Optics design of the ISIS Vertical Section Replacement

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Abstract: The vertical section of the 300 keV injection line has been re-designed. The technique used is to calculate the 3D beam envelopes (6 phase space dimensions) including space charge, axial magnetic field, and acceleration at the dee gaps in the cyclotron. The calculation is first order only, but contains all the relevant physics of that order: in the cyclotron it includes electric focussing, the gap-crossing resonance, and the radial-longitudinal coupling effect of space charge. Because of the coupling effects in the spiral inflector, and the weak magnetic focussing on the first few turns, it is impossible to match the injection line to the cyclotron optics; a factor of 5 growth in emittance is predicted and this agrees with measurements taken in the past on the existing configuration. Space charge limits the achievable beam current in the cyclotron because it drives the vertical incoherent tune to zero; the design’s goal is to achieve 500 $\mu$A, and the present studies and past performance indicate that this is feasible.
1 Introduction

The 12 m long vertical section of the TRIUMF injection line has been in service for 35 years. It is relatively inaccessible, since the top half is through 4 m of concrete roof beams and the bottom half is inside the support structure of the cyclotron. The only accessible part is the 1.4 m “Vertical Removable Section” (VRS). As the whole injection line is entirely electrostatic, it is susceptible to anything that can compromise the insulators. Since the process of buildup of cracked vacuum pump oil, sputtered metal atoms, etc. is cumulative, it was always foreseen that this beamline would have to be re-furbished after some decades.

The as-built design was encumbered with a device needed for commissioning, namely the 11.5 MHz chopper, situated 2 m upstream of the inflector. This meant that besides matching optics to the cyclotron, there was also needed independent quadrupoles to match into and out of the chopper slit. To achieve sufficient flexibility, there were 9 independently controllable quadrupoles in this region just upstream of the inflector.

The chopper has not been used for the past 15 years. It was only ever useful at low intensity, since the short bunches it creates debunch quickly because of space charge for any current over a few 10s of µA. It was therefore decided to remove the chopper. This simplifies the optics and makes tunes more robust against space charge forces, as the beam no longer needs to come to a small size compatible with the slit. The goal was to reduce the number of matching quadrupoles to 4 or 5. The benefit would be simpler tuning.

The goal of the cyclotron upgrade program is to reach 0.5 mA extracted. The highest extracted current achieved with the current injection line was 420 µA in 1988\(^1\). This required raising the dee voltage to 105 kV, to widen the phase acceptance. Higher rf voltage helps in two ways: (1) More extreme late phases can clear the centre post and survive to the second turn, and (2) Electric focusing, which only occurs as the dee voltage is falling, causes early phases to experience vertical defocusing, since on the first few turns magnetic focusing is too small. So the accepted phases only range from near the peak to later. Near peak rf voltage, where electric focusing is smallest, the vertical space charge tune shift can overcome the electric focusing. Thus the phase acceptance narrows as the bunch charge rises.

The rf frequency is 23 MHz, so for 500 µA, the charge in one bunch is 22 pC. At this frequency, the distance a 300 keV H\(^-\) ion travels in one rf period is

\[ \beta \lambda = 0.328 \text{ m}. \]

2 Space Charge Considerations

Can we reach the goal of 500 µA extracted? A conservative value for phase acceptance is 36°. Therefore, to get a feel for the space charge effect, let us imagine a sphere of diameter 3.2 cm, uniformly filled with charge 22 pC, at the injection gap. This will double in size to 6.4 cm after a drift of 6 m. (This can be calculated analytically, since the force follows Coulomb’s law \( F = \frac{kQ}{4\pi\varepsilon_0 r^2} \) and is integrable.) Reversing the time, this implies that a 6.4 cm dia. sphere can be made to reach a minimum of 3.2 cm if given the appropriate focusing force and then left to drift for 6 m.
This underestimates the space charge effect, since the bunches must transversely be much smaller than 6 cm. A reasonable size that would fit through the inflector is 1 cm. Tracking a bunch with diameters 1 cm transverse and 3.2 cm longitudinal yields a transverse doubling distance of only 1.6 m. Therefore, to keep the transverse size below 1.5 cm requires placing focusing lenses every 2 m or less. See Fig. 1.

![Figure 1: Time-reversed calculation of beam envelopes (half-diameters), starting with desired bunch at z = 0. Bunch charge = 22 \mu C, energy = 300 keV. Simple thin lenses of focal length 76 cm placed at z = 1 m and then every 2 m. Notice that the transverse size decreases, as the lengthening bunch alleviates transverse space charge force.](image)

Doing so results in a longitudinal expansion from 3.2 cm to 32 cm (= the bunch spacing, meaning that at this point the beam is effectively DC) in a drift of 14 m. The existing buncher is situated 21 m from the inflector, which is not optimum for 22 \mu C bunches. Repeating the debunch calculation for lower bunch charge, we find that the 21 m distance is optimum for 11 \mu C, or 250 \mu A. Rather than moving the existing bunchers 7 m nearer to the inflector, it is planned to install a rebuncher in the VRS, 6.3 m from the injection gap. This would allow in principle optimal bunching for any bunch charge from zero to > 22 \mu C.

I’ve derived a simple formula\(^2\) for the maximum peak beam current in a cyclotron with weak vertical focusing on the first few turns. The mechanism is that the space charge tune shift becomes comparable to the tune. It is

\[
I_{\text{max}} = \beta \left( \frac{\nu_{y0} b_{\text{max}}}{R_{\infty}} \right)^2 (7.8 \times 10^6 \text{ Amp}).
\]  

The \(\beta\) and unperturbed tune \(\nu_{y0}\) over the first 1/4 betatron oscillation are resp. 0.033 and 0.14, the maximum allowed beam half height is \(b_{\text{max}} \approx 1\) cm, \(R_{\infty} = 10\) m. This yields \(I_{\text{max}} = 5\) mA, so the rf voltage must be large enough to create a phase acceptance of at least 36° to reach 0.5 mA extracted.
3 Envelope Technique, TRANSOPTR

Bunches of beam can be described by their \( 6 \times 6 \) \( \sigma \)-matrix. In the case considered here, there is first a coupling of the transverse directions due to the varying axial field in the vertical section, then the electrostatic inflector also couples the transverse to longitudinal. As a result, the bunches’ axes lie along none of the beam axes. Only linear space charge is calculated in this model. This is done by first calculating the appropriate elliptic integrals to determine the linear part of the forces in the ellipsoid’s natural coordinate system, then transforming by a 3D rotation to the beam axes. This technique is described by deJong and Heighway\(^3\). I have modified the deJong/Heighway code TRANSOPTR to include varying axial fields, inflector, bunching and acceleration by rf gaps.

3.1 Bunching

There is no linear way to take the beam from DC to bunched, so an approximate technique is used. Bunches are launched at the start of the vertical section with the longitudinal correlation parameter \( r_{56} = \frac{\sigma_{56}}{\sqrt{\sigma_{55}\sigma_{66}}} \) set to \(-1\), and the correct length and energy spread to create the minimum bunch length at the point of injection. As noted above, there will exist a combination of voltages for the existing first and second harmonic bunchers plus a new one installed in the VRS, which will create the optimum bunches at the injection gap.

An example of such a calculation is shown in Fig. 2. Here the coherent phase advance per cell is 45°. The beam current is 0.5 mA and this results in the incoherent phase advance being depressed by space charge to 41° at the buncher, and to 16° at the end where bunch length is minimum. The increasing space charge due to the bunching beam causes the transverse size to double, but it happens slowly enough that the beam stays quite well matched.

![Figure 2: Periodic section, as beam bunches.](image-url)
3.2 Axial Magnetic Field

Along the vertical injection line, the magnetic field varies from zero to the final 0.3 T. The technique used to track the envelopes in this region is described in Baartman & Kleeven\textsuperscript{4}. The magnetic field file is in Appendix 1. Note that when this table is read into \textsc{transoptr}, a scale factor of $-1$ is applied to the distance, so that this variable becomes the location along the beamline, up to the inflector entrance, which is at $-11.7$ inches.

3.3 Inflector Orientation

For the inflector in \textsc{transoptr}, we use a “natural” coordinate system, derived as follows. We prefer $+x$ to be radially outward in the cyclotron. Since the beam direction is $+z$, and in the TRIUMF cyclotron the beam rotates ccw, the $+y$ direction is \textbf{down}. In the limit of zero magnetic field, the inflector is an electrostatic $90^\circ$ bend. Therefore, at the inflector entrance, $+y$ is in the direction towards the outer/lower electrode; it is on the North-East side (blue) at the entrance in Fig. 3. To be right-handed, therefore, the inflector $x$-axis is as indicated. In the injection line, conventionally, $+x$ is North, so the axes rotation is $44.5^\circ$. It remains only to determine the sign of this rotation. The ions rotate counterclockwise in the cyclotron as seen from above, i.e. as seen in the direction the injected particles are travelling. The magnetic field in the injection line is in the same direction as it is on the median plane, so ions with a transverse velocity component in the injection line vertical section will also rotate ccw. The sign convention in \textsc{transoptr} turns out to be that a call to subroutine \textsc{rt} with negative argument takes out the coupling from the magnetic field in the vertical line. This means that a negative argument rotates the axes ccw. Hence, the appropriate call to \textsc{rt} at the inflector entrance is an argument of $-44.5^\circ$.

Figure 3: Inflector orientation.
3.4 Inflector/Deflector

The TRIUMF inflector has a height, or electric radius of 11.70 in, and a tilt at exit of 47°. The Baartman/Kleeven Hamiltonian \(^4\) is built into TRANSOPTR to track the envelopes (with space charge as needed) through the inflector. The transfer matrix resulting from single particle tracking is found to agree closely with that found by Root\(^5\) by tracking through the calculated field maps, in spite of the fact that the magnetic field varies sharply at the entrance. TRANSOPTR integrates through the inflector at every calculation; using the calculated transfer matrix would only be correct in the case of negligible space charge force.

The inflector is followed by an electrostatic deflector which is used to properly centre the injected beam. Traditionally, this deflector has been cylindrical. However, I also considered the possibility of using plates curved vertically to augment the vertical focusing. It is as well possible to improve the focal properties of the inflector by curving its plates in this way\(^6\). This is under investigation.

3.5 Acceleration & Electric focusing

The cyclotron’s injection (first) gap is symmetric, focusing equally in \(x\) and \(y\) directions; thereafter the gap extends radially, providing vertical focusing but no radial. This is essential to achieve vertical stability in the first few turns where there is little to no magnetic focusing. In TRANSOPTR, this is handled with a thin lens whose focal strength is proportional to the derivative of the accelerating voltage. The longitudinal direction is similarly simple (\(R_{55} = 1\)) provided the longitudinal coordinate is time. However, if as in TRANSOPTR and commonly in other codes as well, the longitudinal coordinate is \(z = t/(\beta c)\), then \(R_{55} = \beta_f/\beta_i\). TRACE3D documentation erroneously has \(R_{55} = 1\), though the code has recently been corrected\(^7\).

The TRIUMF cyclotron has a built in isochronism error to generate a phase slip to both gradually move the beam from the falling edge of the waveform back onto the peak, and to partially compensate for radial-longitudinal coupling\(^8\). This is built into the calculation by using the phase versus \(E\) as found in Fig. 5 of Dutto et al.\(^8\). See Fig. 4. There is some uncertainty regarding this curve since operationally, this phase slip is empirically tuned by adjusting the trim coils, especially TC0. The result of this tuning is not known in terms of the phase shift versus energy.

All calculations assume a dee voltage of 100 kV, for an energy gain of 400 keV per turn.

3.6 Magnetic focusing

Between gaps, the calculation takes the beam through a flat dipole interspersed with thin vertical focusing lenses whose strength is adjusted to give the correct overall magnetic vertical tune as a function of \(E\). This function is taken from Figs. 2 and 5 of Dutto et al.\(^8\). See Fig. 4. Although Dutto has stated that this does not correspond to the final magnetic field configuration, it is the only record of vertical tune versus \(E\) that apparently exists.
Figure 4: Red: Vertical magnetic tune; negative values actually mean imaginary values. Green: Phase shift due to intentional deviation from isochronism used in the envelope calculations. These are highly idealized versions of curves found in Dutto et al.\textsuperscript{8}

### 3.7 Space Charge in Cyclotron

It is well-known that the space charge effect on bunches circulating in a cyclotron is very different than in a synchrotron. This is because the cyclotron is constantly on transition. The result is that bunches rotate in the median plane, the rate of rotation depending upon the charge density. The stationary distribution is circular (bunch length = radial width). This was first derived by Kleeven\textsuperscript{9}. In general, injected bunches are much longer azimuthally than their radial width. In this case, the only way for the bunch evolution to stabilize is to split into many small droplets. This was experimentally demonstrated in the SIR (Small Isochronous Ring) at MSU\textsuperscript{10}. Perhaps surprisingly, the circular shape of the stationary distribution does not depend upon intensity; higher intensity means the evolution is quicker. The rate at which the vortices rotate is simply twice the Laslett radial tune shift\textsuperscript{11}.

In the linear force envelope model as used in the matching calculation, bunches will rotate, propeller-like, as a whole. This would show up as an out of phase modulation of bunch length and radial width. An example is shown in Fig. 5 for the TRIUMF case (0.5 mA or 22 \( \mu \)C charge per bunch, \( \beta \gamma \epsilon = 0.3 \mu \text{m} \)). The lower plot is for a bunch sufficiently short that space charge effects are strong enough to start the “propeller-like” effect. In the upper plot, the bunch is launched at twice the length and in that case it continues to lengthen linearly with \( R \) as one would expect, because in the negligible space charge case, the phase width of the bunch is invariant. Launching bunches that are shorter still (\( \sim 4 \) mm radius) eventually leads to matched circular bunches. So counter-intuitively, long bunches continue to grow linearly keeping their phase width constant, but very short bunches of the same total charge remain very short with their phase width decreasing monotonically. Since our bunches will be 16 mm in half length, it appears that this effect is not important. However, these are only linear space charge calculations. Any “hot spots” can initiate vortex motion, thus changing the longitudinal bunch profile in irregular ways. It has been observed that at the highest bunch intensities we have used in the TRIUMF cyclotron, the bunches became double-peaked.

In \( H^- \) cyclotrons, the fluctuating radial width of the circulating bunches is of little import. This is because when extracting by stripping, it does not matter if turns overlap; the only important determining factor of extracted beam quality is the correlation between a particle’s \( R \) and \( E \). Since space charge moves particles to higher radius by giving them extra kinetic energy, the effect is not important. This is indicated by the blue curves in Fig. 5.
4 Cyclotron Matching

It is tempting, but turns out to be unproductive, to track a matched beam in the cyclotron backwards to the inflector entrance to discover the beam characteristics needed there. It’s unproductive because the inflector strongly couples all three dimensions. It is therefore not possible to create a matched beam using the injection line optics. Worse, it is not possible to create a matched beam even if we had the freedom to create one at the inflector exit. The reason is that the matched vertical beam size in the cyclotron is larger than would be allowed by the aperture of the injection gap. Also, the tune varies with phase because its origin is the rf electric field. Thus, the matched beam is largest for the early edge of the phase acceptance window and becomes significantly smaller toward the late edge. So all phases are poorly matched, but early phases are most poorly matched.
It is inevitable, therefore, that there will be emittance growth. Both from 5-finger-probe scans in the cyclotron and analysis of the extracted beam, it is concluded that the normalized vertical emittance of the beam circulating in the cyclotron is 0.5 to 1.0 \( \mu m \), whereas the injected beam has emittance 0.1 to 0.15 \( \mu m \). This is a factor 5 growth.

### 4.1 Minimization Objective Function

Instead of backwards runs, the strategy used was to calculate beam envelopes in ISIS, right through the inflector/deflector, and over many turns in the cyclotron. Many times per turn, the squares of the beam sizes are calculated. These are added together in quadrature. The resulting sum is the objective function. To account for the fact that vertical and horizontal tunes are different, the terms are weighted by the appropriate tune. In other words, we minimize the function

\[
\chi = \sum_{i} \nu_x \tilde{x}_i^4 + \nu_y \tilde{y}_i^4
\]

where \( \tilde{x} \) and \( \tilde{y} \) are the radial and vertical beam envelopes. The 4\(^{th} \) power is used instead of squares to ensure that the maximum beam sizes are made as small as possible. If squares are used, one is not sufficiently sensitive to mismatch because as the maximum size grows, the minimum size diminishes and the sum can remain fairly much the same.

In the original design, the cyclotron was to operate in either of two modes: high resolution or high intensity. In the former, the phase acceptance would be restricted and/or third harmonic rf was to be used to allow turns to remain separate, so the widths of individual turns was a primary concern. Third harmonic never came to fruition, and this mode is no longer used.

### 4.2 Dispersion Correction

A subtlety which I think was overlooked in the original design is that in the high intensity mode, the turn width is not of primary importance, so radial-longitudinal coupling due to the gap-crossing resonance is not important. Turns can become harmlessly broad as long as the relation between radius and energy is not spoiled. In other words, to minimize radial mismatch, we do not use the turn widths directly, but instead use the dispersion-corrected turn widths.

The dispersion in a continuous isochronous bend of radius \( \rho \) is \( \eta = \rho / \gamma^2 \), since the radial tune is \( \nu_x = \gamma \). So the dispersion correction for a particle of fractional momentum deviation \( \Delta p / p \) is \( x_\eta = \frac{\rho \Delta p / p}{\gamma^2} \approx \rho \Delta p / p \): \( x = x^* + x_\eta \).

The turn width \( \tilde{x}^* \) with dispersion removed can be found by applying the dispersion matrix to the \( \sigma \)-matrix. We find the following

\[
\tilde{x}^* = \sqrt{\tilde{x}^2 - 2 \tilde{x} x_\eta r_{16} + x_\eta^2},
\]

where \( r_{16} \) is the normalized \( x - \Delta p / p \) correlation parameter.
4.3 Optimization

4.3.1 Method

The minimization is performed using a simulated annealing algorithm, since a simple downhill simplex method is inadequate to discover the global optimum when there are more than 4 variable parameters.

At first, the bunch length was set to 3.2 cm and the 5 parameters $\alpha_x, \beta_x, \alpha_y, \beta_y$ plus $\theta$, the rotation angle of the $x$-$y$ axes, were fitted at the inflector entrance. The function to be minimized was $\chi$ in eqn. 2 above, but augmented with the following term:

$$100 \ \text{Max}\{0, (\text{beam diameter} - \text{half electrode separation})^2\}$$

inside the inflector. This has the effect of making sure the beam nominal size ($4\sigma$) is smaller than half the electrode separation, i.e., the electrode separation is $> 8\sigma$.

The normalized emittance used was $\epsilon_x = \epsilon_y = 13 \mu m$. This a “4rms” emittance, meant to contain 86% of the beam. It is roughly a factor of 2 larger than currently measured for beams of typically 400 $\mu A$ in the injection line. It is in fact the emittance measured for an optimized source at 5 mA, since an attractive option for obtaining 500 $\mu A$ extracted is to run 5 mA unbunched.

Next, a periodic transport was set up and the final 5 matching quadrupoles were placed at variable locations and orientations to determine the optimum configuration that gives both the best match and sufficient tuning flexibility. Besides locations, orientations and strengths of the 5 quads, the phase advance of the periodic section was also used as a variable parameter. Two features of the original design were rediscovered. (1) The unperturbed phase advance per cell was optimally found to be 45$^\circ$ when the cell length was 1 metre. Though phase advance per cell of 60$^\circ$ or 90$^\circ$ is more commonly used at other labs for high intensity transport, it is found in our case that the larger longitudinal space charge forces for transversely smaller beam results in poorer bunching efficiency. (2) No skew quadrupoles were necessary; all the quadrupoles could be oriented with their electrodes at North, East, South, West, without significantly compromising the best match.

Another constraint used was to keep all quadrupole voltages well below 5 kV. This was achieved, but required that the final two quadrupoles have a physical length of 6 inches. All other quadrupoles are 4 inches.

4.3.2 Results

The result is shown in Figs. 6 and 7. In these figures, zero is the location of the inflector. Upstream of this point one can see the bunch coming to a minimum (blue). Downstream of the inflector, the radial beam size grows rapidly; this is due both to space charge and radial-longitudinal coupling due to gap-crossing resonance. However, as noted above, the widening turns are not a problem; the dispersion-corrected widths (black) look very good. The vertical beam envelope (green) is mismatched in the cyclotron. This is due to the unavoidably-too-small beam at the inflector exit.

Fig. 6b shows the case where the deflector plates are curved also in the vertical direction. This has the effect of adding vertical focusing. In all cases tried, this gave an improvement to the matching.
Figure 6: Beam envelopes through the injection line and into the cyclotron versus distance in metres, for the conventional deflector (left), and for one with vertically curved plates (right). Charge per bunch is $22 \mu C$ for a time average current $0.50 \text{ mA}$. This is for bunch injection phase $28^\circ$.

Figure 7: Detail in the region of the final matching quadrupoles, for the conventional deflector.

The maximum envelope sizes are indicative of the eventual circulating emittances, as precessional mixing will occur. These are $\beta \gamma \epsilon_r = 1.5 \mu m$, $\beta \gamma \epsilon_z = 1.7 \mu m$, to be compared with the injected emittances $\beta \gamma \epsilon_x = \beta \gamma \epsilon_y = 0.32 \mu m$. So the emittance growth is still roughly a factor of 5.

The quadrupole settings are given in Appendix B. In the cases of the different bunch injection phases as shown in Appendix B, it is understood that $18^\circ$ is too low, since it would mean that the bunch extends over the phase range $0^\circ$-$36^\circ$. Early phases around $0^\circ$ are poorly focused vertically as shown in the next section. And $38^\circ$ is too high, since it would mean that the bunch extends over the phase range $20^\circ$-$56^\circ$. Bunch phases later than about $50^\circ$ will not allow the full bunch to clear the centre post on the first turn.

I have continued to use 5 “knobs”, but Q93 & Q94 are coupled together as a doublet, and the same for Q95 & Q96. Therefore the final 7 quads are used for matching, but in such a way that there are only 5 knobs. Also, in most cases, Q92 can remain at the periodic quad setting.
Figure 8: Envelope of beam in cyclotron versus energy in MeV, up to turn 20 for 7 starting bunch phases. Red is the radial envelope, green the vertical, blue the longitudinal, and black is the radial envelope with dispersion removed; all are half-sizes in mm.
4.3.3 Injection Phase

Fig. 8 are calculations of the beam envelopes in the cyclotron. To interpret these plots, it should be borne in mind that these are not plots of phase slices of a bunch. In each case, a bunch is injected, but the rf voltage has been linearized so that its slope is not $V \sin \phi$, but $V \sin \phi_0$, $\phi_0$ being the phase of the central particle.

Notice that bunch phases $0^\circ$ and below have too little vertical focusing: at $0^\circ$, the total vertical size is 2 inches at its worst. Above $50^\circ$, the energy gain is too low for the whole bunch to clear the centre post on the first turn. Also notice that the isochronism error does indeed correct the radial-longitudinal coupling (red curve), but only for bunch phase near $10^\circ$. As was stated, however, it is not the red curve, but the black curve that determines the radial beam quality, and it is very well matched at all bunch phases.

4.4 Steering and Collimation

As past operation has demonstrated, there is a sufficient number of “knobs” to adjust the beam position when it exits the inflector: the inflector voltage (mostly vertical), deflector voltage (radial), correction plates (vertical, twice per turn), and trim coil $B_R$ (radial). The steering and collimation in the injection line need to be designed so that the beam can be aligned and sized to fit through the inflector. The reason this is critical is that the inflector electrodes are uncooled; it is not easy to implement cooling, since they operate at nearly 30 kV. Allowed spill is about 1 $\mu$A. This is assured by “skimmers” at the entrance and exit that trip the beam if they intercept more than 1 $\mu$A. These skimmers have a square aperture. The entrance skimmer is aligned with the electrodes, 19 mm on a side.

4.4.1 Collimation

Ideally, one would like to place a collimator and image it optically onto the inaccessible region say halfway through the inflector. There are at least 3 reasons this is not possible.

1. The optics just upstream of the inflector is not fixed, but must change according to current and bunching, etc.
2. Space charge changes the optics such that even for fixed optics, an image location depends upon the bunch charge.
3. The optics is far from simple, coupling $x$ and $y$.

Alternatively, one can attempt to collimate all 4 transverse directions ($x, x', y, y'$). This is effectively the approach that has been used in the past; cooled 13 mm dia. collimators are used throughout the injection line to control halo growth and ensure beam alignment. However, the existing collimators were added after the beamline had been completed, so they were only placed in locations that were accessible without a complete dismantling of the beamline. These locations were far from ideal, especially in the vertical section. Since halo is re-generated more or less continuously by scattering, space charge and bunching, it is desirable to collimate as near to the inflector as possible. Since the periodic section will run near $45^\circ$ phase advance per cell, the collimation can be achieved with two collimators, two cells apart. Space charge depresses the phase advance somewhat, but we are interested in eliminating particles at large transverse amplitude and these see a smaller space charge field on average than the core particles do. The last locations two cells apart are DB89 and DB93. See Fig. 9. As well, a collimator will be placed in DB95. These collimators should be circular, with aperture diameters as given in Table 1.
Table 1: Collimator apertures

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<th>Collimator</th>
<th>Aperture dia. (mm)</th>
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<td>DB89</td>
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<td>DB93</td>
<td>16.4</td>
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<td>DB95</td>
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These apertures were calculated as follows. For the 500\(\mu\)A tune of Fig. 7, the size in the \(y\)-direction is marginal at the inflector entrance; \(\sim 8.5\) mm, while skimmer allows only 9.5 mm. A new tune was developed which compromises slightly the match to the cyclotron, but reduces \(y\) size to 6.4 mm. The beam sizes at the 3 collimator locations were multiplied by 1.3, meaning the full aperture would be 5.2\(\sigma\). A factor 1.3 applied to the beam size at the inflector entrance gives 16.6 mm; a safety margin of 1.2 mm between beam edge and skimmer edge.

### 4.4.2 Steering

Steerers must be placed to centre the beam in the collimators and through the inflector. To centre the beam in CC89, we use steerers in DB85. To centre in CC93, we use steerers in DB89. Centring through the inflector is not as straightforward, as quadrupoles have been situated in such a way as to optimize matching and this layout is not optimum for steering. We therefore compensate by placing rather more steerers than would otherwise be necessary. We use DB93, DB95, as well as placing steerers just upstream of the inflector. The way in which a steering impulse propagates downstream depends on the tune, but not on space charge. Because of the axial magnetic field, an initial kick in the \(x\) direction will result in some movement in the \(y\) direction, and vice versa. These effects are shown in the plots in Fig. 10.

In these figures, the right edge of the plot is the inflector entrance (-11.7 inches). The discontinuity at -18.32 inches is a rotation of 44.5\(^\circ\) to align the axes to the electrodes. So for most of the plot up to -18.3 in., \(x\) is N-S, but to the right of this point, \(x\) is in the direction parallel to the inflector electrode. Up to -18.3 in., \(y\) is E-W, but to the right of this point, \(y\) is in the direction perpendicular to the inflector electrode.

From Fig. 10a, one can see that indeed, the DB89 steerers are optimally placed to position the beam on CC93, which is at \(z = -106\). From Fig. 10b, one can see that the DB93 steerers can centre the beam on CC95, at \(z = -66\), but cannot effectively move the beam in the \(x\) direction at the inflector entrance (blue and red curves). From Fig. 10c, one can see that steerers in DB95 can effectively move the beam in both \(x\) (red) and \(y\) (magenta) directions at the inflector entrance. It happens that the transport from DB95 to the inflector rotates the beam by roughly the same 44.5\(^\circ\) as the inflector axes are rotated.
Figure 9: Drawing of as-designed vertical section.
Figure 10: Steering propagation. The labels are to be interpreted as follows: CB-90N-y is the effect of correction bender 90N (North) in the y (E-W) direction, and so on.
## A Axial Magnetic Field

Table 2: The $D$ column is the distance in inches above the cyclotron’s median plane. The field $B$ is in kG.

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</table>

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The table indicates positions, and voltages of the quadrupoles for 6 different cases.

Table 3: Positions in inches, measured with respect to the cyclotron median plane, and voltages in kV. Sign convention: Positive voltage means focusing in the $x$, or North-South direction.

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<th>Name</th>
<th>$z$(in)</th>
<th>V(kV)</th>
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<td>cyl.</td>
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</table>
C TRANSOPTR code

C.1 Data file:

0.3 0 0 939.3 1. 2.2e-11 !22 pC times 23MHz = 0.5mA
1 5 .1 0.5E-5
253 -539.09 1. -1.
.17 .006 .17 .006 6. .013 !use 4. .009 for 11pC
0.3937008 1. 0.3937008 1. 0.3937008 1. 0. 0.3937008
1 5 6 -1.
10
10. 0. 10. 0
-3.0000 -5. 0. 1
2.3931 -3. 3. 1
1.5917 -0. 5. 1
3.3645 0. 5. 1 !2nd last Q
-3.5864 -5. 0. 1 !last Q
0.1 0.000 0.15 0 !dee voltage
118. 0. 180. 0 !phase
0. -1. 9. 0 !curved deflector
0.0005 0.0001 0.001 0 ! emittance in-rad
1.E-4 20
10 0.0 0.95 10

C.2 System subroutine:

SUBROUTINE tSYSTEM
COMMON /BLOC1/qnpds,q92,q93,q95,q97,q98,VD,phi0,cee,exu
COMMON/PRINT/IPRINT,IQ1,JQ1,IQ2,JQ2,IQ3,JQ3,IQ4,JQ4
npds=qnpds
deff=4.76
dtot=39.37
doub=16.0
d1=(doub-deff)/2.
d2=dtot/2.-d1-deff
ax=1.73
bxu=58.
qp=-3.0
wq=0.
if(exu.ne.0.)then
   call cic(ax,bxu,exu,-ax,bxu,exu)
   wq=2.
endif
call fringeQ(0.18,0.0,0.0,0.,-.33)
c Periodic Section
c if(npds.gt.0)then
do i=1,npds
call DR( d1,0,0)
call EQ( qp, 1.0000, deff,0.0000,0,0)
call DR( d2,0,0)
call DR( d2,0,0)
call EQ( -qp, 1.0000, deff,0.0000,0,0)
call DR( d1,0,0)
call print_transfer_matrix
endo
dendif
c Matching Section
c
call DR(d1,0,0)
call EQuad(q92, 1.0000, deff,wq,"Q092")
call DR(d2,0,0)
call DR(d2,0,0)
call EQuad(q93, 1.0000, deff,wq,"Q093")
call DR(d1,0,0)
call DR(d1,0,0)
call EQuad(-q93, 1.0000, deff,wq,"Q094")
call DRIFT(18.61 ,"")
call EQuad(q95, 1.0000, deff,wq,"Q095")
call DR(d1,0,0)
call DR(d1,0,0)
call EQuad(-q95, 1.0000, deff,wq,"Q096")
call DRIFT(15.69 ,"")
call fit(1,1,1,0.5,10.,0)
call EQuad(q97,1.,deff+2.,wq,"Q097")
call DRIFT(5.7400,"")
call EQuad(q98,1.,deff+2.,wq,"Q098")
call DRIFT(9.6200,"")
call TRIUMF_CYC(cee,vd,phi0,40)
return
end

subroutine TRIUMF_CYC(cee,vd,phi0,nhturns)
c
    c Inputs: cee=0 means cyl. deflector
    c vd= dee voltage
    c phi0= rf phase of beam injected
    c nhturns= number of half turns
    c
    COMMON/PRINT/IPRINT,IQ1,JQ1,IQ2,JQ2,IQ3,JQ3,IQ4,JQ4
    COMMON/MOM/P,BRHO,PMASS,ENERGK,gsq,ENERGKi,charge,current
    COMMON/ZED/ZINIT,Z
    c inflector's x (parallel to electrode surface at entrance) is 44.5 degrees
cw from North. North is x dir'n in vertical line. But a positive theta in
call rt(theta) is cw. So 44.5 must be negative.
    c
    call rt(-44.5,0,0)
call fit(1,3,3,0.25,10.,0)
aa=11.7
    if(abs(AA+Z/2.54).gt.0.01)then
        write(6,*)'ZINIT is wrong. Z must be zero at median plane'
        write(6,*)'Error is ',AA+Z/2.54,' inch'
        stop
    endif
    call INFS(AA, 0 , 1.15,10) !! inflector: INF(AA, RHO, TILT, ...)
b=3.
nsub=6
    rho=brho/(b/10.)/0.0254
    call BE(rho,7.2,0,0,0) ! a drift of 1.3" - simply scaled from the drawing.
    CALL ExB(6.5,rho,.36 ,cee,0,0) ! " deflector: ExB(RadT, RADD, ANGLE,CEE, ...) 
call fit(1,5,5,0.5,10.,0)
    phi=phi0+phase(energk)
    c
    c the first gap
    c
    call rfgap(vd,phi,23.e6,-0.)
    qz2=1.-gsq !this gets the bend to give correct isoch. and radial focus
tunz2=tunz2(energk)
tz2=tunz2-qz2 !have to take out the vertical focus given by the bend
    rho=brho/(b/10.)/0.0254
    do j=1,nsub
        call thinlens2(0,3.141593/rho/nsub*tz2,0,0)
        CALL BE(rho,7.2,0,0,0) ! a drift of 1.3" - simply scaled from the drawing.
        CALL BE(rho,7.2,0,0,0) 
    enddo
    call disp_mat(-rho,0.,0.,0.,0)
    do i=1,nhturns
phi=phi0+phase(energk)
c
the rest of the gaps. Note: rfgap changes brho, so new rho calculated after each
c
call rfgap(2.*vd,phi,23.e6,-1.)
qz2=1.-gsq !this gets the bend to give correct isoch. and radial focus
tunz2=tunez2(energk)
tz2=tunz2-qz2 !have to take out the vertical focus given by the bend
rho=brho/(b/10.)/0.0254
dispe=rho/gsq
do j=1,nsub
c
thin lenses to give the vertical focusing
c
call thinlens2(0.,3.141593/rho/nsub*tz2,0,0)
CALL BE(rho,180./nsub,qz2,0,0)
enddo
call disp_mat(-dispe,0.,0.,0.,0,0)
call fit(1,1,1,0,1.,2)
call fit(1,3,3,0,5,2)
call disp_mat(dispe,0.,0.,0.,0,0)
enddo
RETURN

10 format(5f10.5)
END

function tunez2(energk)
c from fig. 5 of Dutto (1972)
if(energk<.5.)then
  tunez2=(energk-2.)*0.0133
else
  tunez2=0.04
endif
return
end

function phase(energk)
c from fig. 5 of Dutto (1972)
if(energk<.5)then
  phase=166.7*energk-68.3
elseif(energk<2.)then
  phase=15.
elseif(energk<4.)then
  phase=-17.5*energk+50.
else
  phase=-20.
endif
return
end
References


[7] Private communication, Frank Krawczyk, LANL.


