1. Introduction

It is proposed to build a target for beam line 2A that would be capable of accepting a 100 μA beam of 500 MeV protons. However, the target is designed as an annulus of width 5 mm about a central radius of 11.5 mm. Consequently, the incident beam is also required to have that shape.

The production of such an annular beam has been discussed in ref[1], although in that note the central radius of the annulus was approximately 7.5 mm. This annular beam was produced by two AC steering magnets operating 90° out of phase and located between the two 15° dipoles on the west leg of beam-line 2A. REVMOC[2] calculations indicated that a horizontal (x) deflection of 1.65 mr and a vertical (y) deflection of 2.15 mr was required of the primary beam.

In order to produce an annular beam of the required diameter, REVMOC calculations indicate that 2.6 mr of steering are required in the horizontal plane and 3.6 mr of steering are required in the transverse plane. This amount of steering is readily provided by the TRIUMF standard 4-inch steering magnets when operated at DC currents in Amperes numerically equal to the required deflections in mr. However, because the steering magnets must be operated in an AC mode, the immediate implication is that a laminated magnet must be used—that is, hysteresis and eddy current losses must be considered.

This report presents a study of a design that should be suitable for the generation of an annular beam spot at the targets in beam line 2A. It is assumed that the final magnet will be constructed from laminations.

2. Tests with an existing 4-inch steering magnet

The coil of this magnet has 1000 turns of 0.080-in. square copper conductor and was designed for a maximum current of 5 A DC. Its resistance is given as 6.96 Ω. POISSON[3] gives the stored energy for one quarter of the magnet of 35.9 Joules/m at an excitation of 5,000 A-t or 143.6 J/m for a complete magnet. The magnet iron is 6 in. = 0.1524 m long; consequently, its stored energy is estimated to be 21.88 J at full excitation. The stored energy S, inductance L, and current I are related by

$$S = \frac{L\,I^2}{2},$$

and from this the inductance may be estimated as

$$L = \frac{2\,S}{I^2} = \frac{2(21.88)}{(5)^2} = 1.75 \text{ H}.$$

Assuming this value for the inductance of the magnet, its inductive reactance at 60 Hz is then

$$X_L = \omega L = 2\pi(60)(1.75) = 659.734\ \Omega.$$

Thus the impedance of the magnet at 60 Hz is

$$Z = \sqrt{6.86^2 + 659.734^2} = 659.771\ \Omega,$$

that is, the magnet may be considered as a pure inductance.

As a check of this calculation, an existing 4-inch steering magnet was connected to a Variac and the voltage across, and current through, the magnet were (rather imprecisely) monitored. At RMS voltages of 110 V and 51 V the measured RMS currents were 200 mA and 100 mA, respectively. Then, assuming that the magnet may be considered as a resistance of $R = 6.96\ \Omega$ and an inductance L in series, the inductance may be calculated from $Z = V/I$ with $Z = \sqrt{X_R^2 + X_L^2}$. Thus

$$X_L = \sqrt{\frac{V^2}{I^2} - X_R^2} = \begin{cases} \sqrt{(110/0.2)^2 - (6.96)^2} & = 549.956\ \Omega, \\ \sqrt{(51/0.1)^2 - (6.96)^2} & = 509.953\ \Omega, \end{cases}$$

and with $X_L = \omega\, L = 376.991\, L$, we have

$$L = \begin{cases} 1.459 \text{ H.} \\ 1.352 \text{ H.} \end{cases}$$

These inductance values are approximately 20% lower than that calculated from the stored-energy consideration above. Consequently, it is reasonable to assume that the inductance of the existing 4-inch steering magnets is of the order of 1.5 H. It is clear that in order to drive an existing steering magnet with a peak current of 4 A, a peak AC voltage of $(110\sqrt{2})(4/\sqrt{2})/0.2 = 2{,}200$ V would be required.

Were such a voltage used, the resistive power loss in the coil would be

$$P = I_{RMS}^2 R = \left[\frac{I_{peak}}{\sqrt{2}}\right]^2 R = \frac{4^2(6.86)}{2} = 54.9 \text{ W},$$

and the reactive power required at 60 Hz would be

$$Q = X_L I_{RMS}^2 = \omega L I_{RMS}^2 = \omega L \left[\frac{I_{peak}}{\sqrt{2}}\right]^2 = \frac{(2\pi(60)1.5)4^2}{2} = 4.52 \text{ kvar}.$$

However, because eddy-current losses are proportional to the thickness of the laminations, a single 6 inch thick 'lamination' would be expected to have too large a loss of this type. Consequently, an alternate magnet design was sought.

3. An alternate design for the steering magnet

One way to modify the existing design is to decrease the number of turns in the coil and increase the current supplied to it. A quantity of the 0.162-inch, square conductor that was used for the 4-inch steering magnets and mini-quadrupoles of the HEBT is available to construct coils for these new steering magnets. This water-cooled conductor is capable of being driven at a DC current of 100 A. Consequently, a first attempt at a redesign was attempted using this conductor.

The coil of the existing design was replaced with a coil eight turns wide by six turns high of the 0.162-inch conductor. Operation at 100 A peak thus would produce 4,800 A-t of excitation, thus emulating that of the existing steering magnets. The proposed conductor is coated with a nominal thickness of 0.011-inch double Dacron glass (DDG) insulation. Assuming that a single layer of 0.007-inch thick fiberglass tape is wound with a 0.25-inch spacing on the conductor (as with the HEBT steering magnets), the nominal thickness of the conductor plus insulation will be

$$\begin{aligned} d_{nom} &= \text{Conductor dimension} + 2(\text{DDG thickness} + \text{Fiberglass thickness}) \\ &= 0.162 \text{ in.} + 2(0.011 \text{ in.} + 0.007 \text{ in.}) = 0.198 \text{ in.} \end{aligned}$$

Thus, ignoring a ground wrap of 0.028 in., the coil dimension will be approximately 1.60 inches wide by

1.20 inches high. The pole shape and air gap of the existing design were maintained. The thickness of each of the to and bottom yokes were increased from 2.75 inc. to 3.00 in. to make them the same thickness as the outer yokes. Overall outer dimensions were modified to suit the new coil design. A quarter section of the new design is shown in figure 1. Each step on the pole edge is 0.0625 inch deep; the width of the step closest to the midplane is 0.50 inch and that of the other step is 1.00 in.

A POISSON run was made with this particular design. At an excitation of 4,800 A-t the stored energy for the quarter section of a magnet 6 inches long was predicted to be 20.897 J. Thus, assuming a peak excitation current of 100 A, the predicted inductance of the magnet is

$$L = \frac{2S}{I^2} = \frac{2(20.897)}{(100)^2} = 4.179 \text{ mH}.$$

3.1 Resistive losses of the alternate design

The resistance of a coil may be estimated from the following considerations. We take the minimum bending radius of the conductor, R_{min}, to be approximately four time its nominal dimension. Thus

$$R_{min} = 0.75 \text{ in.}$$

We assume a gap G between the coil and the pole and yokes of the magnet and round the pole ends with radii R_{pole} of

$$R_{pole} = R_{min} - G$$

to ensure a constant pole-coil gap. Then, at the center of the first turn, the radius of bend R_1 is

$$R_1 = R_{min} + d_{nom}/2$$

and that at the center of the nth turn R_n is

$$\begin{aligned} R_n &= R_{min} + d_{nom}/2 + (n-1)d_{nom} \\ &= R_{min} - d_{nom}/2 + nd_{nom}. \end{aligned}$$

The pole is square, each side having a (nominal) length of L_{pole}. Consequently, the lengths of the straight sides of the pole are

$$L_{str} = L_{pole} - 2R_{pole}$$

and the length of the nth turn of conductor is

$$l_n = 4L_{str} + 2\pi R_n.$$

Consequently, the length of a layer of N turns is

$$\begin{aligned} L_N = \sum_{n=1}^{N} l_n &= \sum_{n=1}^{N} 4L_{str} + 2\pi R_n \\ &= \sum_{n=1}^{N} 4[L_{pole} - 2R_{pole}] + 2\pi[R_{min} - d_{nom}/2 + nd_{nom}] \\ &= 2N[2(L_{pole} - 2R_{pole}) + \pi(R_{min} - d_{nom}/2 + (N+1)d_{nom}/2)]. \end{aligned}$$

Inserting the following values into this relation

$$\begin{aligned} R_{min} &= 0.750 \text{ in.} & L_{pole} &= 6.000 \text{ in.} \\ G &= 0.125 \text{ in.} & d_{nom} &= 0.200 \text{ in.} \\ R_{pole} &= 0.625 \text{ in.} & N &= 8 \end{aligned}$$

we find the length of an eight-turn layer to be

$$\begin{aligned} L_8 &= 2(8)[2(6.000 - 2(0.625)) + \pi(0.75 - (0.200)/2 + 9(0.200)/2)] \\ &= 16[9.50 + \pi(0.65 + 0.9)] \\ &= 16(14.3695) = 229.915 \text{ in.} \end{aligned}$$

We take the length of an eight-turn layer to be

$$\boxed{L_8 = 240 \text{ in.} = 20 \text{ ft.}}$$

and the length of conductor per coil is estimated to be

$$\boxed{L_{coil} = 6L_8 = 1{,}440 \text{ in.} = 120 \text{ ft.}}$$

The weight of the conductor is given as 0.07473 lb/ft and the resistance at 20°C, R_{20}, is given as 421.1×10^{-6} Ω/ft. Consequently, we estimate

$$\boxed{\begin{aligned} \text{Weight per coil} &= 9.00 \text{ lb} \\ R_{20} \text{ per coil} &= 50.53 \text{ m}\Omega \end{aligned}}$$

Allowing a 30°C rise above ambient of the coil we have

$$\begin{aligned} R_{hot} &= R_{20C}\,[1 + (\text{Temp. coeff}/^\circ\text{C})\Delta T\,(^\circ C)] \\ &= 0.05053[1 + 0.00393(30)] \\ &= 0.0565\ \Omega \text{ per coil.} \end{aligned}$$

Then the peak voltage V_{peak} required for a peak current I_{peak} of 100 A is

$$V_{peak} = I_{peak}R_{hot} = 5.65 \text{ V/coil}$$

and the power dissipated in resistive loss is

$$P_{res} = I^2_{RMS}R = \frac{100^2(0.0565)}{2} = 282.5 \text{ W/coil.}$$

$$\boxed{\text{Resistive power loss} = 0.285 \text{ kW per coil.}}$$

This power is approximately 60% of that calculated in ref[4] for the power loss in the DRAGON 6-inch steering magnets. Given that no cooling problem was found in those magnets, it is felt safe to say that no cooling problem should arise with this new design of steering magnets.

Ignoring line loss, we then have for the *peak* resistive power loss of each magnet

$$\boxed{\begin{aligned} I_{peak} &= 100.0 \text{ A} \\ V_{peak} &= 11.3 \text{ V} \\ P_{peak} &= 0.6 \text{ kW} \end{aligned}}$$

3.2 Core laminations and iron losses

In this section, we follow the treatment of Otter[5] in the calculation of steel quantity, and hysteresis and eddy-current losses. We will also use some of the data given in that report. To set the scene, we quote the following from the leading paragraph of §2 of ref[5].

"... will be built from an M17 steel, using 26 gauge laminations. In the newer terminology this would be termed 47F 168. It is a non-grain oriented steel which is commonly used in transformers in which minimum core loss is not the paramount design parameter. It has the lowest core loss for non-oriented grades. Use of an oriented steel would reduce core losses by a factor between 2 and 3 but the thinner laminations would lead to a higher fabrication cost. ..."

The calculations of Otter were made for a dipole that had a peak flux density of 1.25 T in the steel and one of 1.05 T in the gap. POISSON predictions for the steering magnet considered here indicate a maximum flux density of approximately 0.45 T at the pole root and a peak gap field of approximately 0.112 T at an excitation of 4,800 A-t. (However, the average flux density in the steel is closer to 0.15 T.) Because these values for the steering magnet are considerably lower than those for the dipole of ref[5)], it is felt that the non-oriented steel should be used for the laminations.

3.2.1 Quantity of steel

We assume that the magnet will be constructed according to the dimensions shown in figure 1. With the further assumption of an allowance for punching of 1.0 inch on all sides, the *gross* dimensions of a lamination would be

$$\begin{aligned} \text{Width} &= 2(7.85\text{ in.} + 1.0\text{ in.}) = 17.7\text{ in.} \\ \text{Height} &= 2(6.8125\text{ in.} + 1.0\text{ in.}) = 15.625\text{ in.} \end{aligned}$$

Thus the gross area of a lamination is (17.1 in.)(15.625 in.) = 276.56 in.^{2}. For a magnet length of 6 inches and a lamination factor of 0.95, the number of laminations of 26 gauge steel (0.47 mm thick) per magnet N_l is

$$N_l = \frac{(6\text{ in.})(25.4\text{ mm/in.})(0.95)}{0.47\text{ mm}} = 308$$

and, at a density of 7.6 gm/cc, the weight of each lamination W_l is

$$W_l = (17.7\text{ in.})(2.54\text{ cm/in.})(15.625\text{ in.})(2.54\text{ cm/in.})(0.047\text{ cm})(7.6\text{ gm/cc}) = 637.34\text{ gm} = 1.405\text{ lb.}$$

and the gross weight of steel per magnet W is

$$W = (308\text{ laminations})(1.405\text{ lb./lamination}) = 432.8\text{ lb.}$$

We take

Gross weight of steel per magnet = 435 lb.

To calculate the *net* weight of steel we find from figure 1 that the area of one-quarter of the magnet A_4 is

$$\begin{aligned} A_4 &= (7.85\text{ in.})(3.00\text{ in.}) + (3.8125\text{ in.})(3.00\text{ in.}) + (1.625\text{ in.})(3.00\text{ in.}) \\ &\quad + (0.0625\text{ in.})(1.00\text{ in.}) + (0.0625\text{ in.})(0.50\text{ in.}) \\ &= 39.956\text{ in.}^2 \end{aligned}$$

and the net weight per lamination w_l becomes

$$w_l = 4(39.956\text{ in.}^2)(2.54\text{ cm/in.})^2(0.047\text{ cm})(7.6\text{ gm/cc}) = 368.316\text{ gm} = 0.812\text{ lb.}$$

Thus we estimate the net weight of steel per magnet to be

Net weight of steel per magnet = 250 lb.

3.2.2 Hysteresis losses

Otter gives the core loss for 26 gauge, M17 steel as 0.193 W/lb at a frequency of 60 Hz and 0.475 T. Figure 2 shows the POISSON prediction for the field distribution in the iron yoke at an excitation of 4,800 A-t. There is clearly a variation of flux density throughout the yoke, but a value of 0.475 T is not an unreasonable figure to take as a nominal peak value. Figure 3 of ref[6)] indicates that at that field the ratio of hysteresis loss to total loss for this steel is approximately 66%. On this basis we estimate the hysteresis loss to be 0.66(0.193 W/lb) = 0.1274 W/lb for 26 gauge M17 steel at 60 Hz and 0.475 T. Thus the total hysteresis loss per magnet under these conditions is estimated to be (0.1274 W/lb)(250 lb) = 31.85 W.

Total hysteresis loss per magnet = 35 W.

3.2.3 Eddy current losses

From the above, the eddy current losses are expected to be approximately 34% of the total losses or 0.34(0.193 W/lb) = 0.0656 W/lb. Thus the total eddy current loss is expected to be (0.0656 W/lb)(250 lb) = 16.4 W. We take

Total eddy-current loss per magnet = 20 W.

3.2.4 AC parameters

Assuming that the resistance and inductance values quoted above are correct, then the *total* resistance of two coils in series is $R_{tot} = 2(0.0565\ \Omega = 0.113\ \Omega$. The inductive reactance of the magnet at 60 Hz is

$$X_L = 2\pi(60\text{ Hz})(4.179\times10^{-3}\text{ H}) = 1.575\ \Omega$$

so that the impedance of the magnet, assuming that it may br treated as a resistance and inductance in series, is

$$|Z| = \sqrt{R_{tot}^2 + X_L^2} = \sqrt{(0.113)^2 + (1.575)^2} = 1.579\ \Omega.$$

Again, under this assumption, it is seen that the magnet may be considered to act almost as a pure inductance. The phase angle ϕ is

$$\phi = \tan^{-1}(X_L/R_{tot}) = 85.896°.$$

The required RMS voltage at a RMS current of $I_{RMS} = 100/\sqrt{2}$ A = 70.7 A is

$$V_{RMS} = |Z|I_{RMS} = (100/\sqrt{2}\text{ A})(1.579\ \Omega) = 111.7\text{ V}.$$

The RMS voltage drop across the resistive portion of the load is

$$V_{R,RMS} = I_{RMS}R_{tot} = (100/\sqrt{2}\text{ A})(0.113\ \Omega) = 8.0\text{ V},$$

and that across the inductive portion of the load is

$$V_{L,RMS} = I_{RMS}X_L = (100/\sqrt{2}\text{ A})(1.575\ \Omega) = 111.4\text{ V}.$$

Thus the active power is

$$P = I_{RMS}V_{R,RMS} = (8.0\text{ V})(70.71\text{ A}) = 566\text{ W},$$

the reactive power is

$$Q = I_{RMS}V_{L,RMS} = (70.71\text{ A})(111.4\text{ V}) = 7.88\text{ kvar},$$

and the apparent power is

$$S = V_{RMS} I_{RMS} = (111.7 \text{ V})(70.71 \text{ A}) = \text{ kVA}.$$

The power factor is the

$$\cos\phi = P/S = (0.566 \text{ kW})/(7.90 \text{ kVA}) = 0.072.$$

This low power factor implies an inefficient transfer of energy to the magnet. The power factor may be improved by adding capacitance in series or in parallel with the steerer. We consider each of these, although a capacitance in parallel with the magnet would be the more probable solution.

3.2.5 Addition of a series capacitance

The magnitude of the impedance $|Z_{series}|$ and phase angle ϕ_{series} of a circuit in which a resistance R, an inductance L, and a capacitance C_{series} are connected in series are given by [7)]

$$|Z_{series}| = \sqrt{R^2 + (X_L - X_{C_{series}})^2} = \sqrt{R^2 + (\omega L - 1/\omega C_{series})^2}$$

and

$$\phi_{series} = \tan^{-1}\left[\frac{X_L - X_{C_{series}}}{R}\right] = \tan^{-1}\left[\frac{\omega L - 1/\omega C_{series}}{R}\right]$$

with $X_L = \omega L$ and $X_{C_{series}} = 1/(\omega C_{series})$. Clearly, if $X_L = X_{C_{series}}$ then $|Z_{series}|$ is minimum and $\phi_{series} = 0$—that is, the circuit behaves as a pure resistance. For this to occur at a given (angular) frequency ω_0 requires

$$\omega_0 L = 1/(\omega_0 C) \qquad \text{or} \qquad \omega_0^2 L C_{series} = 1.$$

Thus, for $R = 0.113\ \Omega$, $L = 4.18$ mH, and a frequency of 60 Hz, the required capacitance is

$$C_{series} = \frac{1}{\omega_0^2 L} = \frac{1}{[2\pi(60)]^2(0.00418)} = 1,633\ \mu\text{f}.$$

$$\boxed{C_{series} = 1{,}633\ \mu\text{f}.}$$

Thus, with these values of R, L, and C_{series}, the power factor is unity and the magnitude of impedance is equal to the resistance of the magnet coils (in this case).

3.2.6 Addition of a parallel capacitance

The magnitude of the impedance $|Z_{para}|$ and phase angle ϕ_{para} of a circuit in which a resistance R and an inductance L in series are paralleled by a capacitance C_{para} are given by [7)]

$$|Z_{para}| = \sqrt{\frac{R^2 + X_L^2}{R^2/X_{C_{para}}^2 + (X_L/X_{C_{para}} - 1)^2}} = \sqrt{\frac{R^2 + \omega_0^2 L^2}{\omega_0^2 C_{para}^2 R^2 + (\omega_0^2 L C_{para} - 1)^2}}$$

and

$$\phi_{para} = \tan^{-1}\left[\frac{X_L(1 - X_L/X_{C_{para}}) - R^2/X_{C_{para}}}{R}\right] = \tan^{-1}\left[\frac{\omega_0[L(1 - \omega_0^2 L C_{para}) - C_{para}R^2]}{R}\right]$$

For $\phi = 0$—or a power factor of unity—we require

$$X_L(1 - X_L/X_{C_{para}}) = R^2/X_{C_{para}} \qquad \text{or} \qquad L(1 - \omega_0^2 L C_{para}) = C_{para} R^2$$

so that for our values of R and L we find at 60 Hz

$$C_{para} = \frac{L}{R^2 + \omega_0^2 L^2} = \frac{0.00418}{(0.113)^2 + (2\pi(60))^2(0.00418)^2} = 1,675\ \mu\text{f}.$$

$$\boxed{C_{para} = 1{,}675\ \mu\text{f}.}$$

We note that $C_{para} \approx C_{series}$. This should be the case, for with $R^2 \ll \omega_0^2 L^2$ (as is the case under discussion) then the above expression for C_{para} reduces to that for C_{series}.

We note in passing that, contrary to the result found for a series RLC circuit, the magnitude of the impedance is at a maximum when the power factor is unity. That this is so may be found by differentiating $|Z_{para}|$ with respect to C_{para}. This is simplified by noting that the dependence on C_{para} of $|Z_{para}|$ only occurs in its denominator

$$D = \omega_0^2 C_{para}^2 R^2 + (\omega_0^2 L C_{para} - 1)^2$$

and so only the derivative of $D^{-1/2}$ with respect to C_{para} need be considered. Thus

$$\begin{aligned} \frac{dD^{-1/2}}{dC_{para}} &= -\frac{1}{2} D^{-3/2} \frac{d}{dC_{para}} [\omega_0^2 C_{para}^2 R^2 + (\omega_0^2 L C_{para} - 1)^2] \\ &= -D^{-3/2} [\omega_0^2 C_{para} R^2 + \omega_0^2 L(\omega_0^2 LC - 1)] \end{aligned}$$

and is zero if

$$C_{para} = \frac{L}{R^2 + \omega_0^2 L^2}$$

as was obtained above. With this value of C_{para} the magnitude of the impedance is found to be

$$|Z_{para}| = \frac{R^2 + \omega_0^2 L^2}{R}$$

4. Stability of the power supply

The stability of the power supply must be such that the beam is not allowed to wander outside of the target volume. Were this to happen, damage to the target container could occur. To estimate the required stability of the power supply several REVMOC runs were made in which the amounts of vertical and horizontal steering were increased and decreased by 1% and 10%.

Figure 3 shows the predicted beam profile at the target entrance with *maximum* vertical (3.6 mr)and horizontal (2.6 mr) steering applied. It is to be noted that this is *not* a physical situation; it was run only to find the amounts of steering necessary to provide an annular beam of the required central radius. For this calculation scattering in the stripper foil *only* was included. The result of a similar calculation that includes scattering caused by the 0.005 inch aluminum window located in the beam line downstream of the last 15° dipole and by the 0.010 inch copper window located in the entrance of the target vessel is shown in figure 4. The effect of these additional scatterers is clear.

However, because we are interested in any shifts of the beam centroid caused by power supply instability, the calculations that follow were made considering scattering in the stripper foil *only*.

The procedure used to estimate the centroid shift caused by power supply instability was as follows. A REVMOC run was first made with each of the horizontal and vertical steering increased by 10% from their nominal values. A second run was made with each steering decreased by 10% from nominal values. Two comparison runs, one with each steering increased to 1% above nominal and one with each steering reduced by 1% from nominal, were also made.

Figure 5 shows the predicted shift of the horizontal beam when steering is increased and decreased by 10% of nominal. In this case, the shift of the beam centroid is seen to be approximately 1.2 mm. Figure 6 shows the centroid shift predicted in the vertical plane for the same variation in vertical steering. Again, a shift of approximately 1.2 mm is predicted.

Figures 7 and 8 show similar data for a variation of $\pm 1\%$ from nominal. From these figures it is difficult to obtain an estimate of the centroid shifts. Consequently, the upper portions of the predicted curves are plotted in figures 9 and 10. From these latter figures, it is seen that a centroid shift of approximately 0.12 mm is predicted in each of the horizontal and vertical planes.

A centroid shift of 1.2 mm may be acceptable if the target is properly designed. However, one of 0.12 mm is probably acceptable under any circumstances. Further, given that the stability of a power supply is usually quoted at full output and given that operation of these magnets is expected to be at most 70% of full output, we feel that the maximum current variation should be at most 1%. A regulation of 0.5% should be the design goal.

5. Discussion

This note has presented a design for an AC steering magnet that would be suitable for the production of an annular beam at either of the beam line 2A production targets. The design presented is a modification of that of the existing 4-inch DC steering magnets. New new coils are specified and the overall dimensions of the magnet have been altered so as to accommodate the new coil.

Each of these magnets would be constructed of 308 laminations of 26 gauge M17 steel. Each magnet is predicted to have a (hot) resistance of 56.5 mΩ per coil and an inductance of approximately 4.2 mH.

Figure 11 shows a quarter section of the final magnet based on the more detailed study given in the appendix. Figure 12 shows the dimensions of a complete magnet. Table 1 summarizes the parameters of the magnet.

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Table 1

Final design parameters of the steering magnet

Yoke:	Length	6.00 in.
	Width	16.00 in.
	Height	13.80 in.
	Lamination thickness (M17, 26 gauge)	0.0185 in.
	Number of laminations	308
	Net area/lamination	163.18 in.^{2}
	Weight/lamination	0.83 lb
	Total weight of iron/magnet	256.0 lb
Coil:	Conductor	0.162 in. square
	Turn configuration	8 wide × 6 high
	Nominal width	1.75 in.
	Nominal height	1.40 in.
	Length per coil	122.0 ft.
	Weight per coil	9.12 lb
	Resistance (hot) per coil	57.43 mΩ
	Resistance (hot) per magnet	114.86 mΩ
Cooling:	Flow per coil	0.030 USGPM
	Flow per magnet	0.060 USGPM
Power:	Maximum current I_{RMS}	70.71 A
	Operating frequency f_{oper}	60 Hz
	Total resistance per magnet R_{tot}	0.115 Ω
	Stored energy	134.0 J/m
	Stored energy per magnet	20.4 J
	Inductance L at I_{peak}	4.08 mH
	Inductive reactance X_L at f_{oper}	1.538 Ω
	Inductive voltage $V_{L,RMS} = I_{RMS} X_L$	108.75 V
	Impedance $\|Z\| = \sqrt{R_{tot}^2 + X_L^2}$ at f_{oper}	1.542 Ω
	Phase angle $\phi = \tan^{-1}(X_L/R_{tot})$	85.73°
	Maximum voltage $V_{RMS} = I_{RMS}\|Z\|$	109.04 V
	Active power $P = I_{RMS}^2 R_{tot}$	0.58 kW
	Reactive power $Q = I_{RMS} V_{L,RMS}$	7.69 kvar
	Apparent power $S = V_{RMS} I_{RMS}$	7.71 kVA
	Power factor $\cos\theta = P/S$	7.52×10^{-2}
	Series capactiance for $\cos\theta = 1$ $C_{series} = 1/(\omega^2 L)$	1,725 μf
	Parallel capactiance for $\cos\theta = 1$ $C_{para} = L/(R_{tot}^2 + \omega^2 L^2)$	1,715 μf

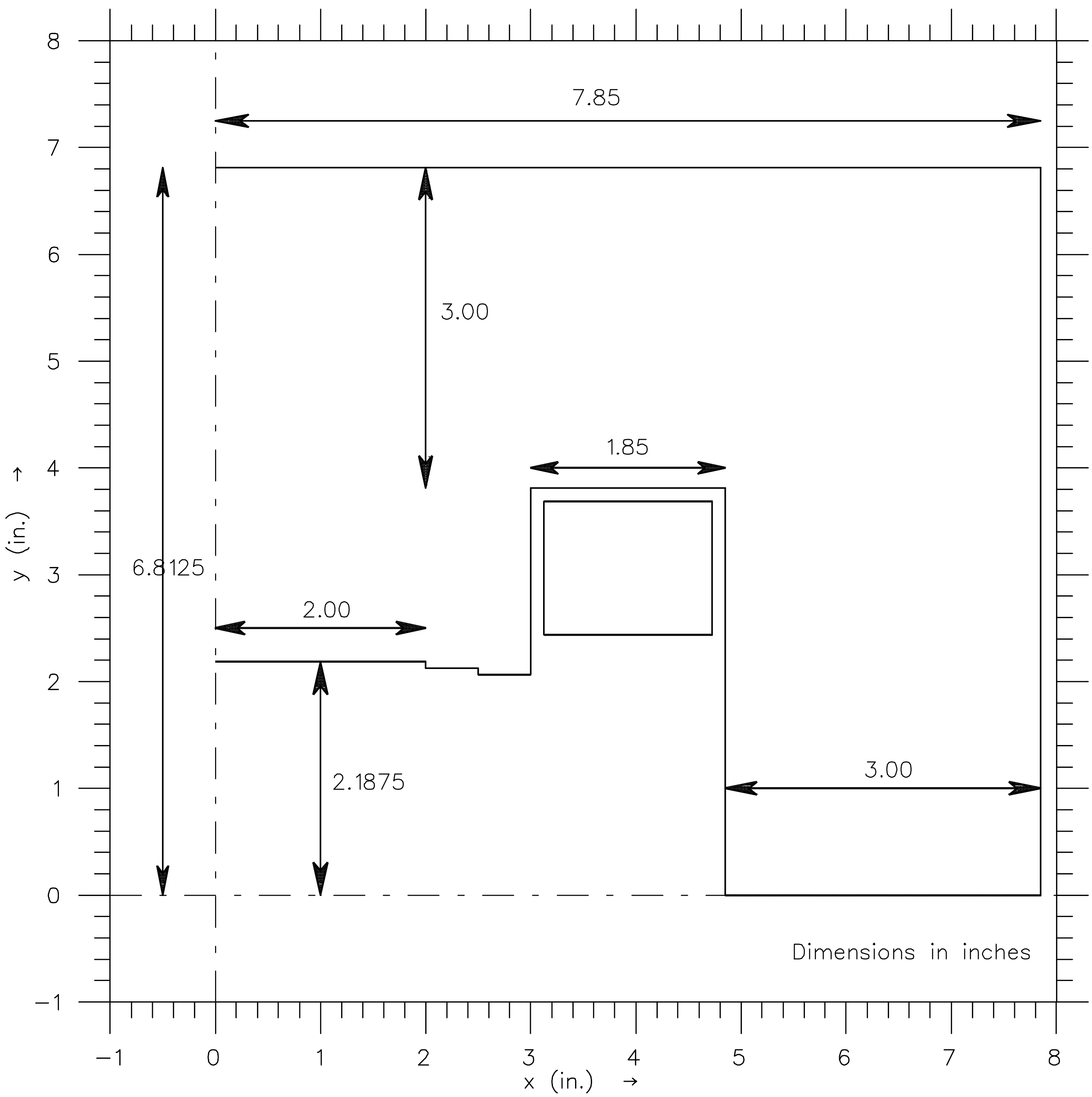

Fig. 1. The proposed design for a 4-in., AC steering magnet for beam line 2A.

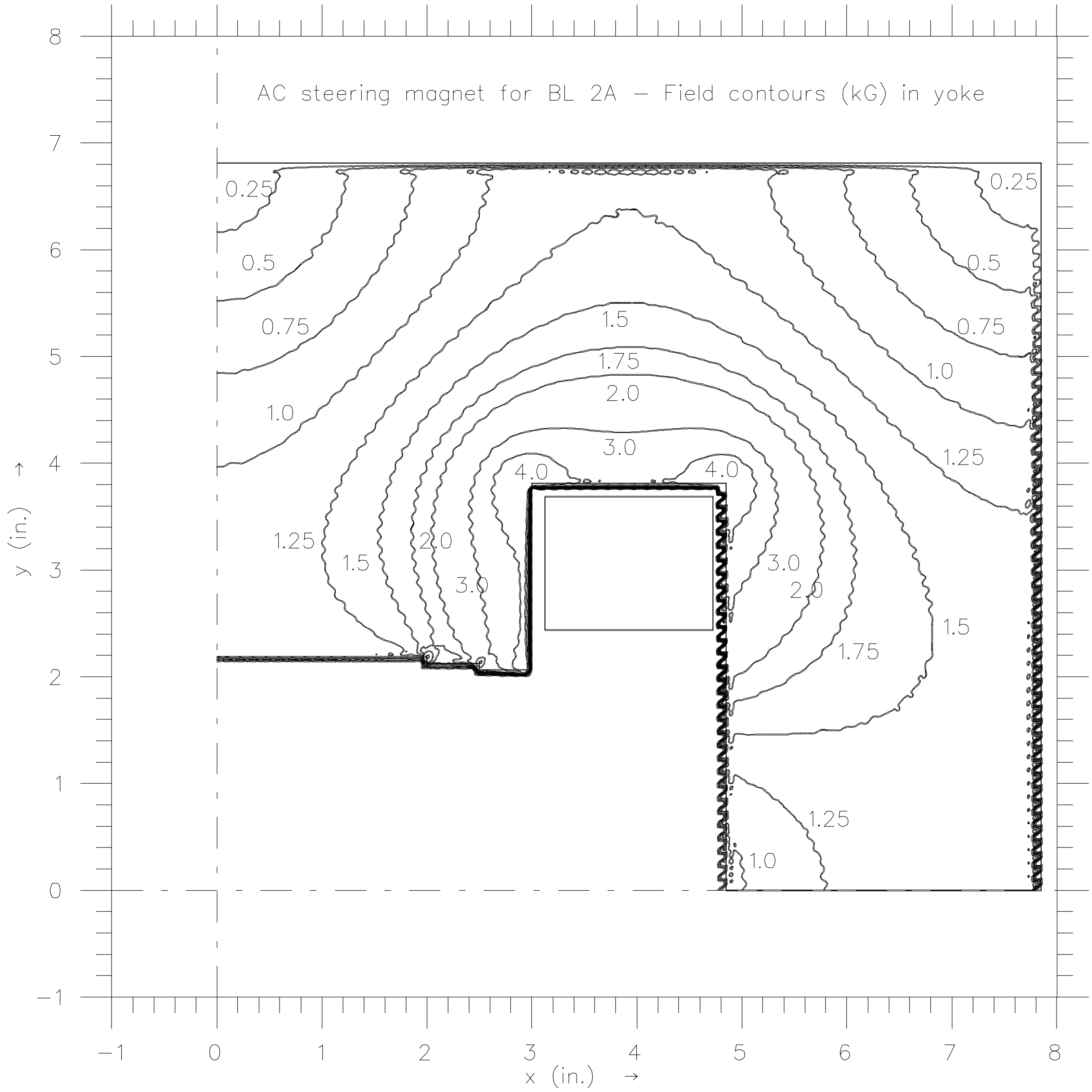

Fig. 2. Contour plot of the flux distribution in the yoke of the proposed steering magnet.

```
            01/01/12 - 2A - 2.6 mr X & 3.6 mr Y - RBennett - X NOM, Y NOM

Space # 1:  Distribution of particles as a function of  X  AT TGTIN  (Element #126)     (along HORIZONTAL axis)
                                                      & Y  at TGTIN  (Element #126)     (along VERTICAL axis)

REAL! Distribution of FINAL RUN FOUND HERE
                                                   COUNTS  = 14999.09
                                                     X PROJECTION

                                                 1  1  1  1  1  1  1  1
                                        2  4  7  1  3  5  7  7  6  4  1  8  5  2
                                     7  2  7  3  3  1  9  7  3  1  4  4  7  3  4  7
                0  0  0  0  0  3  3  6  1  8  3  1  3  8  2  7  5  5  0  1  6  6  6  4  1  0  0  0  0  0  0
             +--|--------------|--------------|--------------|--------------|--------------|--------------|+
   1.513     -                                                                                             -       0
   1.488     |                                                        1                                    |       1
   1.462     |                                                     1                                       |       1
   1.438     |                                                  1     1                                    |       2
   1.413     |                          1     1           2        1                                       |       5
   1.388     |                                   5  2  3  2  5  5  5  1  4  2                              |      34
   1.363     |                             3  5 12  6  8 11  6 10  9  6  4  2  1                           |      83
   1.337     |                 1     1  3 12  9 11 20 23 26 22 19 28 15  9  9  4  1                        |     213
   1.313     |                       2  7 10 14 28 37 32 41 47 32 34 23 24 10 12  2     1                  |     356
Y  1.288     |                       2  5 19 33 30 43 64 55 64 56 44 36 27 21  8  6  1                     |     514 Y
   1.263     -                       3  9 23 31 71 62 74 94 81 80 75 48 40 21  8  4                        -     724
   1.237     |                       4  9 36 42 68 90 94102109 94 90 59 54 26 11  8                        |     896 P
A  1.212     |                       5 16 35 52 77 75115144110120 90 89 65 49 19  3                        |    1064 R
T  1.188     |                    1  8 16 35 69 91105127149136135123 91 72 38 22  7                        |    1225 O
   1.163     |                      10 24 39 72 97121149157140138108 90 77 34 21  6                        |    1283 J
T  1.138     |                 1     7 19 51 76 86118134148175141129115 68 56 19  8  1                     |    1352 E
G  1.112     |                       7 20 37 67 99122149138148134131 87 71 55 23  4                        |    1292 C
T  1.087     |                 1  1  7 20 35 63 97108137168137130104 98 77 46 29  7                        |    1265 T
I  1.063     |                       4 28 37 55 83 96109133143132127 88 78 50 14  6                        |    1183 I
N  1.038     |                       8 12 37 43 80 89115117121113 90 82 56 33 16  4  1                     |    1017 O
   1.013     -                    1  4 13 16 43 71 85 81 89 99 88 84 83 50 32 13  6  1                     -     859 N
  0.9875     |                       1  8 24 27 52 61 67 90 85 66 78 50 30 18  2  1                        |     660
  0.9625     |                       2  6 14 15 35 43 58 38 43 57 42 37 33 11 13  1                        |     448
  0.9375     |                          2  7 13 24 16 37 38 41 34 31 22 10 12  5  1                        |     293
  0.9125     |                          2  5  2 10  9 15 21 15 21 16 14 15  6  3  1                        |     155
  0.8875     |                       1  1  1  1  3  5  6  8  5  6  4  2  7  4  3                           |      57
  0.8625     |                             1              1  5  3     2                                    |      12
  0.8375     |                             1           1                 1                                 |       3
  0.8125     |                                                                                             |       0
  0.7875     |                                                                                             |       0
  0.7625     -                                   1              1                                          -       2
             +--|--------------|--------------|--------------|--------------|--------------|--------------|+
                0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
                .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
                7  7  8  8  8  8  9  9  9  9  0  0  0  0  1  1  1  1  2  2  2  2  3  3  3  3  4  4  4  4  5
                7  9  2  4  7  9  2  4  7  9  2  4  7  9  2  4  7  9  2  4  7  9  2  4  7  9  2  4  7  9  2
                2  7  2  7  3  7  3  7  2  8  3  8  2  7  2  8  3  7  2  7  3  8  2  7  3  8  3  7  2  8  3
                                                     X  AT TGTIN
```

Fig. 3. The beam profile predicted by POISSON with 3.6 mr of vertical and 2.5 mr of horizontal steering. Scales along vertical and horizontal axes are cm. Scattering in stripper foil only is taken into account.

```
      01/01/12 - 2A - 2.6 mr X & 3.6 mr Y - RBennett - X NOM, Y NOM - Windows

   Space # 1:  Distribution of particles as a function of  X  AT TGTIN  (Element #126)     (along HORIZONTAL axis)
                                                         & Y  at TGTIN  (Element #126)     (along VERTICAL axis)

   REAL! Distribution of FINAL RUN FOUND HERE
                                                    COUNTS  = 14963.3
                                                      X PROJECTION

                                                    1  1  1  1  1  1  1
                                       1  2  4  6  9  2  5  6  6  5  3  0  8  5  3  1  1
                1        1  2  4  9  6  7  5  7  8  4  1  1  4  3  6  9  4  6  4  8  0  6  3  1           1
                6  6  6  4  0  6  7  5  5  0  1  9  8  2  6  1  5  6  8  0  3  5  1  6  3  9  5  9  3  7  7
             +--|--------------|--------------|--------------|--------------|--------------|--------------|+
   1.614     -  4        1  0  0  1     0           2     0     0  1  1     1        3                    4-      21
   1.582     |              1                             1     1  1        1                              |       5
   1.550     |                 1           2     2  2  1     1     3  1  1        1        1              1|      17
   1.518     |                       1     1     2  2  1  1  2  6        1     1  1  1                     |      20
   1.486     |           1        1  1  2  2  1  5  4  2  4  5  4  2  1  4     1  1  2     2  1            |      46
   1.454     |                       3  3     6  3  8  4  4 14  5  4  4  3     2        1           1      |      65
   1.422     |                 3  2  2  4  6  4  5  6 16 10 15  8  7  6  7  5  3  3     1     1            |     114
   1.390     |              1  3  1  4  6  6 13 12 21 19 24 19 14 18 14 12  8  7  3  1  1  1               |     208
   1.358     |  1     1     1  1  2  4  4  8  8 24 19 38 33 39 20 36 25 18  5  8  4  1     1     1         |     303
Y  1.326     |        1     1  1  1  6  6 13 21 33 39 75 46 38 54 55 31 22 12  8  2  6  2  1  2           1|     477 Y
   1.294     -        1     1  1  5  7 13 23 42 40 66 62 80 83 69 60 48 36 31 21 12  6  4  3     1         -     715
   1.262     |     2     3        6  8 11 27 41 50 81 91 86 81 92 68 57 52 35 19 12  5  1  3           2  1|     834 P
A  1.230     |        1  1  1  2  8  9 16 21 58 81 97112108143101 97 88 71 38 26 11  6  1     2  2        1|    1102 R
T  1.198     |              1  3  9  9 16 41 54 86 99103143148119102 83 78 42 27 14 12  3  5  1        2   |    1200 O
   1.166     |           1  1  4  4 13 28 43 57 73118140146150137124111 59 50 28 16 14  5  5  3     1      |    1331 J
T  1.134     |  2  1     1  2  7 11 21 31 48 62 95108157154150152132 90 88 49 40 20 10  5  2              1|    1439 E
G  1.102     |        1  2  1  6  8 17 18 36 68102101141146169157135106 70 56 21 16  4  6  1  3            |    1391 C
T  1.070     |  2  1  1     5  4  7 15 30 50 59 83103140161122130122105 81 52 29 14  8  4  2     2        1|    1333 T
I  1.038     |              1  1  6  8 13 28 55 66 94118119114137 92 90 64 46 24 13  9  7  2     2     2  0|    1111 I
N  1.006     |                    6 12 18 31 26 71 95 94106 96 87 95 71 49 41 15 13 10  5  2              1|     944 O
  0.9740     -  1        2  1  4  5  5 18 22 33 52 58 64 80 88 75 73 52 42 30 15  3     1  2        1      -     727 N
  0.9420     |     1     1  1  1  1  9  8 10 15 39 50 57 58 52 69 52 33 33 27 21  7     4  2  1            |     552
  0.9100     |  0              1  7  5  7 12 15 29 30 29 45 37 43 30 33 16 12 11  5  5  5                  |     377
  0.8780     |                 1  2     8 12  9 12 22 25 37 31 22 25 26 12 12  8  2  1  2  1               |     270
  0.8460     |                    1  1  6  3 13  8 11 10 14 25 13 14 11 11  4  3  3     2  1              0|     154
  0.8140     |                    1  4  3  2  8  8  7  6  4  8 11  8  7  7  2  2  2        1  1  1         |      93
  0.7820     |     1           1        2  2  2  4  2  1  3  3  4  8  3  1  2  1     1                 1  1|      43
  0.7500     |                          1        4     4  2  3  2  1  1     1     1  1  2                  |      23
  0.7180     |           1     1  1  1              1     1     2        1     2        1                  |      12
  0.6860     |  0                       1                    3     1        1     2                        |       8
  0.6540     -  5        0  0     1  0  1  1  1  0  2  2     2  1        1     2           1              4-      26
             +--|--------------|--------------|--------------|--------------|--------------|--------------|+
                0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
                .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
                6  6  7  7  7  8  8  8  9  9  9  0  0  0  1  1  1  2  2  2  3  3  3  4  4  4  4  5  5  5  6
                6  9  2  6  9  2  5  8  2  5  8  1  4  8  1  4  7  0  4  7  0  3  6  0  3  6  9  2  6  9  2
                4  6  8  0  2  4  6  8  0  2  4  6  8  0  2  4  6  8  0  2  4  6  8  0  2  4  6  8  0  2  4
                                                      X  AT TGTIN
```

Fig. 4. The beam profile predicted by POISSON with 3.6 mr of vertical and 2.5 mr of horizontal steering. Scales along vertical and horizontal axes are cm. Scattering in stripper foil and in 0.005 inch Al and 0.010 inch Cu windows are taken into account.

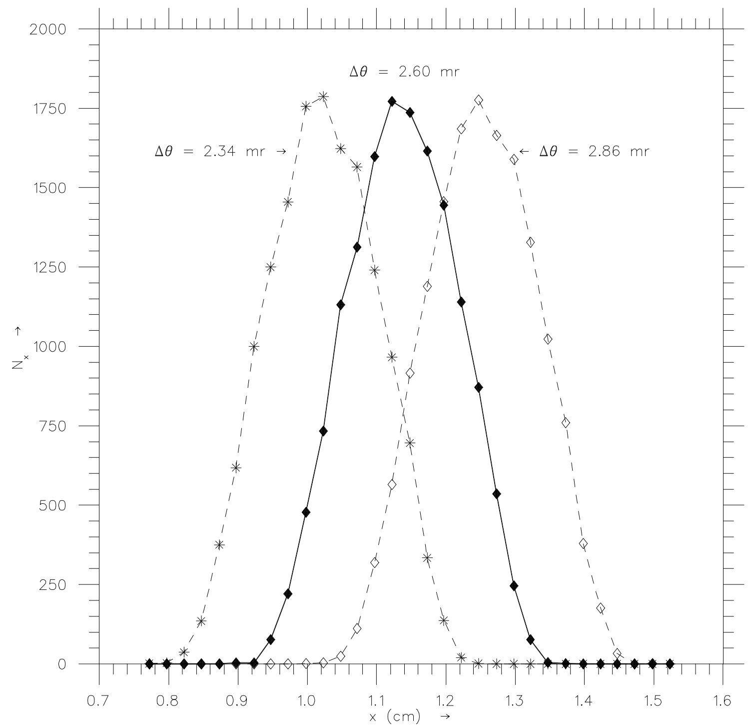

Fig. 5. The effect of 10% changes in horizontal steering on the horizontal beam position.

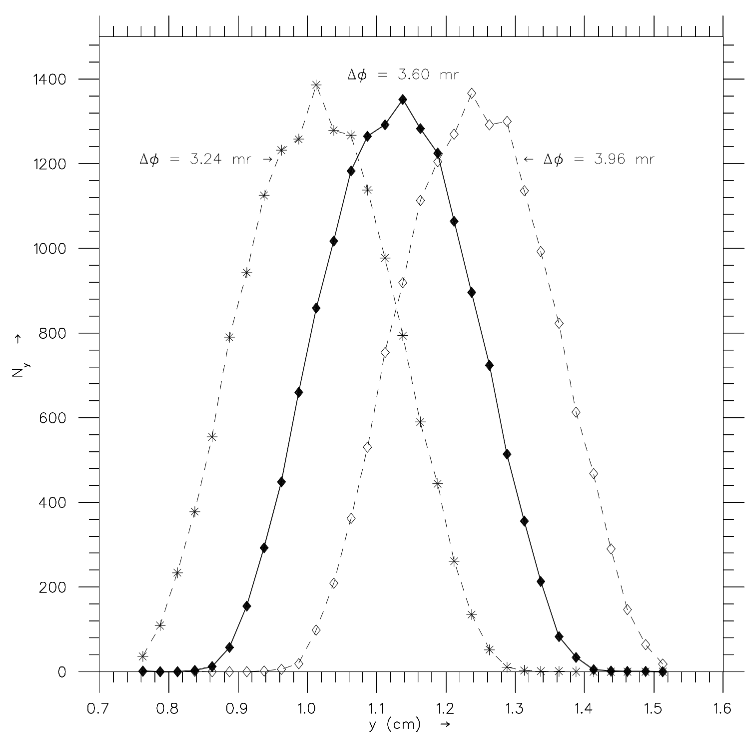

Fig. 6. The effect of 10% changes in vertical steering on the vertical beam position.

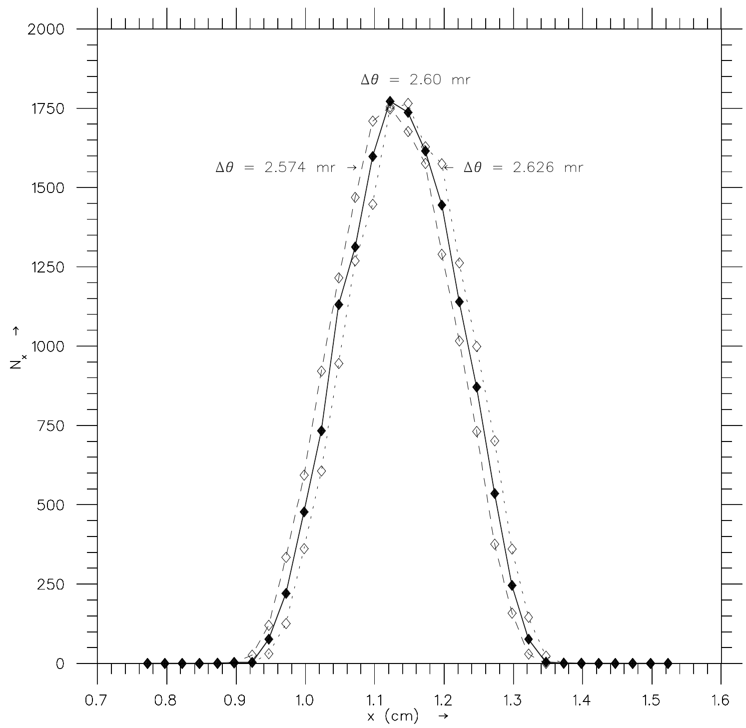

Fig. 7. The effect of 1% changes in horizontal steering on the horizontal beam position.

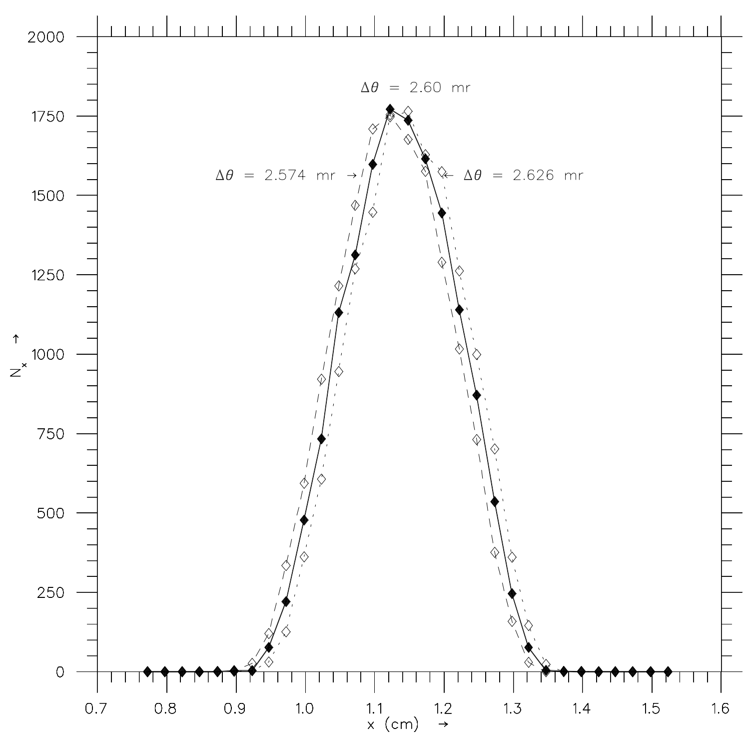

Fig. 8. The effect of 1% changes in vertical steering on the vertical beam position.

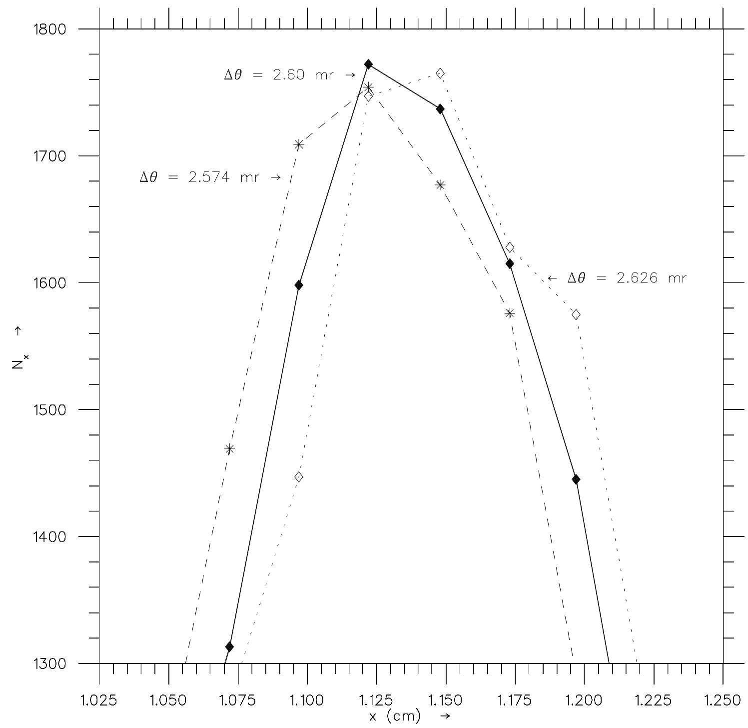

Fig. 9. The effect of 1% changes in horizontal steering on the horizontal beam position.

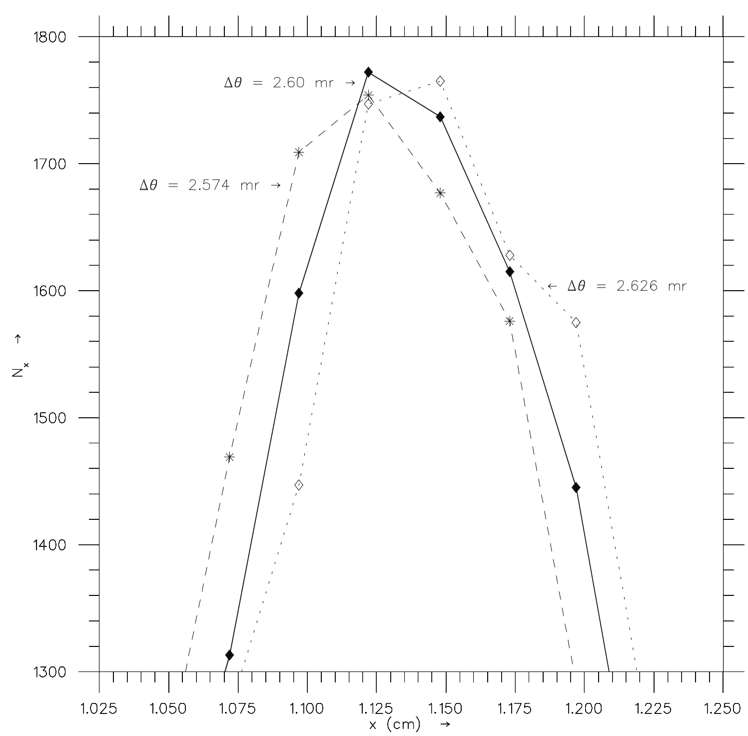

Fig. 10. The effect of 1% changes in vertical steering on the vertical beam position.

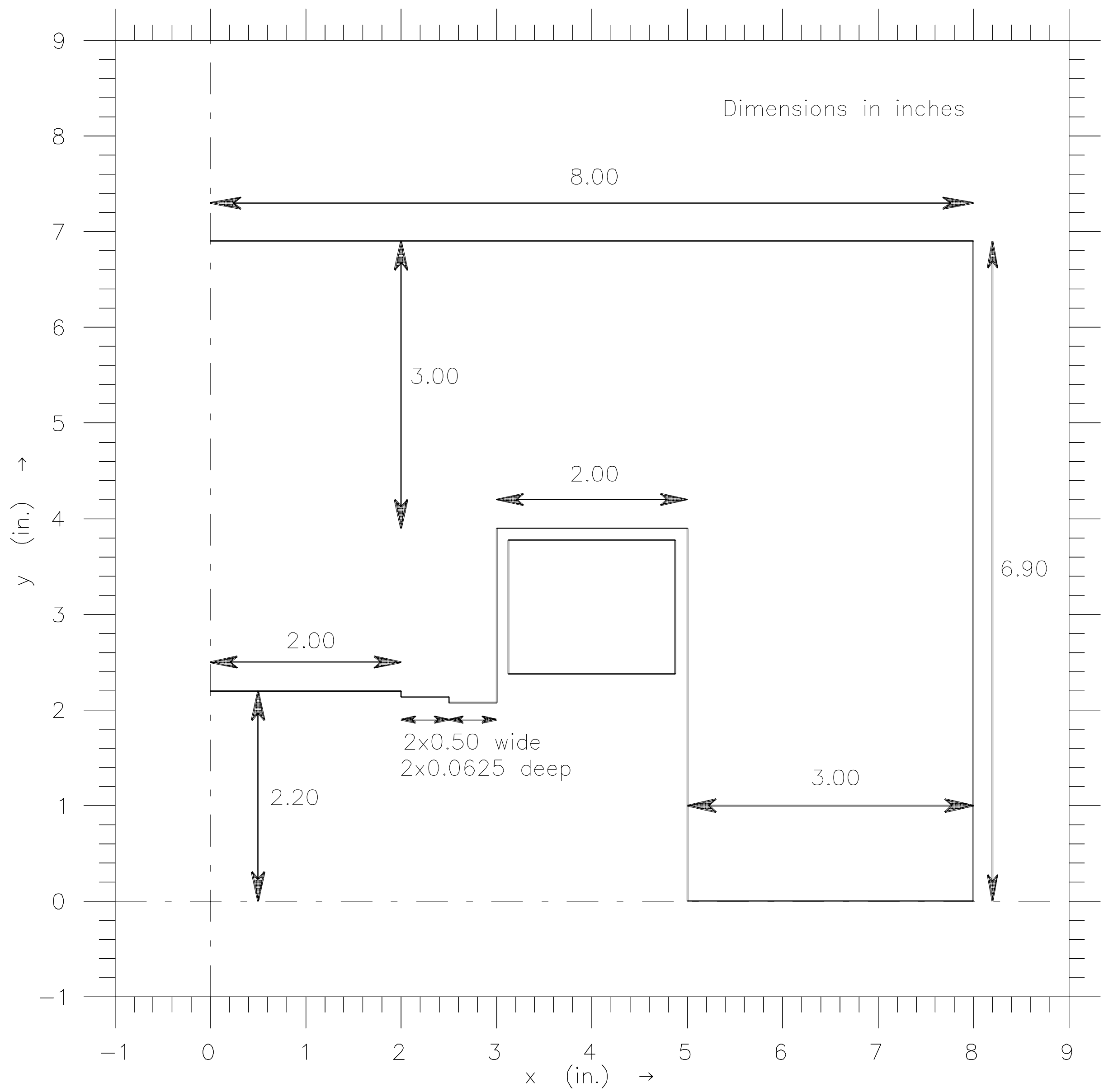

Fig. 11. A quarter section of the proposed design of the final magnet.

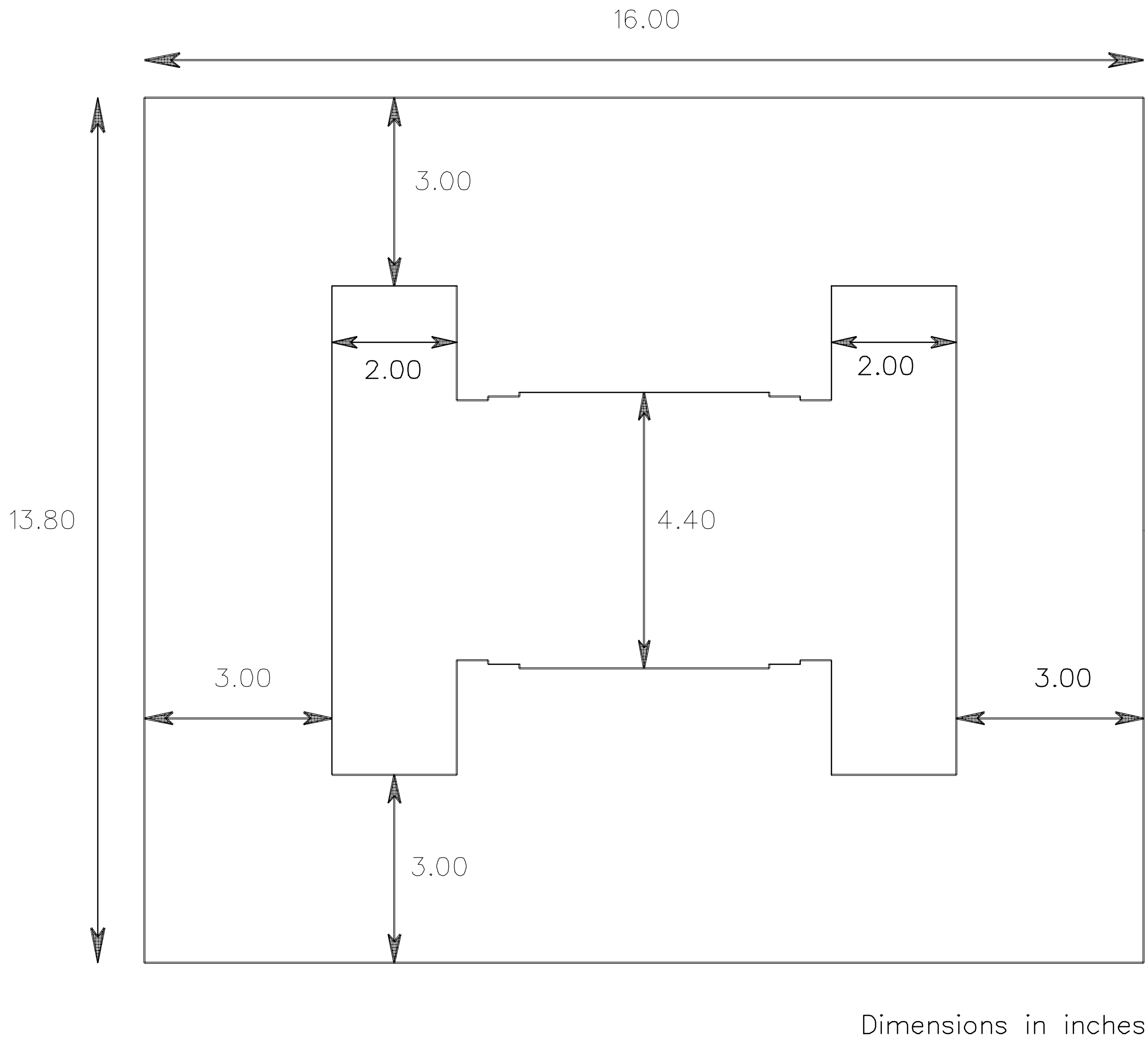

Fig. 12. The overall dimensions of the final steering magnet.

Appendix

In this appendix we present a more detailed calculation of the coil parameters than is presented in the main text.

A1. Coil dimensions

The conductor specified for the coil has the following properties listed in the Anaconda data sheets.

Outer dimension	0.1620 in. (square)
Inner diameter	0.0900 in. (circular)
Copper area	0.01934 in.^{2}
Cooling area	0.006362 in.^{2}
Weight	0.07473 lb/ft
Resistance at 20°C	421.1×10^{-6} Ω/ft
k factor (British units)	0.0622

As stated in the main body, the conductor on hand has been coated with a 0.011 inch thickness of double Dacron glass (DDG) insulation. We also assume an additional thickness of 0.007 inch fiberglass tape is wound on the conductor with a 0.25 inch spacing so that the nominal dimension of an insulated conductor, d_{nom}, is

$$d_{nom} = (0.162 \text{ in.} + 2(0.011 \text{ in.} + 0.007 \text{ in.}) = 0.198 \text{ in.} \approx 0.20 \text{ in.}.$$

We also allow an additional 0.010 inch each for keystoning and interturn spacing. Then the *width* of the coil is obtained from

Wrapped conductor (8×0.20 in.)	1.600 in.
Gapping (7×0.010 in.)	0.070 in.
Ground wrap (4×0.007 in.×2)	0.056 in.
Total	1.726 in.

The *height* of the coil is obtained from

Wrapped conductor (6×0.20 in.)	1.200 in.
Gapping (5×0.010 in.)	0.050 in.
Keystoning (6×0.010 in.)	0.060 in.
Ground wrap (4×0.007 in.×2)	0.056 in.
Total	1.366 in.

We take

Nominal coil width	=	1.75 in.
Nominal coil height	=	1.40 in.

In figure 1 the coil slot is shown as 1.85 inches wide by 1.50 inches high, the height being measured relative to the main flat portion of the pole. Making an allowance of 0.125 inch for coil-yoke insulation leaves 1.375 inches for the vertical dimension of the coil. Although this probably is adequate clearance, it is suggested that the depth of the coil slot be increased by 0.125 inch. Thus either the overall magnet would be increased by 0.25 inch or the thicknesses of the top and bottom yokes each be decreased by 0.125 inch. Of these, the latter appears to be the better solution.

Similarly, the width of the coil slot—specified in figure 1 as 1.85 inches—is inadequate to allow insulation 0.125 inch thick to be placed between the coil sides and the pole and the yoke. Consequently, the width of the coil slot should be increased to (1.75 inches + 2(0.125 inch)) = 2.00 inches or more. The overall width of the magnet is then increased by a minimum of 0.300 inch.

A2. Copper length per coil

The calculations in §3.1 of this report are only slightly modified by the inclusion of the ground wrap dimension in the overall coil dimension. In effect, both R_{min} and G are each increased by the thickness of the ground wrap—0.056 inch. Thus $R_{min} = 0.806$ inch and $G = 0.181$ inch. When these values are inserted into the expression for the length of an eight-turn layer we find

$$\begin{aligned} L_8 &= 2(8)[2(6.000 - 2(0.625)) + \pi(0.806 - (0.200)/2 + 9(0.200)/2)] \\ &= 16[9.50 + \pi(0.706 + 0.9)] \\ &= 16(14.5454) = 232.726 \text{ in.,} \end{aligned}$$

an increase of approximately 3 inches above the length previously calculated. A nominal value of $L_8 =$ 240 in. was used in further calculations to yield a total length of six eight-turn layers of 120 ft. An additional allowance of 2 feet should be added to allow for coil leads. Consequently, we now take the length of conductor per coil to be 122 ft. Then, at a weight of 0.07473 lb/ft, the weight per coil, W_{coil}, becomes 9.12 lb. Given a resistance at 20°C of 421.1×10^{-6} Ω/ft., the resistance per coil at that temperature is $R_{20} = 0.05137\Omega$, and assuming a 30°C temperature rise, the hot resistance of the coil is $R_{hot} = 0.05743\Omega$.

$$\boxed{\begin{aligned} L_{coil} &= 122.0 \text{ ft.} \\ W_{coil} &= 9.12 \text{ lb.} \\ R_{20} &= 51.37 \text{ m}\Omega \\ R_{hot} &= 57.43 \text{ m}\Omega \end{aligned}}$$

Then the peak voltage V_{peak} required for a peak current I_{peak} of 100 A is

$$V_{peak} = I_{peak} R_{hot} = 5.75 \text{ V}$$

and the power dissipated in resistive loss is

$$P_{res} = \frac{V_{peak}}{\sqrt{2}} \frac{I_{peak}}{\sqrt{2}} = 0.288 \text{ kW}.$$

These values of differ little from those given in §3.1. We take for *each* coil

$$\boxed{\begin{aligned} I_{peak} &= 100.0 \text{ A,} \\ V_{peak} &= 5.75 \text{ V,} \\ P_{res} &= 0.30 \text{ kW.} \end{aligned}}$$

A3. Cooling requirements

In these calculations we use the British system of units. For a given (resistive) power loss P_{res} the required flow rate of coolant is given by

$$v \text{ (ft/sec)} = \frac{2.19}{\Delta T(^\circ \text{ F})} \times \frac{\text{Power (kW)}}{\text{Cooling area (in.}^2)} = 0.0304167 \times \frac{P_{res} \text{ (kW)}}{A_c \text{ (in.}^2)}$$

for $\Delta T = 72°\text{F} = 40°\text{C}$. For $A_c = 6.362\times10^{-3}$ in.^{2} and $P_{res} = 0.30$ kW, we have

$$v = 0.0304167 \frac{0.30 \text{ kW}}{6.362\times10^{-3} \text{ in.}^2} = 1.434 \text{ ft/sec.}$$

The volume of flow required per coil is

$$\begin{aligned}\text{Volume/coil} &= v\,\frac{\text{ft}}{\text{sec}} \times 60\,\frac{\text{sec}}{\text{min}} \times A_{H_2O}\,(\text{in.}^2) \times \frac{1}{144}\,\frac{\text{ft}^2}{\text{in.}^2} \times 62.4\,\frac{\text{lb}}{\text{ft}^3} \times \frac{1}{10}\,\frac{\text{IG}}{\text{lb}} \times 1.20095\,\frac{\text{USG}}{\text{IG}} \\ &= 3.1225 \times v\,(\text{ft/sec}) \times A_{H_2O}\,(\text{in.}^2)\ \text{USGPM} \\ &= 3.1225(1.434\,\text{ft/sec.})(6.362\times 10^{-3}\ \text{in.}^2) \\ &= 0.0285\ \text{USGPM.}\end{aligned}$$

Volume per coil	0.030 USGPM
Volume per magnet	0.060 USGPM

A4. Pressure drop

The pressure drop is given by

$$\Delta P \;=\; k\,v^{1.79}\ \text{psi/ft}$$

with k a function of the cooling area. In our case, for a conductor with $k = 0.0622$ and $v = 1.434$ ft/sec we obtain

$$\Delta P = (0.0622)(1.434)^{1.79} = 0.1186\ \text{psi/ft},$$

and the total pressure drop across one coil is

$$\Delta P \;=\; (0.1186\ \text{psi/ft})(122\ \text{ft}) \;=\; 14.47\ \text{psi/coil.}$$

Pressure drop per coil = 14.5 psi.