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Subject A revised conceptual design for a 15° switching dipole for beam line 2A		

1. Introduction

Protons are to be extracted from extraction port 2A to deliver beam to one of two targets of the TRIUMF ISAC facility. At present, only the west target is installed; beam is delivered to it by using two 15° rectangular dipoles. With the installation of the second target it will be necessary to install a ±15° switching magnet at the location of the upstream dipole and move it to a position symmetric to that of the remaining 15° dipole but on the east leg of the beam line.

An earlier report[1)] presented a preliminary design for such a switching magnet. However, on reviewing that report, several inconsistencies were found. In addition, the optics of the beam line have been modified such that the parameters of the switching magnet required modification. As a result, the design of the switching magnet requires further study.

This report presents a modified design of the switching magnet for beam line 2A.

2. Design parameters for the switching magnet

The design proposed here is based on the design of the rectangular dipoles[2)] on the beam line together with the results of magnetic surveys of those magnets. Optics of the beam line now require a magnet that has an effective length 0.943 m (37.126 in.) and that is capable of producing a field of 10.1 kG for 500 MeV protons. As with the existing 15° beam-line dipoles, we design the magnet for a maximum proton energy of 520 MeV (corresponding to a momentum of 1.11633 GeV/c) and field of 10.35 kG.

The measured (straight-line) effective lengths of the rectangular dipoles are $L_{eff} = 37.1$ in. and their iron lengths are $L_{iron} = 33.0$ in. We maintain similar parameters for the switching magnet. Thus we have the following design parameters.

B_0	=	Maximum magnetic field	=	10.35 kG
g	=	Maximum air gap	=	10.16 cm
θ	=	Maximum bend angle	=	±15.0°
s	=	Length of the central trajectory	=	94.269 cm

We first calculate the basic parameters of the magnet.

$$\rho_0 = \text{radius of curvature of the central trajectory} = \frac{s}{\theta} = \frac{(180.0)(0.94269)}{(15.0)(\pi)} = 3.60081 \text{ m} = 141.764 \text{ in.}$$

$$\boxed{\text{Radius of curvature of the central trajectory} = \rho_0 = 3.601 \text{ m} = 141.8 \text{ in.}}$$

We take the effective straight-line length of the magnet to be

$$L_{eff} = 2\,\rho_0 \sin\frac{\theta}{2} = 2(3.60081)(0.13053) = 0.94000 \text{ m} = 37.008 \text{ in.}$$

$$\boxed{\text{Straight-line effective length of the magnet} = L_{eff} = 0.940 \text{ m} = 37.00 \text{ in.}}$$

and assume that the the iron length, L_{iron}, is obtained from

$$L_{eff} = L_{iron} + g$$

so that

$$L_{iron} = L_{eff} - g = 0.9400 - 0.1016 = 0.8384 \text{ m} = 33.008 \text{ in.}$$

$$\boxed{\text{Iron length of the magnet} = L_{iron} = 0.838 \text{ m} = 33.00 \text{ in.}}$$

The above parameters, L_{eff} and L_{iron}, correspond to the measured (straight-line) effective lengths and physical iron lengths of the 15° rectangular dipoles.

3. Pole width and geometry

We propose to fabricate the pole of the switching magnet such that the beam enters the dipole normal to its entrance edge. The exit edge of the magnet will be circular although this may be changed to plane surfaces in further design. We will also insist that the pole width be wide enough such that a uniform field is produced along the trajectory. To this end, given the beam half-width δ, the dipole gap g and pole-edge chamfer c, we require the minimum distance between the central trajectory and the pole edge to be

$$\Delta = \delta + g + c \; .$$

The appendix to this report gives the details of the calculation of the pole profiles. Here we summarize the results of those calculations.

The calculations in the appendix are based on values of $g = 4.0$ in. and $c = \delta = 1.0$ in. Thus $\Delta = 6.0$ in. These, together with the values $L_{eff} = 37.0$ in., $L_{iron} = 33.0$ in. and $\rho_0 = 141.764$ in., produced the design developed in the appendix. It is reproduced below.

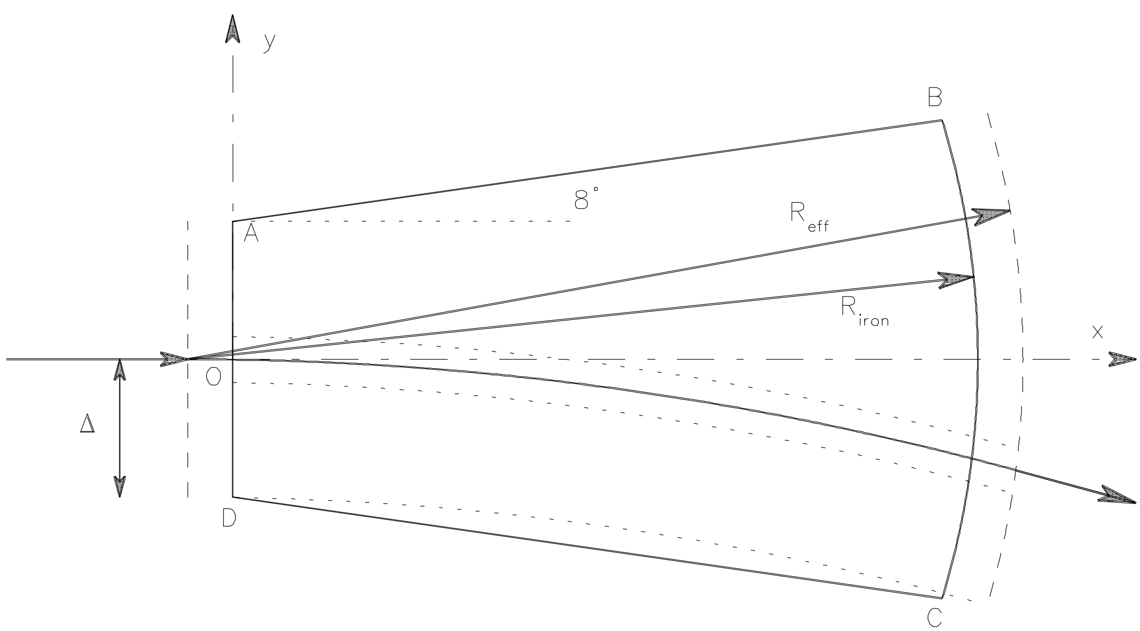

Fig. 1. The pole configuration of the switching magnet with a circular exit edge.

In this figure the incoming beam defines the x-axis and the line $\overline{AD}$ lies along the y-axis. The origin of this coordinate system, O, is at the midpoint of the line $\overline{AD}$. The dashed lines represent the assumed locations of the effective edges of the dipole. The solid curve is the trajectory of the central ray; the dotted curves either side of it indicate the full width of the incident beam. The minimum required separation between the central trajectory and a pole edge is represented by the lower dotted line. Note that the radius vectors of the exit edge of the pole, R_{iron} and R_{eff}, have their centers a distance a = 2.00 in. to the left of the entrance edge.

The dimensions of the straight sides of the pole are defined by the points A, B, C and D. The lengths of the sides of the pole are calculated in the appendix to be:

$$\begin{aligned} \overline{AD} &= 12.00 \text{ in.} \\ \overline{AB} = \overline{DC} &= 31.724 \text{ in.} \end{aligned}$$

4. Ampere-turns per coil

The design of the coil of the dipole must be such that existing, 85 V–650 A power supplies may be used.

Rather than use a maximum field of 10.35 kG, we calculate the required Ampere-turns per coil based on a maximum field of 10.5 kG. Thus

$$NI \text{ per coil} = \frac{1}{2}\left[1.1\,\frac{B_0\,g}{\mu_0}\right] = \frac{1}{2}\,\frac{(1.1)(1.050)(0.1016)}{4\pi\times10^{-7}} = 46,691 \text{ A-t}$$

where we have allowed for a 10% flux leakage. We take

$$\boxed{NI \text{ per coil} = 47{,}000 \text{ Ampere-turns}}$$

and generate the following table

I (Amperes)	100	200	300	400	500	600	700	800	900	1000
N (turns)	470	235	157	118	94	78	67	59	52	47

Because of the existing power supply, we choose

I	=	600 Amperes
Coil configuration		8 turns wide by 10 turns high

5. Coil design

We assume a current density of 3000 A/in.^{2} = 4.65 A/mm^2 and calculate the required conductor area from

$$\text{Conductor area} = \frac{600\ A}{3000\ A/\text{in}^2} = 0.2000 \text{ in.}^2 = 129.03 \text{ mm}^2$$

This is satisfied by Ananconda 0.516 in.-square and Outokumpu #6813 conductors; their parameters are given in the table below.

	Outokumpu #6813		Anaconda 0.516	
OD	[0.5118 in.]	13.0 mm	0.5160 in.	[13.106 mm]
ID	[0.2756 in.]	7.0 mm	0.2870 in.	[7.290 mm]
Copper area	[0.1979 in.^{2}]	129.657 mm^2	0.1940 in.^{2}	[125.161 mm^2]
Cooling area	[0.05965 in.^{2}]	38.485 mm^2	0.06469 in.^{2}	[41.735 mm^2]
Mass	[0.7797 lb/ft]	1.16 kg/m	0.7495 lb/ft	[1.115 kg/m]
Resistance at 20°C	[40.53 $\mu\Omega$/ft]	132.974 $\mu\Omega$/m	41.99 $\mu\Omega$/ft	[137.762 $\mu\Omega$/m]
k (British units)	0.0160		0.01520	

(Anaconda 0.460 in.-square conductor is also possible if a current density of ≈4,000 A/in.^{2} is allowable.)

In what follows, *only Anaconda 0.516 in.-square conductor is considered.* Use of another conductor size would reduce the overall size of the coil slightly (by approximately 0.5 in. if the 0.460 In. square conductor were used, for example) or, alternately, allow for larger coil-pole and coil-yoke clearances.

We assume that each conductor is double-wrapped with insulation that is $t_i = 0.007$ in. thick with a tolerance of 0.0015 in. Then the *total* insulation per conductor has:

Minimum thickness	4(0.007 − 0.0015) in.	=	0.022 in.
Nominal thickness	4(0.007) in.	=	0.028 in.
Maximum thickness	4(0.007 + 0.0015) in.	=	0.034 in.

The tolerance of the outer dimension of the conductor is listed as 0.004 in. so that the dimensions of a *wrapped* conductor are:

Minimum	0.516 in. + 0.022 in. − 0.004 in.	=	0.534 in.
Nominal	0.516 in. + 0.028 in.	=	0.544 in.
Maximum	0.516 in. + 0.034 in. + 0.004 in.	=	0.554 in.

We further allow

a) a gap between layers of 0.010 in. maximum
b) for keystoning, assume 0.010 in.
c) a 4-turn ground wrap of 0.007 in. tape.

Then the *width* of the coil is obtained from

	Maximum	Minimum
Wrapped conductor	4.432 in.	4.272 in.
Gapping (7x0.10)	0.070 in.	
Ground wrap (4x0.178x2)	0.056 in.	0.056 in.
Total	4.558 in.	4.328 in.

The average coil width is 4.443 in. for the 0.516 in. square conductor. We take

Maximum coil width	4.550 in.
Nominal coil width	4.450 in.

The *height* of the coil is obtained from

	Maximum	Minimum
Wrapped conductor	5.540 in.	5.340 in.
Gapping (10x0.10)	0.100 in.	
Keystoning (10x0.010)	0.100 in.	0.050 in.
Ground wrap (4x0.178x2)	0.056 in.	0.056 in.
Total	5.796 in.	5.446 in.

The average coil height is 5.621 in. for the 0.516 in. square conductor. We take

Maximum coil height	5.800 in.
Nominal coil height	5.625 in.

We take the conductor dimension D to be

$$D = \text{Nominal dimension} + 4(\text{Insulation thickness}) + \text{Turn separation}$$

so that for the 0.516 in. square conductor

$$D = 0.516 \text{ in.} + 0.028 \text{ in.} + 0.010 \text{ in.} = 0.554 \text{ in.}$$

If G is the pole-coil separation, the perpendicular distance from the pole to the *outer* edge of the nth conductor is

$$D_n = G + nD + 4t_i$$

with the $4t_i$ term accounting for the (inside) ground wrap. We assume that it is wound as illustrated in the diagram below.

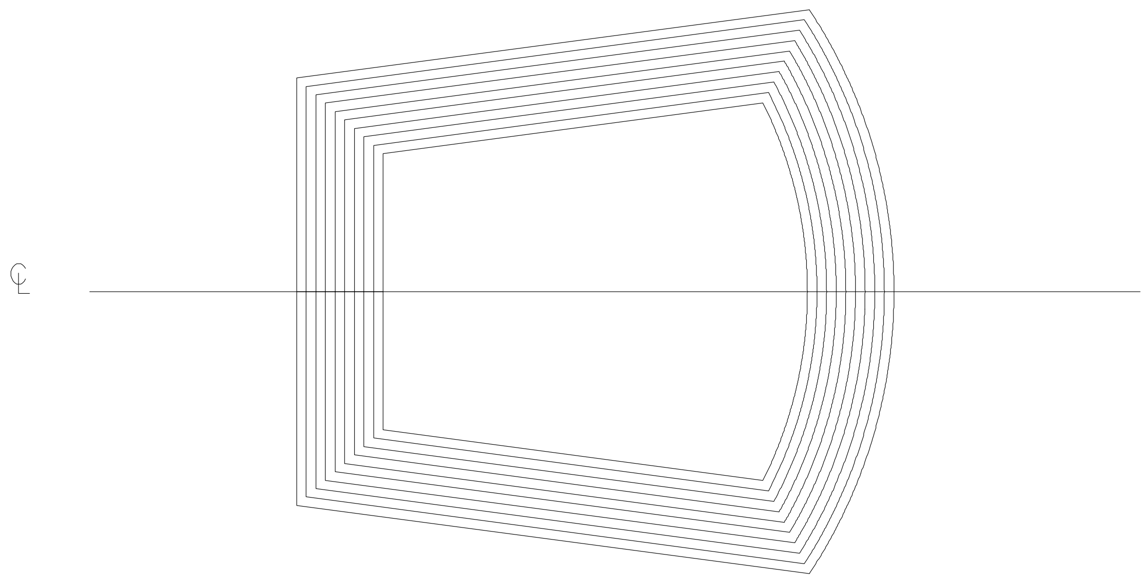

Fig. 2. Illustration of the winding of the coil.

No allowance has been made for finite bends in the conductors in figure 2. The half-length of the coil on the entrance edge of the dipole is the value of y at the intersection of the lines given by the equations

$$\begin{aligned} x &= -D_n \\ y &= x \tan\theta + \Delta + \frac{D_n}{\cos\theta} \end{aligned}$$

with $\theta(= 8^\circ$ in our case) the flare angle of the pole. The point of intersection of these two lines is

$$\begin{aligned} x_1 &= -D_n \\ y_1 &= x_1 \tan\theta + \Delta + \frac{D_n}{\cos\theta} \end{aligned}$$

and thus the length of the nth turn along the entrance side of the magnet is

$$\begin{aligned} l_{in} &= 2\,y_1 \\ &= 2\left\{\Delta + D_n\left[\frac{1-\sin\theta}{\cos\theta}\right]\right\} \end{aligned}$$

The length of a straight longitudinal section of the winding is obtained in a similar manner; it is determined by the above point of intersection and that of the curves given by the following two equations.

$$\begin{aligned}(x+a)^2 + y^2 &= R_n^2 \\ y &= x\tan\theta + K_n\end{aligned}$$

where we have written

$$\begin{aligned}R_n &= R_{iron} + D_n \\ K_n &= \Delta + \frac{D_n}{\cos\theta}\end{aligned}$$

These curves intersect at positive values of x and y of

$$\begin{aligned}x_2 &= \frac{1}{1+\tan^2\theta}\left[\sqrt{[1+\tan^2\theta]R_n^2 - [a\tan\theta - K_n]^2} - (a + K_n\tan\theta)\right] \\ y_2 &= x_2\tan\theta + K_n\end{aligned}$$

and the length of the nth section along the angled pole-edge is

$$l_{str} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\ .$$

The length of the curved section of the nth turn is

$$l_{curv} = 2\,R_n\,\Theta_n$$

where

$$\Theta_n = \tan^{-1}\left[\frac{y_2}{x_2+a}\right]\ .$$

Thus the length of the nth turn is

$$l_n = l_{in} + 2\,l_{str} + l_{curv}$$

and the length of an N-turn layer is simply

$$L_N = \sum_{n=1}^{N} l_n\ .$$

These relationships are easily programmed. Using the following values for the parameters in the above relationships

Parameter	Value	Parameter	Value
t_i	0.007 in.	a	2.000 in.
g	4.000 in.	c	1.000 in.
δ	1.000 in.	Δ	6.000 in.
R_{iron}	33.000 in.	D	0.554 in.

we find for the 0.516 in. square conductor

Turn	x_1 (in.)	y_1 (in.)	x_2 (in.)	y_2 (in.)	θ_n (°)	l_{in} (in.)	l_{str} (in.)	l_{circ} (in.)	l_n (in.)	L_N (in.)
1	−1.082	6.941	30.144	11.329	19.415	13.881	63.066	23.098	100.044	100.044
2	−1.636	7.422	30.513	11.940	20.166	14.844	64.929	24.381	104.155	204.199
3	−2.190	7.904	30.876	12.551	20.895	15.807	66.781	25.667	108.256	312.455
4	−2.744	8.385	31.233	13.160	21.604	16.771	68.622	26.955	112.348	424.803
5	−3.298	8.867	31.585	13.769	22.293	17.734	70.452	28.246	116.431	541.234
6	−3.852	9.348	31.932	14.378	22.963	18.697	72.271	29.540	120.507	661.741
7	−4.406	9.830	32.273	14.985	23.616	19.660	74.080	30.836	124.575	786.317
8	−4.960	10.312	32.610	15.592	24.251	20.623	75.879	32.135	128.636	914.953

We take

$$\boxed{\text{Length of 8-turn layer of 0.516 in. square conductor} = 960 \text{ in.} = 80 \text{ ft} \approx 24.4 \text{ m.}}$$

where extra has been allowed for bends that were not taken into account. The length per coil becomes

$$\boxed{\text{Length of conductor per coil} = 800 \text{ ft} \approx 245 \text{ m.}}$$

Because two coils are required per dipole, then

Total length per dipole	1,600 ft	≈	490 m
Allow 10% for winding losses	160 ft	≈	49 m
Total	1,760 ft	≈	539 m

The required mass of the 0.516 conductor at 0.7495 lb/ft is 1,349 lb [612 kg]. Then order

$$\boxed{\begin{array}{lcl} \text{Total length of 0.516 copper} & = & 1{,}800 \text{ ft} \approx 550 \text{ m.} \\ \text{Total mass of 0.516 conductor} & = & 1{,}400 \text{ lb} \approx 635 \text{ kg.} \end{array}}$$

6. Power requirements

At 20°C the resistance of a coil of the 0.516 in. conductor is

$$R_{20^\circ} = 41.99\times10^{-6}\ \Omega/\text{ft}\times 800 \text{ ft} = 0.03359\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40° C coolant temperature rise). Then the mean coil temperature will be 50°C.

With a 30°C rise above ambient of the coil we then have:

$$R_{hot} = R_{20^\circ}[1 + (\text{Temp. coeff}/^\circ\text{C})dT(^\circ\text{C})]$$

so that for the coil made of the 0.516 in. square conductor

$$R_{hot} = 0.03359[1 + (0.00393)(30)] = 0.03755\ \Omega \text{ per coil}$$

Thus, at a current of 600 A, we obtain

$$\text{Voltage per coil} = 22.53 \text{ V.}$$

Therefore, allowing for a 10% lead loss, we choose a power supply that has

I (A minimum)	600
V (V minimum)	50
P (kW minimum)	30

These requirements are well within the capability of the existing power supply.

7. Cooling requirements

In these calculations we use the British system of units. The power required per coil is

$$\text{Power per coil} = I^2 R_{hot} = (600)(600)(0.03755) = 13.52 \text{ kW.}$$

The required flow rate is given by:

$$v \text{ (ft/sec)} = \frac{2.19}{\Delta T(^\circ \text{ F})}\times\frac{P(\text{kW})}{\text{Cooling area (in.}^2)} = 0.0304167\times\frac{P(\text{kW})}{A_c \text{ (in.}^2)}$$

for $\Delta T = 72°\mathrm{F} = 40°\mathrm{C}$. Given that the cooling area of the 0.516 in. square conductor is $A_c = 0.06469$ in.^{2} [41.735 mm^2] and choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit we have

$$P_{max} = \frac{(2.50)(72)(0.06469)}{2.19} = 5.317 \text{ kW/water circuit},$$

we calculate the number of cooling circuits per coil (excluding lead loss) as

P	=	Total power per coil	=	13.52 kW
Number of circuits	=	P / P_{max}	=	2.54

Because there a total of 10 layers has been assumed and to keep a balanced water flow, this would require that each coil have either two cooling circuits of 5 layers each or five circuits of 2 layers each. The former requires a flow rate of $v = 3.18$ ft/sec and the latter a flow rate of $v = 1.27$ ft/sec. For simplicity we take

$$\boxed{\text{Number of cooling circuits per coil} \quad = \quad 5.}$$

The volume of flow required per circuit is

$$\begin{aligned} \text{Volume/circuit} &= v\,\frac{\text{ft}}{\text{sec}} \times A_{H_2O}\,(\text{in.}^2) \times 60\,\frac{\text{sec}}{\text{min}} \times \frac{1}{144}\,\frac{\text{ft}^2}{\text{in}^2} \times 62.4\,\frac{\text{lb}}{\text{ft}^3} \times \frac{1}{10}\,\frac{\text{IG}}{\text{lb}} \times 1.20095\,\frac{\text{USG}}{\text{IG}} \\ &= 3.1225\,v\,(\text{ft/sec}) \times \text{Cooling area}\,(\text{in.}^2)\ \text{USGPM} \end{aligned}$$

Thus we have the following volumes of flow.

Volume per cooling circuit	0.257 USGPM	(0.972 ℓ/min)
Volume per coil	1.284 USGPM	(4.860 ℓ/min)
Volume per magnet	2.568 USGPM	(9.720 ℓ/min)

8. Pressure drop

The pressure drop is given by

$$\Delta P = k\,v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case, for the 0.516 in. square conductor with $k = 0.0152$ and $v = 1.271$ ft/sec we obtain

$$\Delta P = (0.0152)(1.271)^{1.79} = 0.02335 \text{ psi/ft} = 0.0766 \text{ psi/m}.$$

and the total pressure drop across one cooling circuit is:

$$\boxed{\text{Pressure drop per cooling circuit} \quad = \quad 0.02335 \text{ psi/ft} \times (800/5) \text{ ft} = 3.74 \text{ psi}.}$$

9. Iron dimensions

We propose to construct the top and bottom yokes of pieces of iron that completely cover the pole as illustrated in figure 3 on the following page. Also indicated is the assumed field distribution at the mid-point of the pole.

The top and bottom yokes are trapezoids whose parallel sides are of lengths $l_1 + 2t$ and $l_2 + 2t$. The spacers are parallelograms with one side of length t. The length t is to be determined from the consideration that we want the flux to distribute itself equally between the two spacers. We note that if the spacer thickness is t_s, the dimension $t = t_s/\cos(\theta)$ with $\theta = 8°$ in our case.

We begin by calculating the flux through the pole face. Using the notation from page 2, the pole area is

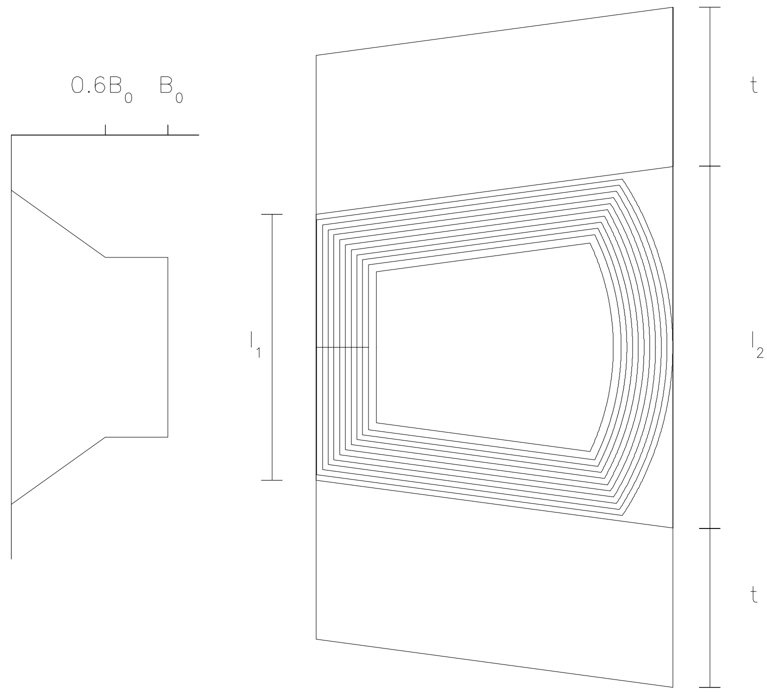

Fig. 3. Cross section of the magnet and the assumed field profile.

given by

$$A_{pole} = \frac{1}{2}(\overline{\mathrm{AD}}+\overline{\mathrm{BC}})x_{int} + \frac{1}{2}R_{iron}^2(\Theta - \sin\Theta)$$

where

$$\Theta = 2\tan^{-1}\left[\frac{|y_{int}|}{x_{int}+2}\right]$$

with x_{int} and y_{int} given by the values of x and y in equation (9) of the appendix. Thus $x_{int} = 31.415$ in., $|y_{int}| = 10.415$ in. and we find $\overline{\mathrm{BC}} = 2(|y_{int}|) = 20.830$ in. Also, $\overline{\mathrm{AD}} = 12.00$ in. so we have

$$\Theta = 2\tan^{-1}\left[\frac{10.415}{33.415}\right] = 34.623^\circ = 0.60429 \text{ radian}$$

and

$$\begin{aligned} A_{pole}\ (\text{in.}^2) &= [(12.000+20.830)(31.415)+(35.000)^2(0.60429-0.56818)]/2 \\ &= [1031.35+44.2366]/2 \\ &= 537.796\ \text{in.}^2 \end{aligned}$$

Then the magnetic flux through the pole is

$$\Phi_{pole} = (BA)_{pole} = (10.500\text{ kG})(537.796\text{ in.}^2) = 5,647\text{ kG-in.}^2$$

$$\boxed{\Phi_{pole} = 5{,}700\text{ kG-in.}^2 = 0.368\text{ T-m}^2}$$

To this we add the flux through the coil slot. As is illustrated in the previous figure, the magnetic field profile has been assumed to rise linearly from zero at the inside edge of the yoke to a value of $0.6B_0$ at the flat part of the pole. At that point the field rises to the full value in the gap the field is assumed to drop abruptly to a value of $0.6B_g$ and to fall linearly to zero at the outer edge of the outer coil. Thus the average field in the coil slot is $0.3B_0$.

The average length of the coil-slot along the slanted pole-edge is taken as the outer length of the fourth turn—that is, the length of the line defined by

$$y = x\tan(8°) + \Delta + (G + 4t_i + 4D)/\cos(8°)$$

and the lines

$$x_1 = -(G + 4t_i + 4D) \qquad \text{and} \qquad x_2 = (R_{iron} - a) + G + 4t_i + 4D$$

with the $4t_i$ term accounting for a 4-layer ground wrap. The intersections of these lines are found to be

$$\begin{aligned} x_1 &= -2.744 \text{ in.} \qquad \text{and} \qquad & y_1 &= 8.385 \text{ in.} \\ x_2 &= 35.744 \text{ in.} \qquad \text{and} \qquad & y_2 &= 13.794 \text{ in.} \end{aligned}$$

for the 0.516 in. conductor. Then the (average) lengths of the side coil-slots, l_{side}, are

$$\begin{aligned} l_{side} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(35.744 + 2.744)^2 + (13.794 - 8.385)^2} = 38.866 \text{ in.} \end{aligned}$$

The width of the side coil-slot is calculated from

$$\begin{aligned} \text{Side coil-slot width} &= \text{Maximum coil width} + 2(\text{Pole-coil separation}) \\ &= 4.550 \text{ in.} + 2(0.500) \text{ in.} = 5.550 \text{ in.} \end{aligned}$$

and the total flux through the side coil-slots becomes

$$\begin{aligned} \Phi_{sides} &= 2(\text{Length of slot})(\text{Width of slot})(0.3\,B_0) \\ &= 2(38.866 \text{ in.})(5.550 \text{ in.})(0.3(10.500 \text{ kG})) = 1359.0 \text{ kG-in.}^2 \end{aligned}$$

$$\boxed{\Phi_{sides} = 1360 \text{ kG-in.}^2 = 0.08774 \text{ T-m}^2.}$$

To estimate the flux through the entrance and exit ends of the dipole we note that the effective length of the dipole is defined as the length at which the field drops from B_0 to zero. The entrance effective edge has been taken to be at $x = -2$ in. and the exit effective edge at $x = 35$ in. Because these numbers are close to the values x_1 and x_2 calculated above, we take the lengths of the entrance and exit coil slots to be $2y_1$ and $2y_2$. The width of these slots we take to be the maximum coil width plus G plus 2(ground wraps); the field therein we take as $B_0/2$. Consequently, the widths of the coil slots on entrance and exit faces are taken as

$$\begin{aligned} \text{Entry and exit coil-slot widths} &= 8D + G + 8t_i \\ &= 8(0.554) \text{ in.} + 0.500 \text{ in.} + 8(0.007 \text{ in.}) = 4.988 \text{ in.} \end{aligned}$$

Then the total flux through the entry coil-slot becomes

$$\begin{aligned} \Phi_{entry} &= (\text{Length of slot})(\text{Width of slot})(0.5\,B_0) \\ &= 2(8.385 \text{ in.})(4.988 \text{ in.})(0.5(10.500 \text{ kG})) = 439.2 \text{ kG-in.}^2 \end{aligned}$$

and that through the exit coil-slot becomes

$$\begin{aligned} \Phi_{exit} &= (\text{Length of slot})(\text{Width of slot})(0.5\,B_0) \\ &= 2(13.794 \text{ in.})(4.988 \text{ in.})(0.5(10.500 \text{ kG})) = 722.5 \text{ kG-in.}^2\ . \end{aligned}$$

$$\boxed{\begin{aligned} \Phi_{entry} &= 440 \text{ kG-in.}^2 = 0.02838 \text{ T-m}^2 \;. \\ \Phi_{exit} &= 725 \text{ kG-in.}^2 = 0.04677 \text{ T-m}^2 \;. \end{aligned}}$$

The total flux through the coil slots is obtained from the sum of the above fluxes. Thus we have

$$\begin{aligned} \Phi_{coilslots} &= \Phi_{sides} + \Phi_{entry} + \Phi_{exit} \\ &= 1360 \text{ kG-in.}^2 + 440 \text{ kG-in.}^2 + 725 \text{ kG-in.}^2 \\ &= 2525 \text{ kG-in.}^2 \end{aligned}$$

$$\boxed{\Phi_{coilslots} = 2525 \text{ kG-in.}^2.}$$

Thus the total flux through the pole and coil slots, Φ_{total}, is estimated to be

$$\begin{aligned} \Phi_{total} &= \text{Flux through pole + flux through coil slots} \\ &= 5{,}700 \text{ kG-in.}^2 + 2{,}525 \text{ kG-in.}^2 \\ &= \; = 8{,}225 \text{ kG-in.}^2 = 0.5306 \text{ T-m}^2. \end{aligned}$$

$$\boxed{\Phi_{total} = 8{,}225 \text{ kG-in.}^2 = 0.5306 \text{ T-m}^2.}$$

The length of the dipole along the longitudinal symmetry axis has been calculated as

$$\begin{aligned} l_{axis} &= R_{iron} + 2(8D + G + 8t_i) \\ &= 33.000 \text{ in.} + 2[8(0.554 \text{ in.}) + 0.500 \text{ in.} + 8(.007 \text{ in.})] = 42.976 \text{ in.} \end{aligned}$$

The length along the outer edge of the dipole is

$$l_{outer} = l_{axis}/\cos(8^\circ) = 42.976/0.99027 \text{ in.} = 43.398 \text{ in.}$$

We average these and set the length of the cross section of the dipole to be

$$l_{av} = 43.187 \text{ in.}$$

With the requirement that the flux divide evenly between the side yokes and with t_y the thickness of the top yoke and B_y the field in it, we have

$$\text{(Cross-sectional area of yoke)(Field in yoke)} = l_{av}\, t_y\, B_y = 8,225/2 \text{ kG-in.}^2$$

or

$$t_y\, B_y = 95.22 \text{ kG-in.}$$

The thickness of the spacers is determined in a similar manner. With t_s its thickness, B_s the field in it and l_{outer} the length of each spacer we have

$$\text{(Area of spacer)(Field in spacer)} = 43.398\, t_s\, B_s = 8,225/2 \text{ kG-in.}^2$$

or

$$t_s\, B_s = 94.76 \text{ kG-in.}$$

Because the field-thickness product is essentially the same for the top and bottom yokes and the spacers, we choose to make them of equal thicknesses. Taking the product as 100 kG-in. for either case, we make the following table.

B_y (kG)	10.	11.	12.	13.
t_y (in.)	10.00	9.091	8.333	7.692

We choose

Yoke field	B_y	12.5 kG	1.250 T
Yoke thickness	t_y	8.00 in.	0.203 m

Thus the length t of figure 3 is $t = t_y/\cos(8°) = 8.079$ in. The lengths l_1 and l_2 of figure 3 are found by considering the intersection of the line

$$y = x\tan(8°) + \Delta + [2(G+4t_i) + 8D]/\cos(8°)$$

with the lines

$$x_{in} = -(G + 8t_i + 8D) \qquad \text{and} \qquad x_{exit} = R_{iron} + G + 8t_i + 8D$$

Those points are found to be

$$\begin{aligned} x_{in} &= -4.988 \text{ in.} \qquad \text{and} \qquad y_{in} = 10.960 \text{ in.} \\ x_{exit} &= 37.988 \text{ in.} \qquad \text{and} \qquad y_{exit} = 17.000 \text{ in.} \end{aligned}$$

The lengths l_1 and l_2 of the parallel sides of the upper and lower yokes are then

$$\begin{aligned} l_1 &= 2y_{in} = 2(10.960) \text{ in.} = 21.920 \text{ in.} \\ l_2 &= 2y_{exit} = 2(17.000) \text{ in.} = 34.000 \text{ in.} \end{aligned}$$

Then the maximum width of the dipole is

$$\begin{aligned} \text{Maximum width} &= l_2 + 2t \\ &= 34.000 \text{ in.} + 2(8.079 \text{ in.}) = 50.158 \text{ in.} \end{aligned}$$

and its minimum width is

$$\begin{aligned} \text{Minimum width} &= l_1 + 2t \\ &= 21.920 \text{ in.} + 2(8.079 \text{ in.}) = 38.078 \text{ in.} \end{aligned}$$

The overall length of the dipole has been calculated as

$$\text{Dipole length} = R_{iron} + 2(8D + G + 8t_i) = 42.976 \text{ in.}$$

We take

Maximum dipole width	50.200 in.
Minimum dipole width	38.100 in.
Overall dipole length	43.000 in.

The height of the pole is calculated from

$$\begin{aligned} \text{Pole height} &= \text{Maximum coil height} + \text{Chamfer} + 0.5 \text{ in.} \\ &= 5.800 \text{ in.} + 1.000 \text{ in.} + 0.500 \text{ in.} = 7.300 \text{ in.} \end{aligned}$$

and that of the side yokes from

$$\begin{aligned} \text{Side-yoke height} &= 2(\text{Pole height}) + \text{Gap} \\ &= 2(7.300 \text{ in.}) + 4.000 \text{ in.} = 18.6 \text{ in.} \end{aligned}$$

The table on the following page summarizes the calculations that have been made.

	0.516 in. conductor	
Coil-slot width	5.550 in.	141.7 mm
Pole height	7.300 in.	185.4 mm
Side-yoke height	18.600 in.	474.4 mm
Maximum dipole width	50.200 in.	1275.1 mm
Minimum dipole width	38.100 in.	967.7 mm
Dipole length	43.000 in.	1092.2 mm

10. Iron weight

We are now in a position to estimate the amount of iron required. We have

$$\begin{aligned} \text{Area of top or bottom yoke} &= (\text{Average length of parallel sides})(\text{Longitudinal length}) \\ &= (50.200 + 38.100)(43.000)/2 = 1,898.5 \text{ in.}^2 \end{aligned}$$

with the area of each spacer obtained in a similar manner

$$\begin{aligned} \text{Area of spacer} &= (\text{Short side})(\text{Long side})\sin(\text{Contained angle}) \\ &= (8.079 \text{ in.})(43.398 \text{ in.})\sin(82) = 347.2 \text{ in.}^2 \ . \end{aligned}$$

We take

$$\boxed{\begin{array}{lclcl} \text{Area of top or bottom yoke} &=& 1{,}900 \text{ in.}^2 &=& 1.2258 \text{ m}^2 \\ \text{Area of spacer} &=& 350 \text{ in.}^2 &=& 0.2258 \text{ m}^2 \end{array}}$$

Thus we have

Section	Area (in.^{2})	Height (in.)	Volume (in.^{3})
Top yoke	1,900.0	8.0	15,200.0
Bottom yoke	1,900.0	8.0	15,200.0
Vertical Yoke	350.0	18.6	6,510.0
Vertical Yoke	350.0	18.6	6,510.0
Top pole	540.0	7.3	3,942.0
Bottom pole	540.0	7.3	3,942.0
Total			51,304.0

We take the total volume of iron required to be

$$\text{Total volume of iron} = 51,500 \text{ in.}^3 = 29.803 \text{ ft}^3 = 0.8439 \text{ m}^3$$

and using a density of 0.2833 lb/in.^{3} the iron mass is obtained from

$$\begin{aligned} \text{Iron mass} &= (\text{Iron volume})(\text{Density}) \\ &= (0.2833 \text{ lb/in.}^3)(51,500 \text{ in.}^3) \\ &= 14,590 \text{ lb.} \end{aligned}$$

We take

$$\boxed{\text{Dipole weight} = 14{,}600 \text{ lb} = 6{,}625 \text{ kg.}}$$

11. POISSON studies

The parameters derived in the above were used as initial input to the program POISSON[3]. In further analysis, the simplifying assumption was made that a transverse cut through the magnet at the (longitudinal) midpoint of the pole would produce a reasonable expectation of the field distribution to be expected in the iron.

An immediate result was that at an excitation of 47,000 A-t, POISSON predicted a field along the longitudinal centerline of approximately 11.4 kG. To obtain a central field of 10.5 kG it was necessary to reduce the excitation to 43,000 A-t—a ratio of excitations of 1.09. However, because POISSON is a two-dimensional code, the program treats a magnet as if it were of infinite length—thus ignoring the effect of fringe fields. The flux from the fringe fields also must be carried by the magnet yoke and, consequently, the field strength in the yoke is expected to be higher than that calculated by POISSON. As a rough estimate, we would expect the field in the yoke to be a factor of L_{eff}/L_{iron} than that calculated by the program. Because $L_{eff}/L_{iron} = 37/33 = 1.12$ is not too different from the ratio of the excitations noted above, further calculations were made at an excitation of 47,000 A-t, the assumption being that the calculated yoke fields will be indicative of those existing in the yoke during nominal running operation.

The first POISSON run was made for a magnet with poles having a 1 in. chamfer and yokes 8 in. thick. This resulted in a prediction that the central field drooped as one moved away from the longitudinal centerline of the magnet. The run was repeated with the chamfer replaced with an edge shim identical to that used in the other beam line 2A dipoles[2]. Runs similar to the latter were made with a pole profile that included an edge shim but with yoke thicknesses of 8.5 in. and 9.0 in. Figure 4 is a plot of the predicted variation of the transverse field as a function of distance from the (longitudinal) centerline of the magnet.

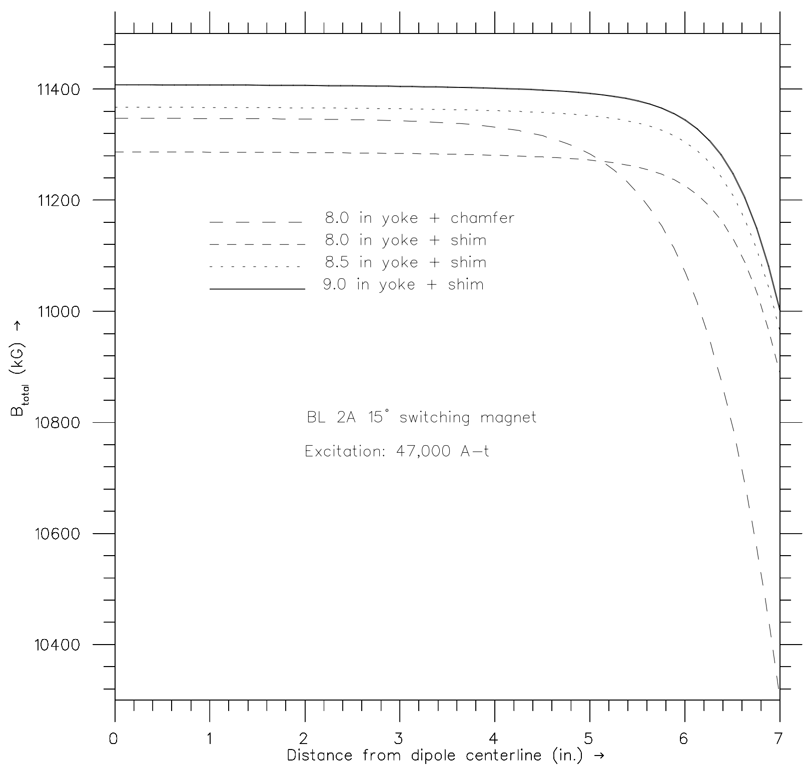

Fig. 4. Variation of midplane field as a function of distance from the dipole centerline.

We define the 'linear region' as the distance from the dipole centerline at which the predicted field is 99.9%

of that predicted on the centerline. Although not clearly evident in figure 4, the limits of this linear region are $x = \pm 3.75$ in. for the case of a chamfered pole and $x = \pm 4.75$ in. for each of the other cases. Given that a constant pole width (of $x = \pm 8.32$ in.) was used for all cases shown in figure 4, it is clear that a better field uniformity is obtained from the use of an edge shim rather than a simple pole chamfer.

To put the above more in perspective, we note that at the center of the magnet the central trajectory lies approximately 1.25 in. either side of the magnet centerline. Thus the outer edge of the beam extends a distance 2.25 in. from the centerline. The data shown in figure 4 thus indicate that a simple pole chamfer should be adequate. However, at the dipole exit the beam centroid lies approximately 4.6 in. either side of the centerline. There the pole width has been calculated to be $x = \pm 10.4$ in. Assuming that the linear region is proportional to the pole width, a simple chamfer would produce a linear region of $x = \pm 10.4(3.75/8.32) = \pm 4.7$ in., a distance insufficient to cover the entire beam width. With a pole shim, however, the linear region would extend to $x = \pm 5.9$ in., just adequate to encompass the full beam. Consequently, we restrict further study to a dipole with pole shims.

The size of the linear region is not the only criterion in the design of the dipole, however. We must also consider the fields in the iron. Figure 5 below shows the computed fields in the yoke of the magnet with a pole 16.64 in. wide at the midpoint, pole shims as above and an 8 in. thick yoke.

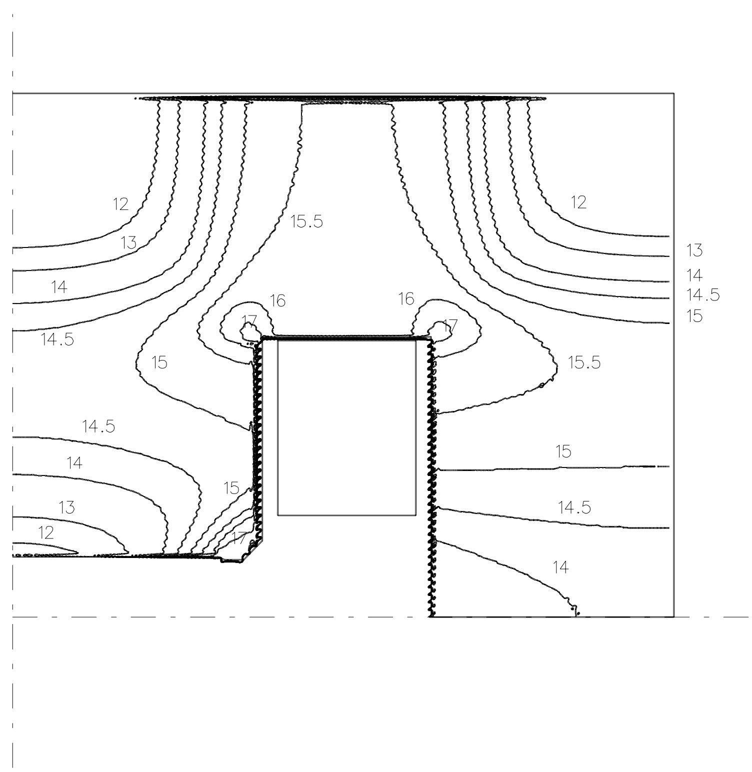

Fig. 5. Contours in kG of the field in the yoke of the 15° dipole; yoke thickness is 8.0 in.

It is seen from this figure that the predicted field in the yoke above the coil is quite high—of the order of 15.5 kG. As well, the field predicted in its pole averages approximately 14.5 kG. Each of these values is higher than the design value. Further, if the assumption that these calculations at 47,000 A-t account for the extra flux that the yoke must carry because of fringe fields is incorrect, these fields would increase by at least 10% to unacceptably high values. Consequently, the effect of increasing the yoke thickness was studied.

Figures 6 and 7 show the predicted fields in the yokes of dipoles with yoke thicknesses of 8.5 in. and 9.0 in.

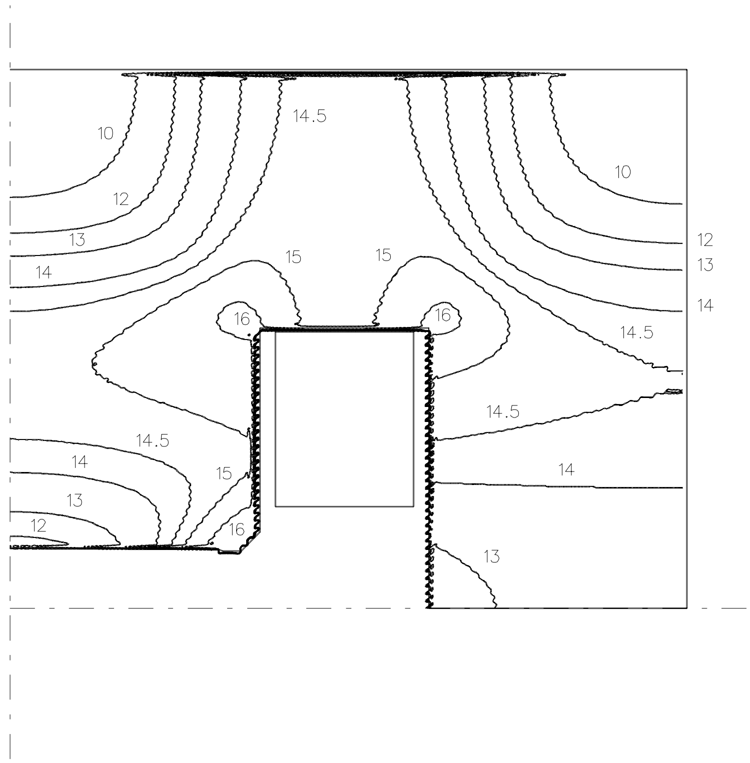

Fig. 6. Contours in kG of the field in the yoke of the 15° dipole; yoke thickness is 8.5 in.

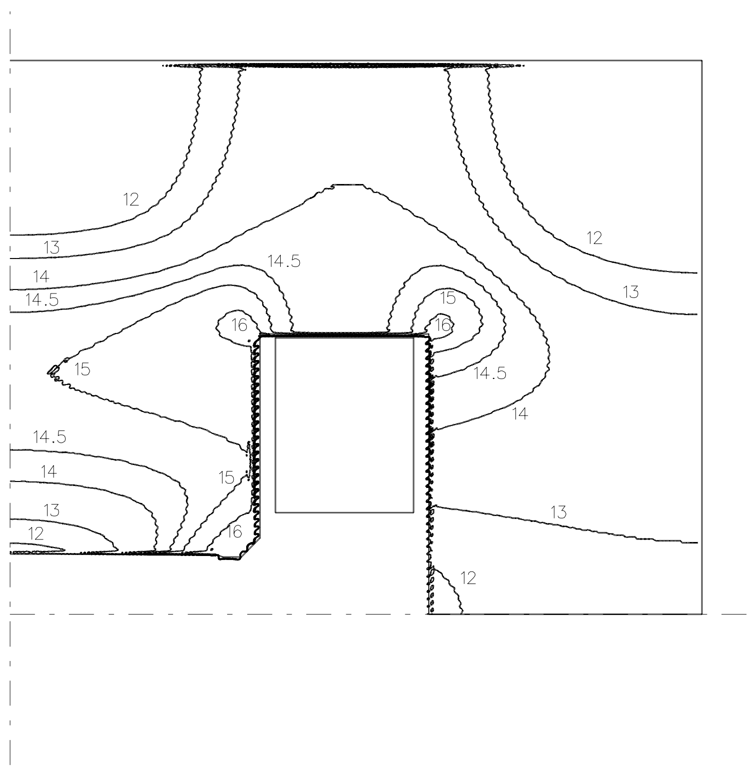

Fig. 7. Contours in kG of the field in the yoke of the 15° dipole; yoke thickness is 9.0 in.

From these figures it is seen that, as expected, as the yoke thickness is increased the predicted field in the yoke above the coil slot decreases to acceptable values. However, the predicted field in the pole is seen to increase. Again, this is to be expected because the pole width has been held constant and, although the yoke thickness has been increased so as to carry the additional flux, the pole width has not. Thus a study of the effect of increasing the pole width was necessary.

Figure 8 shows the predicted fields in the yoke of dipoles with a yoke thickness of 9.0 in. and with a pole $2{\times}0.6$ in. wider than that considered in figure 7.

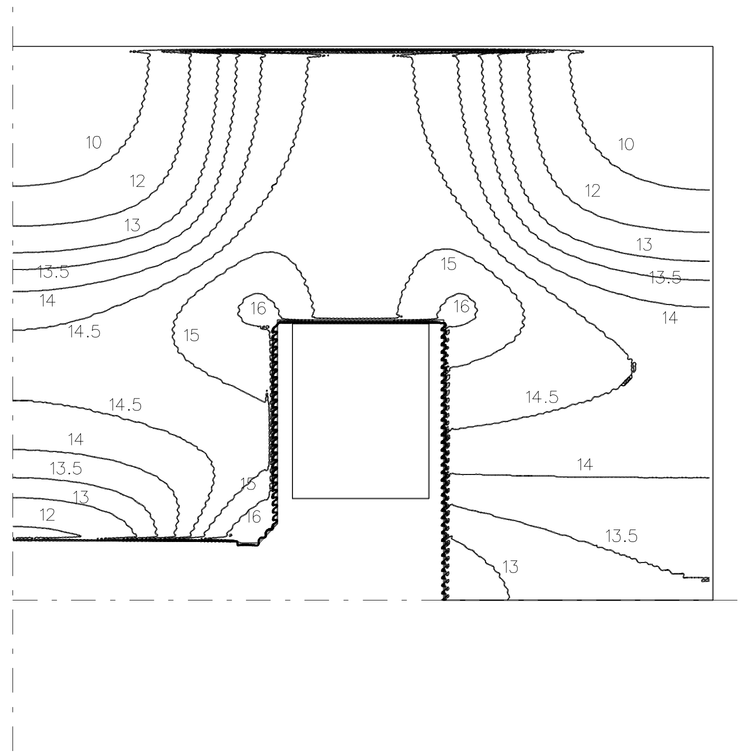

Fig. 8. Contours in kG of the field in the yoke of the 15° dipole; yoke thickness is 9.0 in., pole width increased by 1.2 in. (at the pole center).

It is seen that increasing the pole width does reduce the field in the pole somewhat. However, the field in the yoke above the coil is high. The results of a second run with a tapered pole, its base width the same as in figure 8, is shown in figure 9. From that figure it is seen that the predicted field in the pole remains reasonably low while, at the same reducing the field in the yoke in the region of the coil slot.

Consequently, it is recommended that the yoke thickness of the 15° switching magnet be increased to 9.0 in. and the pole width be increased by 1.2 in. This, clearly, will reflect in an increased weight of the dipole. Further, because the pole has been made wider, a larger amount of copper will be required for the coil.

11.1 Recalculation of the coil parameters

For reasons that shortly will become apparent, in this subsection we extend our considerations to the three viable coil configurations—7 wide by 12 high, 8 wide by 10 high and 10 wide by 8 high—that meet the required NI per coil at a current of 600 A. The table on the next page is a repeat of the coil calculations of §5 for the wider pole. From that table we see that the length of a 7-turn layer is estimated to be 800 in., that of an 8-turn to be 931 in. and that of a 10-turn later to be 1204 in. Adding approximately 48 in. to

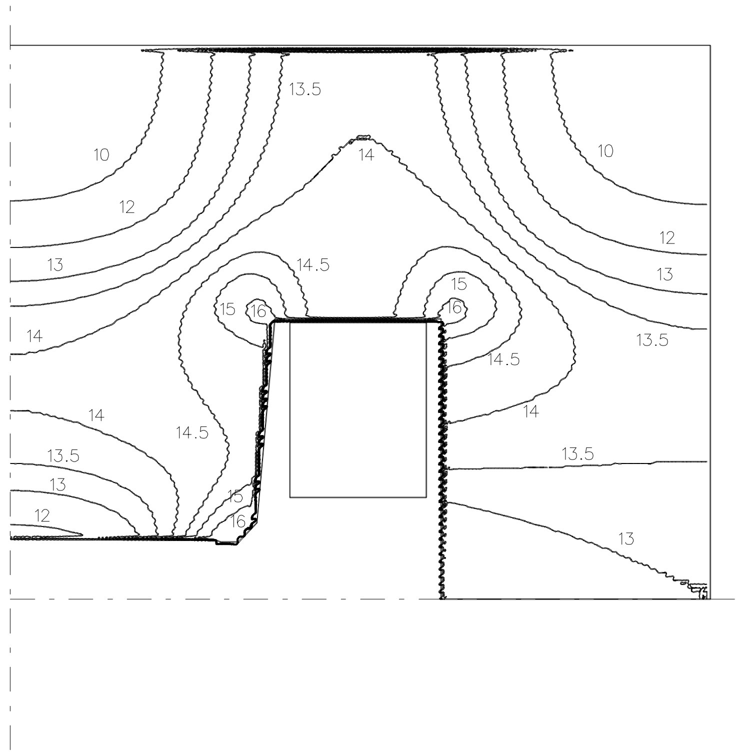

Fig. 9. Contours in kG of the field in the yoke of the 15° dipole; yoke thickness is 9.0 in., pole side width increased 0.6 in. at base by taper.

Turn	x_1 (in.)	y_1 (in.)	x_2 (in.)	y_2 (in.)	θ_n (°)	l_{in} (in.)	l_{str} (in.)	l_{circ} (in.)	l_n (in.)	L_N (in.)
1	−1.082	7.541	29.937	11.900	20.436	15.081	62.648	24.312	102.041	102.041
2	−1.636	8.022	30.298	12.510	21.173	16.044	64.495	25.599	106.139	208.180
3	−2.190	8.504	30.653	13.120	21.890	17.007	66.331	26.888	110.227	318.407
4	−2.744	8.985	31.003	13.728	22.586	17.971	68.157	28.180	114.308	432.714
5	−3.298	9.467	31.347	14.336	23.263	18.934	69.971	29.475	118.380	551.094
6	−3.852	9.948	31.686	14.943	23.922	19.897	71.775	30.772	122.445	673.539
7	−4.406	10.430	32.021	15.550	24.563	20.860	73.570	32.073	126.502	800.042
8	−4.960	10.912	32.351	16.155	25.188	21.823	75.355	33.375	130.553	930.595
9	−5.514	11.393	32.676	16.760	25.797	22.786	77.130	34.681	134.598	1,065.193
10	−6.068	11.875	32.997	17.365	26.390	23.750	78.897	35.989	138.636	1,203.828

these estimates (to allow for bends not taken into account), we find a 7×12 layer requires 840 in. = 70 ft of copper, a 8×12 layer requires 972 in. = 81 ft of copper and a 10×8 layer requires 1,248 in. = 104 ft of copper. Thus we have

	Feet of copper required per			Add	Total
	layer	coil	dipole	10%	length
7×12	70	840	1,680	168	1,850
8×10	81	810	1,620	162	1,800
10×8	104	832	1,664	166	1,830

Continuing with calculations for the various coil configurations, we find the following for a current of 600 A.

	R_{20} (mΩ)	R_{hot} (mΩ)	Voltage (V per coil)	Total voltage (V)	Power (kW)
7×12	35.272	39.430	23.658	52.05	31.23
8×10	34.012	38.022	22.813	50.19	30.11
10×8	34.936	39.055	23.433	51.55	30.93

In the above table the total voltage (per magnet) includes a 10% allowance for lead loss. From the above it is seen that from a viewpoint of power consumption there is little to choose between the various arrays.

For cooling, we use the value of $P_{max} = 5.317$ kW/water circuit that was obtained in §7 to determine the cooling requirement of each configuration. Thus

	Power (kW) per coil	P/P_{max}	Number of ccts	v (ft/sec)	Vol per cct (USGPM)	ΔP per cct (psi)
7×12	14.195	2.670	3	2.224	0.449	17.798
8×10	13.688	2.574	5	1.287	0.260	3.868
10×8	14.060	2.644	4	1.653	0.334	7.773
10×8	14.060	2.644	2	3.305	0.668	47.680

The two calculations for the 10×8 array are for two possible cooling arrangements. Relative to the use of an 8×10 coil array, it is seen that the use of a 7×12 array or a 10×8 array reduces the number of cooling circuits required. Consequently, the chances of leaks in the cooling system are also reduced. On the other hand, the use of a 10×8 array increases the overall width of the dipole by 4 turns or approximately 2.25 in. At the same time the heights of the poles and vertical yokes are reduced by similar amounts. Similarly, again relative to the use of an 8×10 coil array, a 7×12 array increases the heights of the poles and vertical yokes by 4 turns while allowing a reduction equivalent to 2 turns in the overall width of the magnet.

Because there is little difference in the power requirements for the configurations studied and despite the possibility of fewer water leaks with the 7×12 and 10×8 arrays, we consider only the 'middle of the road' 8×10 coil array in what follows.

Note added in passing: We note that old, undated (but probably ca 1960s) copper tables from Anaconda give a length of 60 ft as the nominal length per coil of the 0.516 in. square conductor. Assuming that the calculated 81 ft of copper per 8-turn layer is correct, the 162 ft required per cooling circuit would require 3 reels of conductor. Thus a total of 2×5×3 reels—or 1,800 ft—of copper are required for the dipole, in agreement with that suggested in §5. However, when copper is purchased for the coils, the length of copper per reel must be ascertained.

11.2 Recalculation of the iron parameters

Figure 10 shows a section through the horizontal midplane of the dipole derived from the results of the POISSON calculations. Also shown is a section through the dipole at the entrance of the yoke. Based on the dimensions shown in that figure we recalculate an estimate of the iron required for the dipole.

To calculate the (new) pole area we use the relations of §9. We calculate the values $x_{int} = 31.230$ in. and $|y_{int}| = 10.989$ in. to determine $\overline{\mathrm{BC}} = 2(|y_{int}|) = 21.978$ in. and $\Theta = 2\tan^{-1}[y_{int}/(x_{int}+2)] = 36.598° = 0.63875$ radian. Further, $\overline{\mathrm{AD}} = 13.2$ in. and $R_{iron} = 35.00$ in. Thus

$$\begin{aligned} A_{pole} &= [(13.200 + 21.978)(31.230) + (35.00)^2(0.63875 - 0.59619)]/2 \\ &= [1098.6089 + 52.1276]/2 \;=\; 575.368 \text{ in.}^2 \end{aligned}$$

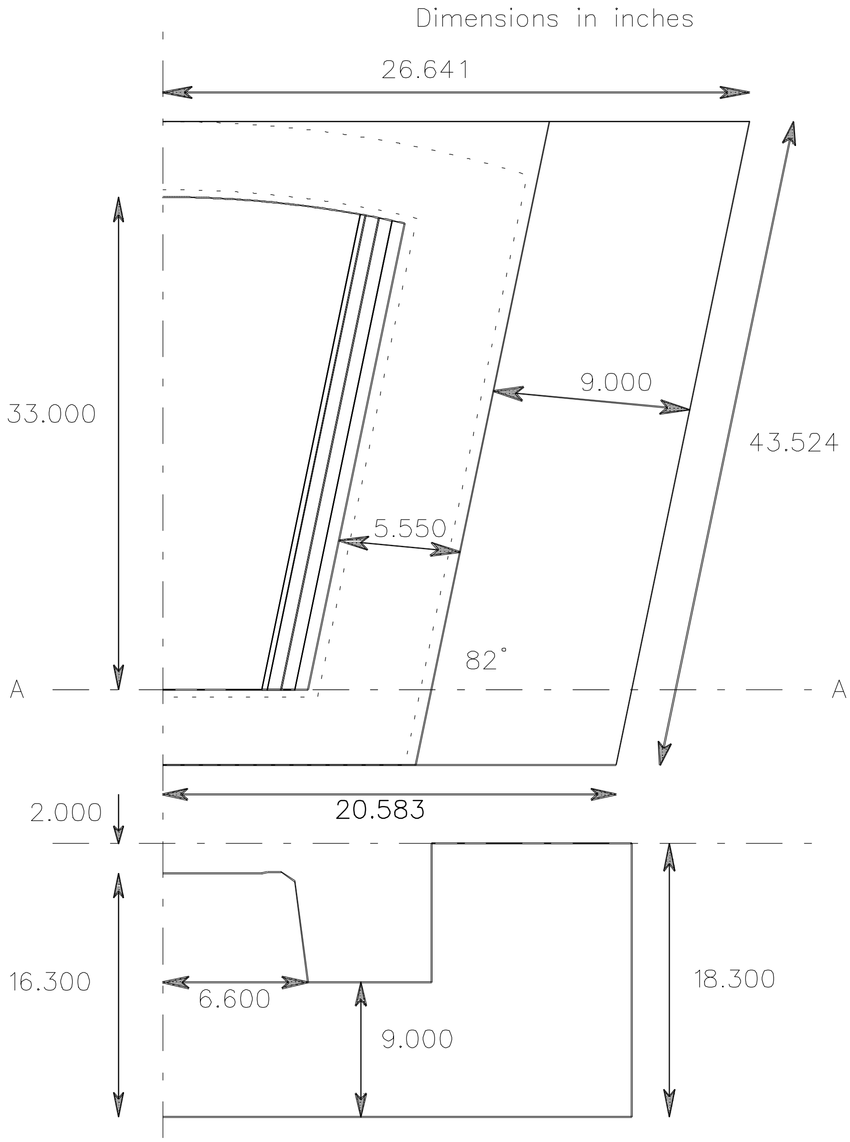

Fig. 10. Top: Cross-section of dipole through the magnetic midplane. Bottom: Section of dipole through A–A at the dipole entrance.

The areas of the vertical side yokes are

$$A_{side} = (43.524 \text{ in.})(9.000 \text{ in.}) = 391.716 \text{ in.}^2$$

and those of the top and bottom yokes are

$$\begin{aligned} A_{top} &= \text{(average width)(overall length)} \\ &= 2(26.641 \text{ in.} + 20.583 \text{ in.})[33.000 \text{ in.} + 2(0.500 \text{ in.} + 4.550 \text{ in.})] \ / \ 2 \\ &= 2,035.3544 \text{ in.}^2 \end{aligned}$$

From these areas we may calculate the iron volumes and weights.

Section	Area (in.^{2})	Height (in.)	Volume (in.^{3})
Top yoke	2,035.4	9.0	18,318.6
Bottom yoke	2,035.4	9.0	18,318.6
Vertical Yoke	391.7	18.6	7,285.6
Vertical Yoke	391.7	18.6	7,285.6
Top pole	575.4	7.3	4,200.4
Bottom pole	575.4	7.3	4,200.4
Total			59,609.2

We take the total volume of iron to be 60,000 in.^{3} = 34.722 ft^3. At a density of 0.2833 lb/in.^{3}, this volume has a weight of 16,998 lb. We take

$$\boxed{\text{Weight of iron in dipole} = 17{,}000\ \text{lb} = 7{,}710\ \text{kg.}}$$

Figure 11, below, shows the details of the pole shim that was used in these calculations.

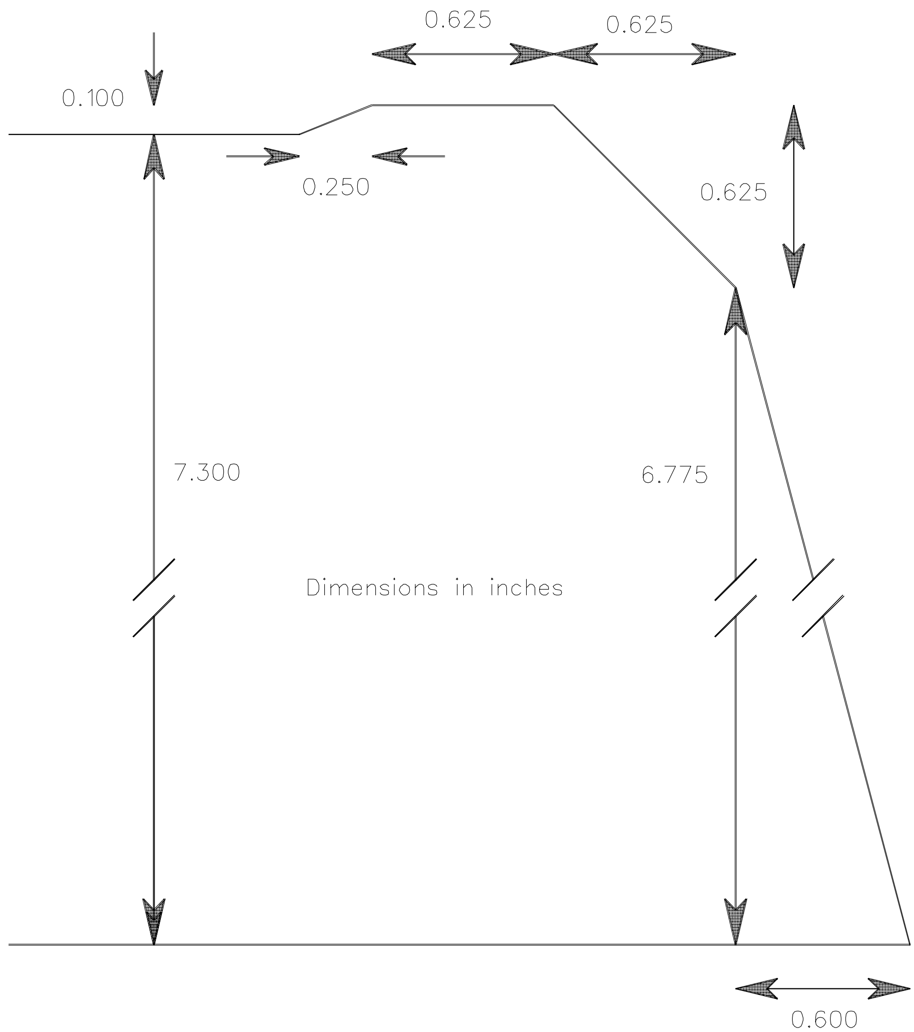

Fig. 11. Details of the pole shim used in the calculations.

12. Discussion

This note has presented a conceptual design for a ±15° switching magnet for use on beam line 2A. Although not included in this note, the possibility of the use of other conductor sizes was considered. One, an Outokumpu #6813 conductor is essentially identical to the Anaconda 0.516 in. square conductor considered here. Consequently, the results given here would apply equally to a magnet with coils made of the Outokumpu copper.

Also considered were three configurations for the coil winding. That chosen, an 8×10 array, was considered the best choice albeit a 7×12 array *might* result in a slightly smaller magnet that requires a slight increase in power consumption.

However, as noted earlier, use of a smaller, Anaconda 0.4600 in. square conductor is possible. This would result in a slightly smaller total weight of iron and copper. Relative to the larger 0.516 in. conductor, it is estimated that approximately 350 fewer pounds of copper and 1,300 fewer pounds of iron are necessary for a magnet made with the smaller conductor. Assuming a raw material cost of $10/lb for copper and $1/lb for iron, this would reflect in a raw material cost of approximately $5,000 less for a magnet made with the smaller conductor.

On the other hand, a magnet made with the larger conductor is estimated to require approximately 12 kW less power when operating. Thus, in the long term it may be less expensive to use the larger conductor.

Because the dipole always will operate at or about the maximum field for which it was designed, it is recommended that the switching magnet be constructed with a coil made of the 0.516 in. square conductor in order reduce the long-term operational costs.

Given the design parameters, a preliminary dimensioning of the dipole was obtained. These were then subject to an investigation with the program POISSON. These latter studies indicated the need to increase both the yoke thickness and the pole width.

Table 1 summarizes the results of each of these calculations.

References

1. G. M. Stinson, *A conceptual design for a 15° switching dipole for beam line 2A*, TRIUMF Report, TRI-DNA-96-4, January, 1996.
2. G. M. Stinson, *A revised conceptual design for a 15° dipole on beam line 2A*, TRIUMF Report, TRI-DNA-96-10, December, 1996.
3. M. T. Menzel and H. K. Stokes, *User's Guide for the POISSON/SUPERFISH Group of Codes*, Los Alamos National Laboratory Report LA-UR-87-115, January, 1987.

Table 1

Summary of ±15° switching magnet design parameters

		Preliminary dimensions	Final dimensions
Yoke:	Iron length on centerline	33.000 in.	33.000 in.
	Iron width (maximum)	50.200 in.	53.282 in.
	Iron width (minimum)	38.100 in.	41.166 in.
	Iron thickness	8.000 in.	9.000 in.
	Coil-slot width	5.550 in.	5.550 in.
	Side-yoke height	18.600 in.	18.600 in.
Pole:	Width at entrance	12.000 in.	13.200 in.
	Height at center	7.300 in.	7.300 in.
	Chamfer at 45°	1.000 in.	—
	Edge shim (total width)	—	1.500 in.
Iron:	Total mass	14.60×10^3 lb	17.00×10^3 lb
Dipole:	Overall width (maximum)	50.200 in.	53.282 in.
	Overall height	34.600 in.	34.600 in.
	Overall length (incl. coil)	43.100 in.	43.100 in.
Coil:	Conductor OD	0.516 in.	0.516 in.
	Conductor ID	0.287 in.	0.287 in.
	Nominal coil width	4.450 in.	4.450 in.
	Nominal coil height	5.625 in.	5.625 in.
	Total coolant flow	2.568 USGPM	2.600 USGPM
	Turn configuration	8 wide × 10 high	8 wide × 10 high
	Resistance (hot) per coil	37.55×10^{-3} Ω	
		38.02×10^{-3} Ω	
	Cooling circuits per coil	5	5
Copper:	Total length per magnet	1,600 ft	1,620 ft
	Total mass per magnet	1,200 lb	1,214 lb
	Total length to order	1,800 ft	1,800 ft
	Total mass to order	1,400 lb	1,400 lb
Power:	Total current	600.0 A	600.0 A
	Total Voltage	50.0 V	51.0 V
	Power	30.0 kW	30.6 kW

Appendix

A1. Circular exit pole

To study the geometry of the pole we establish an $x - y$ coordinate system with its origin at the center of the entrance face of the dipole. Further, we assume that the difference between its effective and physical lengths is equally divided between the exit and entry. Thus, defining

$$a = \frac{L_{eff} - L_{iron}}{2} = 2 \text{ in.} \ ,$$

we take the *effective* exit edge of the dipole to be a circle of radius L_{eff} centered at $(x, y) = (-a, 0)$. We ensure that the iron edge of the dipole remains a constant distance from the effective edge by taking the iron edge to be a circle of radius $(L_{eff} - a)$ also centered at $(-a, 0)$. Writing $z = x + a$, the equation of these pole edges may be written as

$$z^2 + y^2 = \alpha^2 \qquad \begin{cases} \alpha = L_{eff} \text{ for effective edge} \\ \alpha = L_{eff} - a \text{ for iron edge} \end{cases} . \tag{1}$$

For reasons that will shortly become apparent, it is useful in further analysis to consider the more general equation

$$z^2 + (y + \beta)^2 = \gamma^2 \ . \tag{2}$$

With

$$\beta = \rho_0 \qquad \text{and} \qquad \gamma = \rho_0 \ ,$$

equation (2) represents the central trajectory of the beam—a circle of radius ρ_0 centered at $(x, y) = (-a, -\rho_0)$. With

$$\beta = \rho_0 + \delta \qquad \text{and} \qquad \gamma = \rho_0 \ ,$$

it represents the trajectory of a particle with central momentum that is displaced a distance δ *below* the central ray. Finally, with

$$\beta = \rho_0 \qquad \text{and} \qquad \gamma = \rho_0 - \Delta \ ,$$

equation (2) represents the trajectory of a particle in an orbit that maintains a constant distance Δ below the central trajectory.

Equations (1) and (2) may be solved to find their intersection. Because of the symmetry of the problem we consider a solution for which x is positive and y is negative. For this case the general solution is

$$y = \frac{\gamma^2 - \beta^2 - \alpha^2}{2\beta} \tag{3}$$

$$z = \frac{\sqrt{2\gamma^2(\alpha^2 + \beta^2) + 2\alpha^2\beta^2 - \alpha^4 - \beta^4 - \gamma^4}}{2\beta} \tag{4}$$

We are now in a position to calculate the pole size. The following parameters are available.

Parameter	Symbol	Value
$L_{eff} - L_{iron}$	2a	4.0 in.
Beam half-width	δ	1.0 in.
Dipole gap	g	4.0 in.
Chamfer	c	1.0 in.
Distance of central trajectory from dipole edge	Δ	6.0 in.
Radius of curvature of central ray	ρ_0	141.764 in.
Radius of curvature of effective edge of dipole	L_{eff}	37.0 in.
Radius of curvature of iron edge of dipole	$L_{eff} - a$	35.0 in.

The point of intersection of the central ray and the effective edge of the dipole is obtained from equations (3) and (4) by setting $\gamma = \beta = \rho_0$ and $\alpha = L_{eff}$ to yield

$$y_{c,eff} = \frac{\rho_0^2 - \rho_0^2 - L_{eff}^2}{2\rho_0} = -\frac{(37.0)^2}{2(141.764)} = -4.8284 \text{ in.}$$

$$z_{c,eff} = \frac{L_{eff}}{2\rho_0}\sqrt{4\rho_0^2 - L_{eff}^2} = \frac{37.0}{2(141.764)}\sqrt{4(141.764) - (37.0)^2} = 36.6836 \text{ in.}$$

As a check we note that the bend angle θ is obtained from

$$\theta = \tan^{-1}\left[\frac{z_{c,eff}}{\rho_0 - |y_{c,eff}|}\right] = 14.9969^\circ$$

Similarly, the point of intersection of the central ray and the pole side that is a constant distance Δ below the central ray is obtained from those equations by setting $\gamma = \rho_0 - \Delta$, $\beta = \rho_0$ and $\alpha = L_{eff}$. This yields

$$y_{c,side} = -\frac{L_{eff}^2 + 2\Delta\rho_0 - \Delta^2}{2\rho_0} = -\frac{(37.0)^2 + 2(6.0)(141.764) - (6.0)^2}{2(141.764)} = -10.7015 \text{ in.}$$

$$\begin{aligned} z_{c,side} &= \frac{\sqrt{(L_{eff}^2 - \Delta^2)(4\rho_0^2 - 4\Delta\rho_0 - L_{eff}^2 + \Delta^2)}}{2\rho_0} \\ &= \frac{\sqrt{[(37.0)^2 - (6.0)^2][4(141.764)^2 - 4(6.0)(141.764) - (37.0)^2 + (6.0)^2]}}{2(141.764)} \\ &= 35.4186 \text{ in.} \end{aligned}$$

We then take the pole side to be defined by the line through the points $(x_1, y_1) = (0, -\Delta)$ and $(x_2, y_2) = (z_{c,side}, y_{c,side})$. The equation of this line is obtained from

$$\frac{y + \Delta}{x} = \frac{y_{c,side} + \Delta}{z_{c,side}} = \frac{-10.7015 + 6.0}{35.4186} = -0.140685$$

or

$$y = -0.140685\,x - 6 \tag{5}$$

The angle of the pole with respect to the positive x-axis is $\phi = \tan^{-1}(-0.140685) = -8.008^\circ$. Rather than use this somewhat unusual value we take $\phi = -8.0^\circ$; the equation of the pole side is then

$$y = x\tan(-8.0) - 6 = -0.140541\,x - 6 = mx + b \tag{6}$$

and ask for the point of its intersection with the physical pole edge that is defined by

$$(x + a)^2 + y^2 = (L_{eff} - a)^2 = \rho^2\ . \tag{7}$$

The solution of equations (6) and (7) for positive values of x is

$$x = \frac{1}{1 + m^2}\left[-(a + mb) + \sqrt{(1 + m^2)\rho^2 - (ma - b)^2}\right] \tag{8}$$

and inserting the appropriate numerical values we find

$$\left.\begin{aligned} x &= 31.4145 \text{ in.} \\ y &= -10.4150 \text{ in.} \end{aligned}\right\}\ . \tag{9}$$

The resulting pole configuration is shown in the figure below.

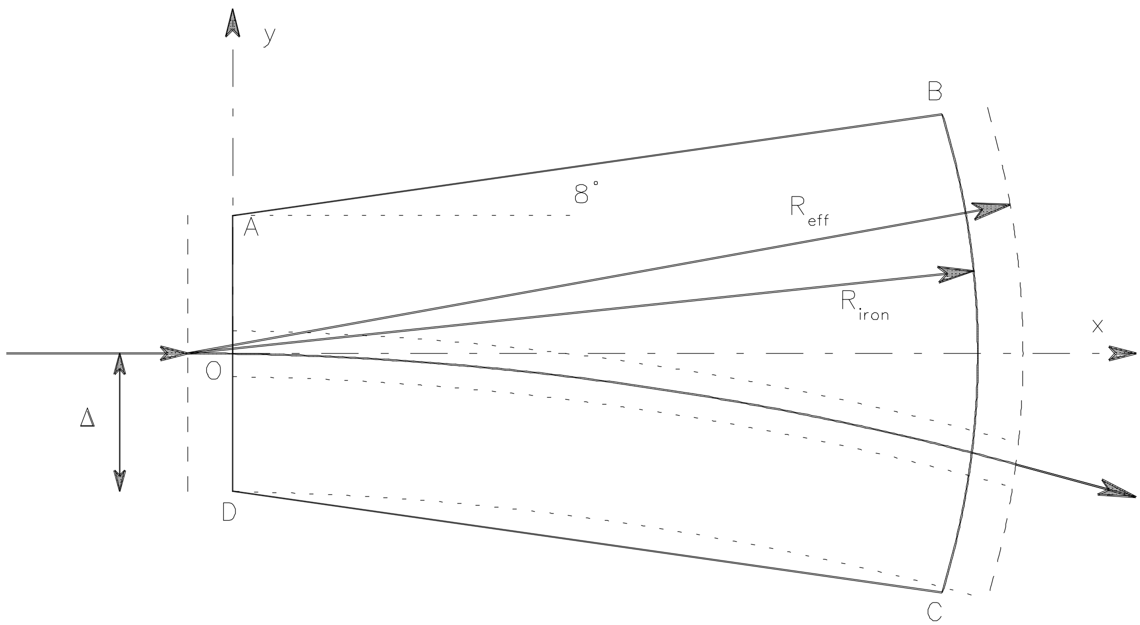

Fig. A1. The pole configuration of the switching magnet with a circular exit edge.

In this figure the incoming beam defines the x-axis and the line $\overline{AD}$ lies along the y-axis. The origin of this coordinate system, O, is at the midpoint of the line $\overline{AD}$. The dashed lines represent the assumed locations of the effective edges of the dipole. The solid curve is the trajectory of the central ray; the dotted curves either side of it indicate the full width of the incident beam. The minimum required separation between the central trajectory and a pole edge is represented by the lower dotted line. Note that the radius vectors of the exit edge of the pole, R_{iron} and R_{eff}, have their centers a distance $a = 2.00$ in. to the left of the entrance edge.

The corners of the pole are defined by the points A, B, C and D. These points have the coordinates (0, 6), (31.415, 10.415), (31.415, −10.415) and (0, −6) respectively. From these we obtain the lengths of the sides of the magnet. We have

$$\begin{aligned}\overline{AD} &= \sqrt{(0-0)^2 + (6-(-6))^2} \\ &= 12.00 \text{ in.}\end{aligned}$$

$$\begin{aligned}\overline{AB} = \overline{DC} &= \sqrt{(31.415-0)^2 + (10.415-6)^2} \\ &= 31.724 \text{ in.}\end{aligned}$$

$$\begin{aligned}R_{iron} &= L_{eff} - a \\ &= 35.00 \text{ in.}\end{aligned}$$

We note at this point that the normal to the *effective* pole edge—that is, the line joining the points $(-a,\ 0)$ and $(y_{c,eff},\ z_{c,eff})$—makes an angle of $-7.5°$ with respect to the x-axis whereas that of the exiting central ray is $-15°$ with respect to the x-axis. Consequently, from an optics point of view, a dipole of this design is considered to have an entry angle of $0°$ and an exit angle of $7.5°$.

A2. An alternate pole profile

The (exit) pole profile that consists of straight-line segments can be designed in many ways. One of the simplest is to taylor the exit pole face such that the beam exits normal to it. This, then, requires that the effective pole face at the magnet exit make a 75° angle with respect to the x-axis.

From equations (3) and (4) we find the point of intersection of the central ray and the *iron* edge of the dipole by setting $\gamma = \beta = \rho_0$ and $\alpha = L_{eff} - a$. This yields

$$\begin{aligned} y_{c,iron} &= -\frac{(L_{eff}-a)^2}{2\rho_0} \\ &= -\frac{(35.0)^2}{2(141.764)} = -4.3206 \text{ in.} \\ z_{c,iron} &= \frac{L_{eff}-a}{2\rho_0}\sqrt{(2\rho_0)^2-(L_{eff}-a)^2} \\ &= \frac{35.0}{2(141.764)}\sqrt{(2(141.764))^2-(35.0)^2} = 34.7323 \text{ in.} \\ x_{c,iron} &= z_{c,iron} - a = 32.7323 \text{ in.} \end{aligned}$$

The bend angle θ_{iron} of this portion of the trajectory is obtained from

$$\theta_{iron} = \tan^{-1}\left[\frac{z_{c,iron}}{\rho_0 - |y_{c,iron}|}\right] = 14.1819^\circ ,$$

indicating that the remaining 0.8° of bend takes place between the iron edge and the effective edge of the magnet.

The equation of a line through $(y_{c,iron}, z_{c,iron})$ with a slope of 75° is

$$y = x\tan(75) - 126.4792 = 3.73205x - 126.4792 \tag{10}$$

Equations (6) and (10) intersect at

$$\begin{aligned} x &= 31.1107 \text{ in.} \\ y &= -10.3723 \text{ in.} \end{aligned}$$

and equation (10) intersects the iron edge $x = 33.0$ in. at

$$\begin{aligned} x &= 33.0000 \text{ in.} \\ y &= -3.3215 \text{ in.} \end{aligned}$$

Figure A2 shows the pole profile for this case. As for figure A1, the solid lines indicate the central trajectory and the physical pole. The dashed lines at the entrance and exit are the assumed positions of the effective edges of the dipole. Full (horizontal) extent of the beam is indicated by the two dotted trajectories either side of the central ray and the clearance between the central ray and the pole side is shown by the lower dotted line.

Corners of the pole shown in figure A2 are defined by the points A, B, C, D, E and F. From the coordinates given above we may calculate the lengths of the sides of the magnet. We have

$$\begin{aligned} \overline{AF} &= \sqrt{(0-0)^2 + (6-(-6))^2} \\ &= 12.00 \text{ in.} \end{aligned}$$

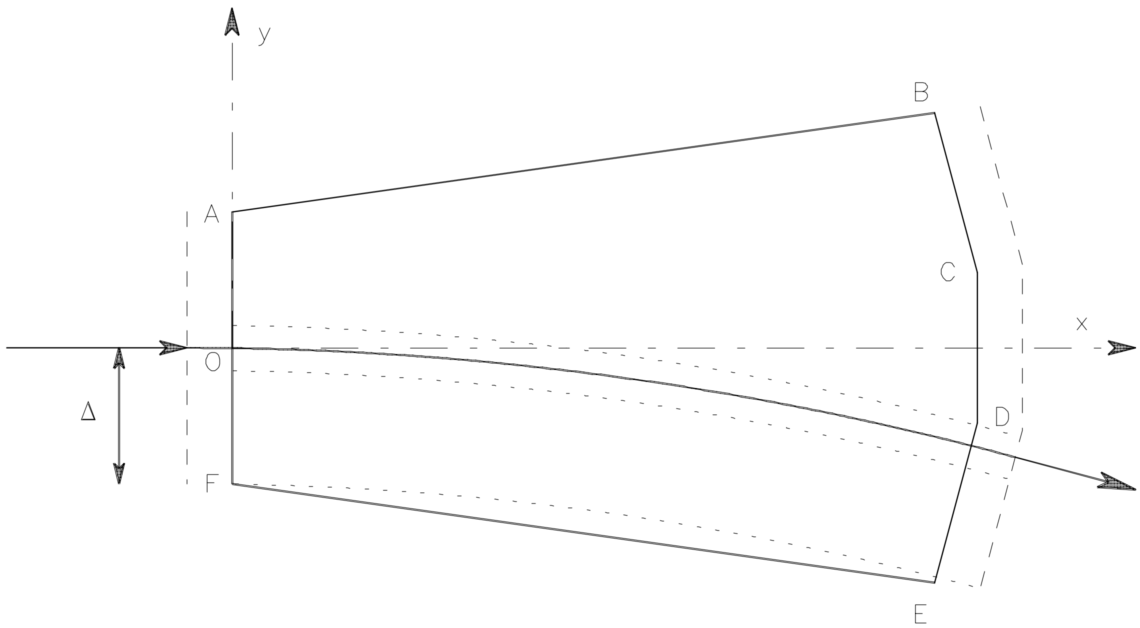

Fig. A2. The pole configuration of the switching magnet with a non-circular exit edge.

$$\begin{aligned} \overline{AB} = \overline{EF} &= \sqrt{(31.1107-0)^2+(10.3723-6)^2} \\ &= 31.4164 \text{ in.} \end{aligned}$$

$$\begin{aligned} \overline{BC} = \overline{DE} &= \sqrt{(31.1107-33.0)^2+(10.3723-3.3215)^2} \\ &= 7.2995 \text{ in.} \end{aligned}$$

$$\overline{CD} = 2(3.3215) = 6.6430 \text{ in.}$$

From the above figure it would appear that the upper edge of the beam strikes the iron edge of the pole close to the point labelled D with the consequence that there may be some field inhomogeneity in this region. To clarify this we note that the equation of the upper ray is given by

$$(x+a)^2 + (y+\rho_0)^2 = (\rho_0 + delta)^2$$

and this trajectory intersects the pole boundary $\overline{CD}$ at $x = 33in.$ and

$$\begin{aligned} y &= -\rho_0 + \sqrt{(\rho_0+\delta)^2 - (x+a)^2} \\ &= -141.764 + \sqrt{(142.764)^2 - (35.0)^2} \\ &= -3.3568 \text{ in.} \end{aligned}$$

For all practical purposes, this point $(x, y) = (33.0, -3.3568)$ is identical the point of intersection of the lines $\overline{CD}$ and $\overline{DE}$ $(x, y) = (33.0, -3.3215)$ that was calculated on the previous page. Consequently, there may a problem of field uniformity at the outer edge of the beam. Should this be the case, a simple solution is to angle the pole edge $\overline{DE}$ at an angle of 82.5° with respect to the x-axis—thus producing a magnet with normal entrance and 7.5° exit angles.