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Author GM Stinson		*Page* 1 *of* 9
Subject Generic B–I and I–B fits to measured MEBT and HEBT quadrupole B–I data		

1. Introduction

B–I data has been taken [1)] for eighteen of the CRNL type L1 and type L2 quadrupoles that are to be used in the HEBT section of the ISAC facility at TRIUMF. We list here the results of fitting the measured B–I data using the program PLOTDATA [2)].

2. Fitting results for the type L1 and type L2 quadrupoles

B–I curves were taken quadrupoles at excitation currents of 20, 40, 60, 80, and 100 Amperes for sixteen type L1 quadrupoles and for two type L2 quadrupoles that will be installed as the first eighteen quadrupoles of the HEBT beam line. For each individual quadrupole the measured field was fitted to a cubic polynomial in current. Thus the B–I relationship is given by

$$B(\mathrm{kG}) = \alpha_H \left[\frac{I}{10}\right]^3 + \beta_H \left[\frac{I}{10}\right]^2 + \gamma_H \left[\frac{I}{10}\right] + \delta_H \ .$$

In addition, an average over the entire ensemble of each of the coefficients was also computed. Table 1(a) lists the fitted coefficients for each of the type L1 quadrupole as well as the computed average coefficients. Listed in table 1(b) is similar data for the type L2 quadrupoles.

3. Generic B-I and I-B fits for the type L1 and type L2 quadrupoles

The (maximum) field gradients required for the HEBT quadrupoles are given in ref [3)]. When these are converted to pole-tip fields, it is found that the maximum requirement for a pole-tip field is of the order of 4 kG. However, the range of currents to obtain the measured field data produces fields up to 8.7 kG. Useful data for the HEBT quadrupoles occurs for currents at and below 60 A—at which current the measured field is approximately 5.4 kG for the quadrupoles measured. Further, for each quadrupole, there are only three data points to use in any fitting of B–I data. Because these quadrupoles will, on average, be running at fields lower than 4 kG, the present data was considered inadequate.

Fortunately, similar data had been taken for the quadrupoles that were used in the MEBT portion of the line. For those quadrupoles, B–I data was taken at currents from 10 A to 100 A in 10 A steps. Because the quadrupoles used in the MEBT are of the same type that will be used in the HEBT, it was decided to use the previous data to generate generic B–I and inverse I–B relationships. These were then applied them to the HEBT quadrupoles to see how well the generic fits reproduced the measured data. To this end the following procedure was adopted.

Measurements were selected from an arbitrary group of seven MEBT quadrupoles and, for each of these, quadratic fits were made to the B–I and I–B data for currents from 10 A to 60 A in 10 A steps. The coefficients obtained for the individual quadrupoles were then averaged. These averaged values were then used as coefficients to define what is termed here as generic B–I and I–B relations.

For completeness, we list in table 2 the coefficients obtained for the quadratic fit of the individual B–I data. As noted above, these were obtained by fitting an equation of the form

$$B = \alpha_M \, I^2 + \beta_M \, I + \gamma_M \ ,$$

with B in kG and I in Amperes, to the measured B–I data. Using the average value of each of these coefficients produces the generic B–I relationship

$$\begin{aligned} B &= <\alpha_M> I^2 + <\beta_M> I + <\gamma_M> \\ &= -7.27296 \times 10^{-5}\, I^2 + 9.46413 \times 10^{-2}\, I - 2.11857 \times 10^{-2} \end{aligned} \quad (1)$$

that gives the field B in kG for currents I expressed in Amperes. Table 3 lists, as a function of current, the measured fields for these MEBT quadrupoles. Also tabulated are the fields calculated from the above equation and the percentage difference between the measured and those calculated fields. From table 3 it is seen that fields calculated from the average parameters agree with the measured fields to within 1% over the current range 10 A $\leq I \leq$ 60 A.

Similarly, quadratic I–B fits were made to the MEBT quadrupole data. Average coefficients were then extracted to produce an I–B fit of the form

$$\begin{aligned} I &= <\alpha_I> I^2 + <\beta_I> I + <\gamma_I> \\ &= 1.01210 \times 10^{-1}\, B^2 + 1.052614 \times 10^{+1}\, B + 2.56048 \times 10^{-1} \end{aligned} \quad (2)$$

giving the required current I in Amperes to produce a given field B in kG.

Our expectation is then that fields calculated for the HEBT using equations 1 and 2 will fit their measured fields equally well.

4. Application of the generic fits to the HEBT quadrupoles

Relationships (1) and (2) above were then applied to the data obtained from the measurements of the first eighteen HEBT quadrupoles. In table 4(a) we compare the measured fields of each HEBT quadrupole with that calculated from equation (1) for currents of 20 A, 40 A, and 60 A. For each current, and therefore calculated field $B_{calculated}$, we list the measured field $B_{measured}$ and the percentage difference between these two fields. From the data presented in table 4(a) two points are evident.

1. The fields calculated using the generic relationship of equation (1) agree within 1% with those fields that were measured for the entire group of 18 quadrupoles. It is to be noted that this is true *regardless* of the type of quadrupole. All quadrupoles are type L1 except Q7 and Q9; the latter are type L2.
2. The generic fit consistently *overestimates* the field at all currents. If the percentage differences are averaged over all quadrupoles and all three currents, we find that, on average, the generic fit overestimates the quadrupole field by 0.50%.

Table 4(b) lists, as a function of current, the measured fields B_{meas} and the current I_{calc}, calculated from equation (2) above, required to produce that field. Also listed is the percentage difference between I_{calc} and the current $I_{measured}$ that was recorded when B_{meas} was measured. From this table we make the following observations.

1. The currents calculated using the generic relationship of equation (2) agree within 1% with those currents that were used to obtain the measured fields for the entire group of 18 quadrupoles. Again, this is true regardless of the type of quadrupole.
2. The generic fit consistently *underestimates* the current at all fields. If the percentage differences are averaged over all quadrupoles and all three currents, we find that, on average, the generic fit underestimates the required quadrupole current by 0.52%.

Given that some 'knobbing around' will probably be required to set the final tune for the beam line, we feel that these generic relationships will be suitable for setting up the beam line.

4.1 Further quadrupole measurements

As this report was being written, B–I measurements for four additional quadrupoles became available. These quadrupoles, designated AQ3 through AQ5, are to be the four quadrupoles immediately upstream of the gas target of the DRAGON experimental location. In table 5(a) we compare the measured field of each of these quadrupoles with that calculated from equation (1) for currents of 20 A, 40 A, and 60 A. As in table 4(a), for each current—and therefore calculated field $B_{calculated}$—the measured field $B_{measured}$, and the percentage difference between these two fields are listed. Similarly, in table 5(b) we list, as a function of current, the measured field B_{meas} and the current I_{calc} required current given by equation (2) to produce that field.

The conclusions drawn from the data presented in tables 5(a) and 5(b) are, as expected, the same as those drawn from the data of tables 4(a) and 4(b).

5. Discussion

This report presents a summary of an attempt to produce generic B–I and I–B relationships that would be suitable for the HEBT quadrupoles. By extension, these relationships should be suitable for all of the CRNL type L1 and type L2 quadrupoles. The generic B–I relationship is

$$B = -7.27296 \times 10^{-5}\, I^2 + 9.46413 \times 10^{-2}\, I - 2.11857 \times 10^{-2}$$

and the generic I–B relationship is

$$I = 1.01210 \times 10^{-1}\, B^2 + 1.052614 \times 10^{+1}\, B + 2.56048 \times 10^{-1} \ .$$

In each of these relationships, fields are in kG and currents in Amperes. Because these expressions were developed over limited current and field ranges, they are restricted to current and field values of

$$0 \text{ A} \leq I \leq 60 \text{ A} \qquad \text{and} \qquad 0 \text{ kG} \leq B \leq 5.5 \text{ kG} \ .$$

From a comparison of the measured and calculated B–I and inverse I–B data we conclude

1. The fields calculated using the generic relationship of equation (1) agree within 1% with those fields that were measured for the entire group of 22 quadrupoles. This is true *regardless* of the type of quadrupole.

2. The generic B–I fit consistently *overestimates* the field at all currents. Averaged over all quadrupoles and all three currents, the generic fit overestimates the quadrupole field by 0.5%.

3. The currents calculated using the generic relationship of equation (2) agree within 1% with those currents that were used to obtain the measured fields for the entire group of 22 quadrupoles. Again, this is true regardless of the type of quadrupole.

4. The generic fit consistently *underestimates* the current at all fields. Averaged over all quadrupoles and all three currents, the generic fit underestimates the required quadrupole current by 0.5%.

Consequently, we feel that the generic B–I and I–B relationships given above are suitable for use in the tuning og the beam line.

References

1. D. Evans, *Private communication*, TRIUMF, August, 2000.
2. J. L. Chuma, *PLOTDATA Command and Reference Manual*, TRIUMF Report TRI-CD-87-03b, June, 1991.
3. R. Laxdal, *Design Specification for ISAC HEBT*, TRIUMF Report TRI-DN-99-23, TRIUMF, May, 2000.

Table 1(a)

Coefficients of B–I fit for type L1 HEBT quadrupoles

Fit is of the form:

$$B(\text{kG}) = \alpha_H \left[\frac{I}{10}\right]^3 + \beta_H \left[\frac{I}{10}\right]^2 + \gamma_H \left[\frac{I}{10}\right] + \delta_H$$

α_H	β_H	γ_H	δ_H	Quadrupole
−0.0060104	0.0707411	0.6270236	0.3504004	Q1
−0.0061146	0.0718125	0.6295831	0.3410002	Q2
−0.0061250	0.0718929	0.6307854	0.3384004	Q3
−0.0062812	0.0740268	0.6211785	0.3480000	Q4
−0.0061250	0.0714464	0.6368932	0.3329994	Q5
−0.0060625	0.0708214	0.6348928	0.3340000	Q6
−0.0060729	0.0708304	0.6369522	0.3290004	Q8
−0.0061667	0.0724465	0.6295590	0.3402006	Q10
−0.0062187	0.0734910	0.6193573	0.3479997	Q11
−0.0060313	0.0705983	0.6338208	0.3338007	Q12
−0.0060104	0.0698483	0.6417377	0.3218005	Q13
−0.0062083	0.0726070	0.6285480	0.3375995	Q14
−0.0061146	0.0716161	0.6301906	0.3355999	Q15
−0.0062396	0.0735624	0.6225837	0.3453996	Q16
−0.0060625	0.0704821	0.6402144	0.3275998	Q17
−0.0060938	0.0711875	0.6327499	0.3340000	Q18
−0.0061211	0.0717132	0.6310044	0.3373626	Averages

Table 1(b)

Coefficients of B–I fit for type L2 HEBT quadrupoles

Fit is of the form:

$$B(\text{kG}) = \alpha_H \left[\frac{I}{10}\right]^3 + \beta_H \left[\frac{I}{10}\right]^2 + \gamma_H \left[\frac{I}{10}\right] + \delta_H$$

α_H	β_H	γ_H	δ_H	Quadrupole
−0.0061875	0.0793750	0.5885000	0.3934000	Q7
−0.0061563	0.0792947	0.5837140	0.3986003	Q9
−0.0061719	0.0793348	0.5861070	0.3960002	Averages

Table 2

Individual coefficients for MEBT quadratic fits

Quadrupole	α_M	β_M	γ_M
Q7A	−0.712498E-04	0.0944846	−0.0278998
Q8B	−0.607143E-04	0.0935414	−0.0071999
Q8A	−0.716070E-04	0.0944096	−0.0148999
Q25	−0.735713E-04	0.0946471	−0.0163999
Q11B	−0.839286E-04	0.0957236	−0.0331999
Q23	−0.758930E-04	0.0949954	−0.0199001
Q11A	−0.721429E-04	0.0946871	−0.0288000
Average	−7.27296E-05	9.46413E-02	−2.11857E-02

Table 3

$B_{measured}$, $B_{average}$, and difference [$= 100(B_{measured} - B_{average})/B_{measured}$]: MEBT quadrupoles

Current (A)	Quadrupole	$B_{measured}$ (kG)	$B_{average}$ (kG)	Difference (%)
10	Q7A	0.911	0.918	−0.763
	Q8B	0.923	0.918	0.547
	Q8A	0.923	0.918	0.547
	Q25	0.924	0.918	0.654
	Q11B	0.918	0.918	0.005
	Q23	0.924	0.918	0.654
	Q11A	0.912	0.918	−0.653
20	Q7A	1.831	1.843	−0.631
	Q8B	1.838	1.843	−0.247
	Q8A	1.843	1.843	0.025
	Q25	1.845	1.843	0.133
	Q11B	1.844	1.843	0.079
	Q23	1.847	1.843	0.241
	Q11A	1.834	1.843	−0.466
30	Q7A	2.743	2.753	−0.350
	Q8B	2.744	2.753	−0.313
	Q8A	2.753	2.753	0.015
	Q25	2.757	2.753	0.160
	Q11B	2.762	2.753	0.340
	Q23	2.762	2.753	0.340
	Q11A	2.747	2.753	−0.204
40	Q7A	3.638	3.648	−0.278
	Q8B	3.638	3.648	−0.278
	Q8A	3.647	3.648	−0.030
	Q25	3.651	3.648	0.079
	Q11B	3.663	3.648	0.407
	Q23	3.658	3.648	0.271
	Q11A	3.644	3.648	−0.112
50	Q7A	4.519	4.529	−0.222
	Q8B	4.519	4.529	−0.222
	Q8A	4.528	4.529	−0.023
	Q25	4.535	4.529	0.131
	Q11B	4.546	4.529	0.373
	Q23	4.543	4.529	0.307
	Q11A	4.526	4.529	−0.067
60	Q7A	5.384	5.395	−0.213
	Q8B	5.386	5.395	−0.176
	Q8A	5.391	5.395	−0.083
	Q25	5.396	5.395	0.010
	Q11B	5.406	5.395	0.195
	Q23	5.405	5.395	0.176
	Q11A	5.392	5.395	−0.064

Table 4(a)

HEBT quadrupoles: $B_{measured}$, $B_{calculated}$, and difference $[= 100(B_{measured} - B_{calculated})/B_{measured}]$

Quad	$I_{measured} = 20$ A, $B_{calculated} = 1.84255$ kG		$I_{measured} = 40$ A, $B_{calculated} = 3.64810$ kG		$I_{measured} = 60$ A, $B_{calculated} = 5.39546$ kG	
	$B_{measured}$ (kG)	Diff (%)	$B_{measured}$ (kG)	Diff (%)	$B_{measured}$ (kG)	Diff (%)
Q1	1.836	−0.357	3.619	−0.804	5.341	−1.020
Q2	1.835	−0.411	3.631	−0.471	5.362	−0.624
Q3	1.835	−0.411	3.634	−0.388	5.367	−0.530
Q4	1.833	−0.521	3.628	−0.554	5.364	−0.587
Q5	1.840	−0.138	3.646	−0.058	5.382	−0.250
Q6	1.835	−0.411	3.633	−0.416	5.362	−0.624
Q7	1.832	−0.576	3.647	−0.030	5.407	0.213
Q8	1.834	−0.466	3.636	−0.333	5.367	−0.530
Q9	1.828	−0.796	3.632	−0.443	5.390	−0.101
Q10	1.836	−0.357	3.638	−0.278	5.371	−0.456
Q11	1.827	−0.851	3.619	−0.804	5.343	−0.982
Q12	1.832	−0.576	3.627	−0.582	5.354	−0.774
Q13	1.833	−0.521	3.636	−0.333	5.367	−0.530
Q14	1.832	−0.576	3.630	−0.499	5.361	−0.643
Q15	1.830	−0.686	3.625	−0.637	5.353	−0.793
Q16	1.831	−0.631	3.629	−0.526	5.358	−0.699
Q17	1.838	−0.247	3.642	−0.167	5.376	−0.362
Q18	1.832	−0.576	3.628	−0.554	5.356	−0.737

Table 4(b)

HEBT quadrupoles: $B_{measured}$, $I_{calculated}$, and difference $[= 100(I_{calculated} - I_{measured})/I_{calculated}]$

Quad	$I_{measured} = 20$ A			$I_{measured} = 40$ A			$I_{measured} = 60$ A		
	B_{meas} (kG)	I_{calc} (A)	Diff (%)	B_{meas} (kG)	I_{calc} (A)	Diff (%)	B_{meas} (kG)	I_{calc} (A)	Diff (%)
Q1	1.836	19.923	−0.385	3.619	39.676	−0.817	5.341	59.363	−1.073
Q2	1.835	19.912	−0.440	3.631	39.811	−0.475	5.362	59.607	−0.659
Q3	1.835	19.912	−0.440	3.634	39.845	−0.390	5.367	59.665	−0.561
Q4	1.833	19.891	−0.550	3.628	39.777	−0.561	5.364	59.630	−0.620
Q5	1.840	19.967	−0.166	3.646	39.980	−0.051	5.382	59.839	−0.268
Q6	1.835	19.912	−0.440	3.633	39.833	−0.418	5.362	59.607	−0.659
Q7	1.832	19.880	−0.606	3.647	39.991	−0.022	5.407	60.130	0.216
Q8	1.834	19.901	−0.495	3.636	39.867	−0.333	5.367	59.665	−0.561
Q9	1.828	19.836	−0.827	3.632	39.822	−0.447	5.390	59.932	−0.113
Q10	1.836	19.923	−0.385	3.638	39.890	−0.277	5.371	59.712	−0.483
Q11	1.827	19.825	−0.882	3.619	39.676	−0.817	5.343	59.387	−1.033
Q12	1.832	19.880	−0.606	3.627	39.766	−0.589	5.354	59.514	−0.816
Q13	1.833	19.891	−0.550	3.636	39.867	−0.333	5.367	59.665	−0.561
Q14	1.832	19.880	−0.606	3.630	39.800	−0.504	5.361	59.595	−0.679
Q15	1.830	19.858	−0.716	3.625	39.743	−0.646	5.353	59.503	−0.836
Q16	1.831	19.869	−0.661	3.629	39.788	−0.532	5.358	59.561	−0.738
Q17	1.838	19.945	−0.276	3.642	39.935	−0.163	5.376	59.770	−0.385
Q18	1.832	19.880	−0.606	3.628	39.777	−0.561	5.356	59.537	−0.777

Table 5(a)

HEBT quadrupoles: $B_{measured}$, $B_{calculated}$, and difference $[= 100(B_{measured} - B_{calculated})/B_{measured}]$

Quad	$I_{measured} = 20$ A, $B_{calculated} = 1.84255$ kG		$I_{measured} = 40$ A, $B_{calculated} = 3.64810$ kG		$I_{measured} = 60$ A, $B_{calculated} = 5.39546$ kG	
	$B_{measured}$ (kG)	Diff (%)	$B_{measured}$ (kG)	Diff (%)	$B_{measured}$ (kG)	Diff (%)
AQ1	1.836	−0.357	3.640	−0.222	5.372	−0.437
AQ2	1.833	−0.521	3.637	−0.305	5.367	−0.530
AQ3	1.835	−0.411	3.638	−0.278	5.374	−0.399
AQ4	1.834	−0.466	3.637	−0.305	5.366	−0.549

Table 5(b)

HEBT quadrupoles: $B_{measured}$, $I_{calculated}$, and difference $[= 100(I_{calculated} - I_{measured})/I_{calculated}]$

Quad	$I_{measured} = 20$ A			$I_{measured} = 40$ A			$I_{measured} = 60$ A		
	B_{meas} (kG)	I_{calc} (A)	Diff (%)	B_{meas} (kG)	I_{calc} (A)	Diff (%)	B_{meas} (kG)	I_{calc} (A)	Diff (%)
AQ1	1.836	19.923	−0.385	3.640	39.912	−0.220	5.372	59.723	−0.463
AQ2	1.833	19.891	−0.550	3.637	39.878	−0.305	5.367	59.665	−0.561
AQ3	1.835	19.912	−0.440	3.638	39.890	−0.277	5.374	59.746	−0.424
AQ4	1.834	19.901	−0.495	3.637	39.878	−0.305	5.366	59.654	−0.581