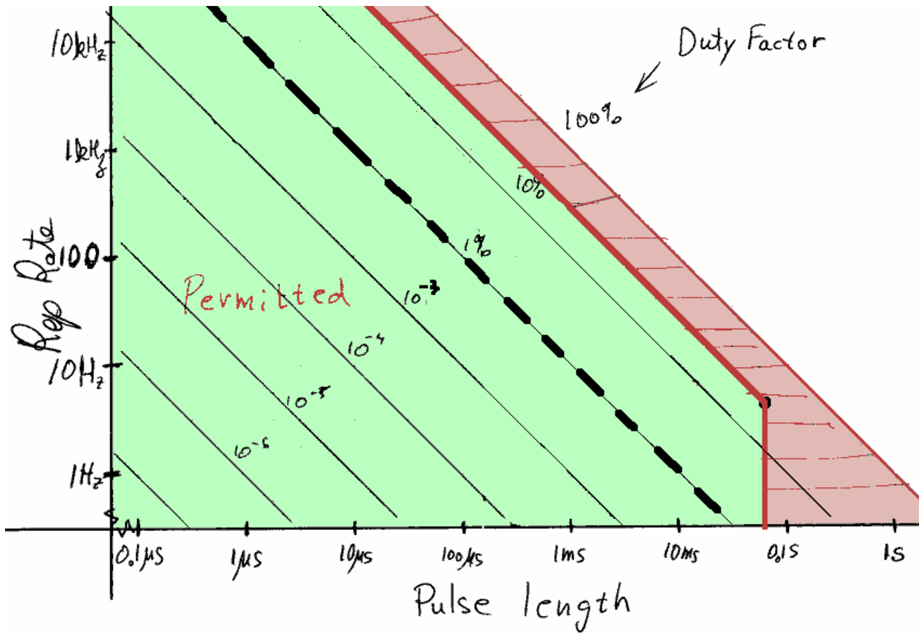
Material	0.3MeV	10MeV	50MeV
Aluminum	0.40	21	77
Copper	0.14	7	20
Tungsten	0.16	6	15
Al_2O_3	0.40	13	40
YAG	0.15	4	

This means that for the 300 keV case, move the vertex upwards by a factor of 2 or so.

The model also breaks down for rastering. For very large beam spots, the vertex also moves upwards (same reason).

EMBD Dump

A good example is EMBD dump. Here, (Chao, TRI-DN-12-03) the spot will be round with rms size $7\,$mm. We find for Cu, $\tau = 0.22\,$s, $\Delta t = 65\,$ms. See below. Maximum duty for a 100kW beam would be $65/220 = 30\%$. In fact, the design calls for only $1\,$kW capacity, so maximum duty will be 1% (dashed line), and no danger of local melting for any pulse length.



ISAC: Protons on Ta

The same formalism applies to any energetic particle. 500 MeV protons have a stopping power probably similar to tungsten. (I cannot find any for Ta.)

I use $100\,\mu$A and an rms beam size of 1 mm. This is known to cause local melting and so is a good check of the model.

We find $\tau = 20$ ms. $\Delta t = 170$ ms if ambient $T = 300$ K, but of course this target is used near the melting point and this can reduce Δt to values smaller than τ, causing melting.

To avoid melting, we need to raster this beam. What is the lowest allowed frequency? The simplest scheme is annular. Assume a radius of 5 mm, circumference of about 30 mm. The beam has a 95% size of 4 mm and needs to move out of the way in 20 ms or 0.2 m/s, so a frequency of $0.2/0.03 = 7$ Hz. Thus, the 12 Hz ANAC ac steerers should have been just fine. Perhaps it was a fatiguing issue.

If frequency is large compared with this number, it is as if the beam is as large as the rastered spot. At 70 Hz, there should be no longer a fatiguing issue. This is effectively like $\sigma_{x,y} = 5$ mm, though the gaussian approximation is probably not very good. We can also defocus the beam and make it this size. In either case, we find $\tau = 0.5$ sec. This means beam trips of time length small compared to this number would be harmless, but beam trips of half a second or longer are thermally stressful.