1. Introduction

It is proposed to build a target for beam line 2A that would be capable of accepting a 100 μA beam of 500 MeV protons. However, the target is designed as an annulus of width 5 mm about a central radius of 11.5 mm. Consequently, the incident beam is also required to have that shape.

An earlier report [1)] presented the design of a magnet that would be suitable to produce the required annular beam. The thrust of that design was to reproduce the existing 4-inch steering magnets in a form suitable for AC excitation. An alternate approach suggested was to use the stator of an electric motor to achieve the required goals [2)].

This report presents a study of the usefulness of such a design.

2. Theory of operation

We begin with a brief discussion of the theory behind the operation of such a proposed magnet. This begins with the development of the field produced by a single wire and continues to the production of a pure dipole field by a continuous distribution of current.

The procedure followed is that used in the development of superconducting coils for a dipole for a large accelerator. Such coils run parallel to the beam over the length of the magnet and run transverse to the beam over a relatively short length. Because these magnets are long with respect to their transverse dimensions, field calculations may be made with reasonable accuracy in two dimensions.

The following two subsections are a condensation of the treatment presented by Meβ and Schmüser [3)]. The reader is referred to that article and references therein for a complete treatment of the subject.

2.1 Multipole expansion for a single conductor

We choose a coordinate system in which the beam direction is along the $+z$-axis of a cylindrical coordinate system (r, θ, z). A single, infinitely-long straight conductor carrying a current I parallel to the $+z$-axis is located the coordinate $r = a$, $\theta = \phi$. We wish to find the field generated by this current at a point $P(r, \theta)$ as indicated in figure 1 below.

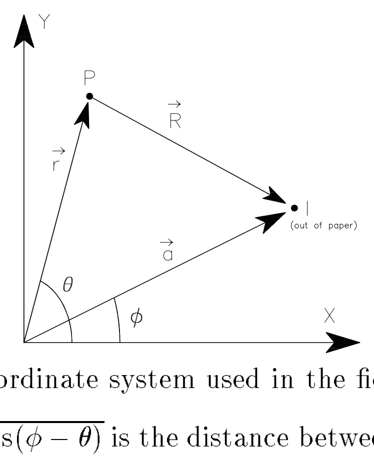

Fig. 1. The coordinate system used in the field calculation.

In this figure $|\vec{\boldsymbol{R}}| = \sqrt{a^2 + r^2 - 2\,a\,r\cos(\phi - \theta)}$ is the distance between the point $P(r, \theta)$ and the conductor. Because the current is directed parallel to the z-axis, the vector potential generated by it has only a z-component that is given by

$$A_z(r,\theta) \;=\; -\,\frac{\mu_0 I}{2\,\pi}\ln\left[\frac{R}{a}\right]$$

Because we are interested in the case $r < a$ we write

$$\begin{aligned}
R^2 &= a^2 - 2\,a\,r\cos(\phi-\theta) + r^2 \\
&= a^2\left[1 - 2\,a\,r\cos(\phi-\theta)\,\frac{r}{a} + \frac{r^2}{a^2}\right] \\
&= a^2\left[1 - \frac{r}{a}\left(e^{i(\phi-\theta)} + e^{-i(\phi-\theta)}\right) + \frac{r^2}{a^2}\,e^{i(\phi-\theta)}e^{-i(\phi-\theta)}\right] \\
&= a^2\left[1 - \frac{r}{a}\,e^{i(\phi-\theta)}\right]\left[1 - \frac{r}{a}\,e^{-i(\phi-\theta)}\right]
\end{aligned}$$

so that

$$\ln\left[\frac{R}{a}\right] \;=\; \frac{1}{2}\ln\left[1-\frac{r}{a}\,e^{i(\phi-\theta)}\right] + \frac{1}{2}\ln\left[1-\frac{r}{a}\,e^{-i(\phi-\theta)}\right].$$

But for an arbitrary complex number z with $|z| < 1$,

$$\ln(1-z) \;=\; -\,z - \frac{1}{2}\,z^2 - \frac{1}{3}\,z^3 - \ldots - \frac{1}{n}\,z^n - \ldots$$

so we have

$$A_z(r,\theta) \;=\; \frac{\mu_0 I}{2\,\pi}\sum\nolimits_{n=1}^{\infty}\frac{1}{n}\left[\frac{r}{a}\right]^n\cos(n(\phi-\theta))\;.$$

Thus for $r < a$ we find the components of the magnetic field $\vec{\boldsymbol{B}}$ to be

$$\begin{aligned}
B_\theta &= -\,\frac{\partial A_z}{\partial r} \;=\; -\,\frac{\mu_0 I}{2\,\pi\,a}\sum\nolimits_{n=1}^{\infty}\left[\frac{r}{a}\right]^{n-1}\cos(n(\phi-\theta))\;, \\
B_r &= \frac{1}{r}\,\frac{\partial A_z}{\partial \theta} \;=\; \frac{\mu_0 I}{2\,\pi\,a}\sum\nolimits_{n=1}^{\infty}\left[\frac{r}{a}\right]^{n-1}\sin(n(\phi-\theta))\;, \\
B_z &= 0\;.
\end{aligned}$$

Thus we see that a single line current generates multipole fields of all orders of n.

2.2 The generation of pure multipole fields

Now consider an arrangement of currents on a cylinder of radius a. We show that a pure multipole field of order $n = m$ is obtained inside the cylinder if the current density follows a function of the azimuthal angle ϕ given by

$$I(\phi) \;=\; I_0\cos(m\phi)\;.$$

In this case for $r < a$ the vector potential generated by this current distribution is given by

$$A_z(r,\theta) \;=\; \frac{\mu_0 I_0}{2\,\pi}\sum\nolimits_{n=1}^{\infty}\frac{1}{n}\left[\frac{r}{a}\right]^n\int_0^{2\,\pi}\cos(m\phi)\cos(n(\phi-\theta))\,d\phi\;.$$

But

$$\cos(n(\phi-\theta)) \;=\; \cos(n\,\phi)\cos(n\,\theta) + \sin(n\,\phi)\sin(n\,\theta)$$

and

$$\int_0^{2\,\pi}\cos(m\,\phi)\cos(n\,\phi) \;=\; \pi\,\delta_{m,n} \qquad\text{and}\qquad \int_0^{2\,\pi}\cos(m\,\phi)\sin(n\,\phi) \;=\; 0\;,$$

so that A_z, B_θ, and B_r reduce to

$$\begin{aligned} A_z(r,\theta) &= \frac{\mu_0 I_0}{2}\,\frac{1}{m}\left[\frac{r}{a}\right]^m \cos(m\,\theta)\ , \\ B_\theta(r,\theta) &= -\frac{\mu_0 I_0}{2\,a}\left[\frac{r}{a}\right]^{m-1} \cos(m\,\theta)\ , \\ B_r(r,\theta) &= -\frac{\mu_0 I_0}{2\,a}\left[\frac{r}{a}\right]^{m-1} \sin(m\,\theta)\ . \end{aligned}$$

We now note the special cases of $m = 1$, 2, and 3 as they apply in a *Cartesian* coordinate system. We have, for $m = 1$,

$$\begin{aligned} B_x &= B_r\cos(\theta) - B_\theta\sin(\theta) \\ &= 0\ , \\ B_y &= B_r\sin(\theta) + B_\theta\cos(\theta) \\ &= -\frac{\mu_0 I_0}{2\,a} \ =\ \text{constant.} \end{aligned}$$

Thus a constant field in the y-direction—a pure dipole field—is produced.
For $m = 2$ we obtain a pure quadrupole field. Thus

$$\begin{aligned} B_x &= g\,r[\sin(2\,\theta)\cos(\theta) - \cos(2\,\theta)\sin(\theta)] \\ &= g\,r\sin(\theta) \ =\ g\,y\ , \\ B_y &= g\,r[\sin(2\,\theta)\sin(\theta) + \cos(2\,\theta)\cos(\theta)] \\ &= g\,r\cos(\theta) \ =\ g\,x\ , \\ \text{with } g &= -\frac{\mu_0 I_0}{2\,a^2}\ . \end{aligned}$$

For $m = 3$ we find a pure sextupole field. Thus

$$\begin{aligned} B_x &= \frac{1}{2}\,g'\,r^2[\sin(3\,\theta)\cos(\theta) - \cos(3\,\theta)\sin(\theta)] \\ &= \frac{1}{2}\,g'\,r^2[2\sin(\theta)\cos(\theta)] \ =\ g'\,x\,y\ , \\ B_y &= \frac{1}{2}\,g'\,r^2[\sin(3\,\theta)\sin(\theta) - \cos(3\,\theta)\cos(\theta)] \\ &= \frac{1}{2}\,g'\,r^2[\cos^2(\theta) - \sin^2(\theta)] \ =\ \frac{1}{2}\,g'(x^2 - y^2)\ , \\ \text{with } g' &= -\frac{\mu_0 I_0}{a^3}\ . \end{aligned}$$

3. The approximation of a continuous current distribution by a discrete distribution

In practice it is difficult to produce a continuous current distribution such as that indicated above. For superconducting magnets such a distribution is approximated by current shells. This technique is discussed in detail in ref[3]. An alternate approach is to approximate a continuous current distribution by winding a motor stator such that the number of turns per slot are proportional to the cosine of the azimuthal angle θ. Such an approach is reported by Benaroya and Ramler[4]. Their design requirement was to provide a deflection of approximately 16 mr for the 21.6 MeV deuteron beam of the Argonne National Laboratory cyclotron. We shall use their approach in what follows.

Our ultimate goal is to approximate a continuous current distribution by a series of discrete windings in

$m = 360/\Delta\theta$ slots that are equally spaced at intervals of $\Delta\theta$ degrees around the circumference of a circle of radius a. The first slot will be at an angle of $\theta = \Delta\theta/2$. Rather than having the continuous current distribution $I = I_0 \cos(\theta)$ that was used above, we wind the exciting coil such that the number of turns per slot, $n(\theta)$, varies with θ and hold the current, I, constant. Thus the number of Ampère-turns per slot becomes $n(\theta)I = (N\cos(\theta))I = NI\cos(\theta)$ where N is a number to be determined from the design requirements. The geometry of the problem is illustrated in figure 2 below.

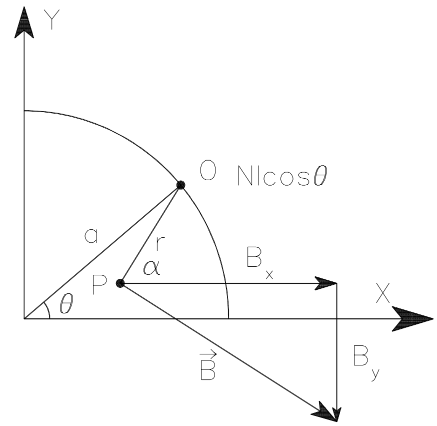

Fig. 2. The geometry used for a discrete current distribution.

In this figure the point O represents a slot at an angle θ on a stator of internal radius a. Its coordinates in the Cartesian coordinate system are $(x, y) = (a\cos(\theta), a\sin(\theta))$ and the slot contains $N\,I\cos(\theta)$ Ampère-turns. For convenience, we take the direction of current flow to be along the positive z-axis—that is, out of the page. We wish to find the magnetic field $\vec{\boldsymbol{B}}(\theta)$ produced by this current at the point $P(x, y)$ a distance r from the point O. Thus $\overline{OP} = r$.

By Ampère's law, the field at the point P has the magnitude

$$\oint \vec{\boldsymbol{B}}(\theta) \cdot d\vec{\boldsymbol{l}} = \mu_0\, N\, I \cos(\theta)$$

so that

$$B(\theta) = \frac{\mu_0\, N\, I}{2\,\pi\, r} \cos(\theta)$$

and has a direction perpendicular to the line $\overline{OP}$. If the angle between $\overline{OP}$ and the horizontal axis is α, then the horizontal and vertical components of $\vec{\boldsymbol{B}}(\theta)$ are

$$\begin{aligned} B_x(\theta) &= B(\theta)\sin(\alpha) \\ &= \frac{\mu_0\, N\, I}{2\,\pi\, r}\, \frac{a\sin(\theta) - y}{r} \cos(\theta) \\ B_y(\theta) &= -\,B(\theta)\cos(\alpha) \\ &= -\,\frac{\mu_0\, N\, I}{2\,\pi\, r}\, \frac{a\cos(\theta) - x}{r} \cos(\theta) \\ \text{with } r^2 &= (a\sin(\theta) - y)^2 + (a\cos(\theta) - x)^2 \end{aligned}$$

For a continuous current distribution the total field produced at P is obtained by integration. Thus

$$B_y = -\,\frac{\mu_0\, N\, I}{2\,\pi} \int_0^{2\,\pi} \frac{a\cos(\theta) - x}{r^2} \cos(\theta)\, d\theta\ .$$

[We note at this point that for the special case of the point P being the origin, $r = a$ and we have

$$B_y = -\frac{\mu_0 N I}{2\pi}\int_0^{2\pi}\frac{a\cos^2(\theta)}{a^2}\,d\theta = -\frac{\mu_0 N I}{2\pi a}\left[\frac{\sin(2\theta)}{4}+\frac{\theta}{2}\right]_0^{2\pi} = -\frac{\mu_0 N I}{2a}.$$

Similarly, we find

$$B_x = \frac{\mu_0 N I}{2\pi}\int_0^{2\pi}\frac{a\sin(\theta)\cos(\theta)}{a^2}\,d\theta = \frac{\mu_0 N I}{2\pi a}\left[\frac{1}{2}\sin^2(\theta)\right]_0^{2\pi} = 0.$$

That is, we find the values found in §2.2 for B_y and B_x with $I_0 = NI$.]

However, because we are dealing with a discrete winding pattern the above integration must be replaced by a summation. We replace the integral with a sum over θ from $\theta = \Delta\theta/2$ to $\theta = 360 - \Delta\theta/2$ in steps of $\Delta\theta$, where $\Delta\theta$ is the angular separation between the stator slots. We also divide the equations for B_y and B_x by NI and write the vertical field produced at the point P as

$$\frac{B_y^*}{NI}\Delta\theta = -\frac{\mu_0}{2\pi}\sum_{\theta=\Delta\theta/2}^{360-\Delta\theta/2}\frac{a\cos(\theta)-x}{r^2}\cos(\theta)\,\Delta\theta$$

and that for the horizontal field produced at P as

$$\frac{B_x^*}{NI}\Delta\theta = \frac{\mu_0}{2\pi}\sum_{\theta=\Delta\theta/2}^{360-\Delta\theta/2}\frac{a\sin(\theta)-y}{r^2}\cos(\theta)\,\Delta\theta.$$

The question, then, is to determine over what ranges of x and y does the summation procedure give a good approximation to the values of B_x and B_y calculated for a continuous distribution of current. That is, over what range of x and y may we write

$$\frac{B_y^*}{NI}\Delta\theta = \frac{\mu_0}{2a}$$
$$\text{and } \frac{B_x^*}{NI}\Delta\theta = 0\ ?$$

It is convenient to use units of Gauss, centimeters, and degrees. Consequently, we write

$$\mu_0 = 4\pi\times10^{-7}\ \text{Tesla-meter/A} = 0.4\pi\ \text{Gauss-cm/A}$$

and rewrite the above relation relation for B_y^* as

$$\frac{B_y^*}{NI} = \frac{0.2\pi}{a}\frac{180}{\Delta\theta(°)\,\pi} = \frac{36}{a\,\Delta\theta(°)}$$

with $\Delta\theta$, the angular separation of the stator slots, now expressed in degrees and a, the inner radius of the stator, expressed in cm.

A simple numerical integration routine was written to calculate B_y^* and B_x^*. To check the program, an attempt to reproduce Table 1 of ref[4] was made using data given in that paper. Agreement to within ±0.0002 G/A for B_y^*/NI was attained for slot separations of 5°, 10°, and 15°. The absolute values of B_x^*/NI were in good agreement, but often there was a sign difference. However, for a slot separation of 30°, results differed significantly. For example, in ref[4] the value for B_y^*/NI of 0.4853 G/A is given at the origin whereas the routine written for this report calculates a value of 0.4199 G/A at that point. This latter value is exactly the value one obtains on evaluating the above expression for B_y^*/NI with $a = 1.125$ in. $= 2.8575$ cm and $\Delta\theta = 30°$. The implication is that one (or both) calculation(s) is (are) incorrectly reported or in error.

However, because a slot separation of 10° is common—if not standard—the agreement between the two calculations is deemed adequate.

4. Application to a steering magnet for TRIUMF

For use at TRIUMF a magnet with a 4 inch bore would be suitable. Thus, if we assume an inner bore of 4.25 in. and a stator-slot angular pitch of 10°, from the above we would expect to find

$$\frac{B_y^*}{NI} = \frac{36}{2.54(4.25/2)}\frac{1}{10} = 0.666975 \text{ G/A} .$$

Table 1 presents the results of a numerical integration for this case for a number of points within the magnet aperture.

Table 1

Numerical integration for B_y^*/NI and B_x^*/NI for an internal radius of $a = 2.125$ inches = 5.3975 cm and a 10° pitch of stator slots. The units of B_y^*/NI and B_x^*/NI are Gauss/Ampère.

x (in.)	Y distance from magnet center (in.)									
	0		0.25		0.5		0.75		1.00	
	B_x^*	B_y^*	B_x^*	B_y^*	B_x^*	B_y^*	B_x^*	B_y^*	B_x^*	B_y^*
0	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
0.25	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
0.25	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
0.50	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
0.75	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
1.00	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670
1.25	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	−0.0001	0.6670
1.50	0.0000	0.6670	0.0000	0.6670	0.0000	0.6670	−0.0001	0.6673	0.0036	0.6672
1.75	0.0000	0.6655	−0.0021	0.6665	−0.0016	0.6725	0.0266	0.6662	−0.1010	0.5443
2.00	0.0000	0.5231	−0.2084	0.7130	0.5378	0.9298				

x (in.)	Y distance from magnet center (in.)							
	1.25		1.50		1.75		2.00	
	B_x^*	B_y^*	B_x^*	B_y^*	B_x^*	B_y^*	B_x^*	B_y^*
0	0.0000	0.6670	0.0000	0.6670	0.0000	0.6673	0.0000	0.6757
0.25	0.0000	0.6670	0.0000	0.6670	−0.0005	0.6668	−0.0054	0.6385
0.50	0.0000	0.6670	0.0000	0.6670	0.0013	0.6656	0.0830	0.7928
0.75	0.0000	0.6670	0.0001	0.6668	0.0031	0.6783		
1.00	0.0000	0.6669	0.0009	0.6692	−0.0769	0.6180		
1.25	0.0013	0.6679	−0.0322	0.6392				
1.50	−0.0393	0.6346	−0.0322	0.6392				

Data in the above table predicts a vertical field uniform to approximately 0.15% over a region of radius 1.25 inches. Such a volume should be more than adequate for the purpose of this magnet.

The magnet constructed according to the recipe given in ref[4)] had an iron length of 9 inches and a measured effective length of 11.2 inches. This effective length is essentially the iron length plus one (inner) bore diameter. Consequently, if we were to construct a magnet with an iron length of 9 inches we would expect to find an effective length of approximately 13 inches.

To calculate the field required in a magnet for use at TRIUMF we note that the maximum angular deflection required is 3.6 mr [1]. We will design a magnet for a maximum deflection of 5 mr. Because of this small angle we take the arc length of a trajectory in the magnet, s, to be equal to the effective length, L_{eff}, of the magnet. Thus the radius of curvature of a particle in the magnet ρ becomes

$$\rho = \frac{s}{\theta} = \frac{L_{eff}}{\theta} = \frac{13 \text{ in.}}{0.005 \text{ radian}} = 2{,}600 \text{ in.} \approx 66 \text{ m.}$$

Thus, for 500 MeV protons, the required field is

$$B\ (\text{kG}) = \frac{(B\rho)_0}{\rho} = \frac{33.356\, p(\text{GeV/c})}{\rho(\text{m})} = \frac{33.356(1.09007)}{66} = 0.551 \text{ kG.}$$

But we have

$$\frac{B_y^*}{N\,I} = \frac{36}{2.54(4.25/2)}\,\frac{1}{10} = 0.666975 \text{ G/A}$$

so that

$$N\,I = \frac{B_y^*}{0.666975 \text{ G/A}} = \frac{551 \text{ G}}{0.666975 \text{ G/A}} = 826 \text{ A.}$$

The above calculations have been done for a coil in air. To account for the iron yoke we note that for a current I inside a hollow iron cylinder of radius R the effect of the iron on the inner field is equivalent to that of an image current I' located at a radius a' with

$$I' = \frac{\mu - 1}{\mu + 1}\, I \qquad \text{and} \qquad a' = \frac{R^2}{a}\ .$$

The current I' is in the same direction as the current I and therefore increases the inner field. For our case that current is at the same radius as the that of the inner conductor. Assuming an infinite permeability for the iron, we have in effect twice the actual circulating current. Thus the value of $N\,I$ calculated above requires multiplication by a correction factor $k_1 = 0.5$ to account for the iron yoke.

Further, a correction for the finite length of the conductors needs to be applied. That used in ref [4] is $k_2 = 1/\sin(\gamma)$, where γ is the angle subtended by a line joining the end of the yoke to the axial midpoint of the magnet and a perpendicular from the axial midpoint of the magnet to its inner circumference. In our case

$$\gamma = \tan^{-1}\frac{9.0/2}{2.125} = 64.722^\circ$$

so that

$$k_2 = \frac{1}{\sin(\gamma)} = 1.106\ .$$

Consequently, the design value for the number of Ampère-turns required is

$$(N\,I)_{design} = k_1\, k_2\, (N\,I)_{theory} = (0.5)(1.106)(826) \approx 460 \text{ A-t}\ .$$

This is (almost) exactly twice the number of Ampère-turns required for the magnet of ref [4] (they required 232 A-t).

The value of N in the above expression is determined by noting that there are two identical coils, one each for horizontal and vertical deflection, orientated 90 mechanical degrees with respect to each other. Consequently, the total number of turns in any one slot is $n = N\cos\theta + N\sin\theta$. The maximum number of turns per slot occurs at $\theta = 45^\circ$ and, using $n = 144$ as in ref [4], we find $N = 102$. Thus an excitation current of (460 A-t)/(102 turns) = 4.5 Ampères is required. The winding pattern of the TRIUMF magnet would be that used in ref[4].

We are now in a position to specify further design parameters. Given that we have assumed an iron length the same as that used in ref[4] and that the bore of our magnet is twice that of that reference, it is logical to assume an iron yoke dimension twice that of ref[4]. Thus we begin by assuming a yoke with an inner diameter of 4.25 inches and an outer diameter of 10 inches. We shall assume a width of a stator slot to be 0.28 inch. The depth of a stator slot will be determined by the conductor used.

It is not known if laminations of these dimensions are available commercially. At the worst they would have to be a special order. However, we proceed on the assumption that such laminations may be obtained.

4.1 Conductor selection

In ref[4] the coils were wound with 23 AWG conductor. The current carrying capacity at 700 circular mils per Ampère of this conductor is listed in the ARRL Handbook[5] as 0.728 Ampères. It is also noted that this capacity is a satisfactory design figure for small transformers; no value is given for continuous duty for wires in conduits or bundles.

In our opinion the windings of a magnet of the design considered here are more like those of a transformer than those of a bundle. Consequently, we shall use the current capacities listed in ref[5] for (small) transformers as our criteria in the choice of conductor.

Our design is for a peak current of 4.5 A or an RMS current of 3.18 A. Further, on the assumption that a maximum deflection of 3.6 mr (rather than the design value of 5 mr) will be the probable operating point, we would expect a normal peak operating current of 3.25 A or an RMS current of 2.29 A. The current capacity at 700 circular mils per Ampère is given in ref[5] as 3.69 A for 16 AWG wire and as 2.32 A for 18 AWG wire. With the rider that a more detailed engineering design may find that a smaller gauge wire is useable, we shall assume these wire gauges in what follows.

We choose Belden Poly-Thermaleze conductor as that to be used for the coils. Its properties are given in ref[6] as

> "Dual-coated magnet wire. Base coat is a cross-linked, modified polyester; top coat is an amide-imide polymer. Rated for 180°C usage. Has exceptional ability to resist solvents and abuse in different windings. Complies with J-W-1177A specifications. MC 35-C(Heavy)."

This conductor is used in the winding of the coils of the standard 4-inch steering magnets at TRIUMF. The catalog gives the number of turns per linear inch of 16 AWG wire as 18.6 and that for 18 AWG wire as 23.2. Thus we take the diameter of 16 AWG wire to be approximately 0.055 inch and that of 18 AWG to be approximately 0.045 inch. Assuming a slot width of 0.280 inch, 5 turns of 16 AWG wire or 6 turns of 18 AWG wire could be wound across the slot, leaving a minimum of 0.005 inch for insulation between the stator and the coil itself.

4.2 Slot depth

To determine the required depth of the slots we consider how the conductor may be wound. Two possible ways of winding two layers of wire of diameter D are shown in figure 3. The close-packed method has a vertical dimension of $D(1+\cos 30)$ per two layers of conductor; that of the stacked method is $2D$. However, if n is the number of turns in the upper layer, there are $(n-1)$ turns in the lower layer for a total of $(2n-1)$ turns per two layers of a close-packed array whereas there are $2n$ turns per two layers of a stacked array. The estimated wire diameters obtained allow us to estimate the minimum depths of a slot required for the two wire sizes by calculating the number of two-turn arrays required to meet the maximum of 144 turns per slot.

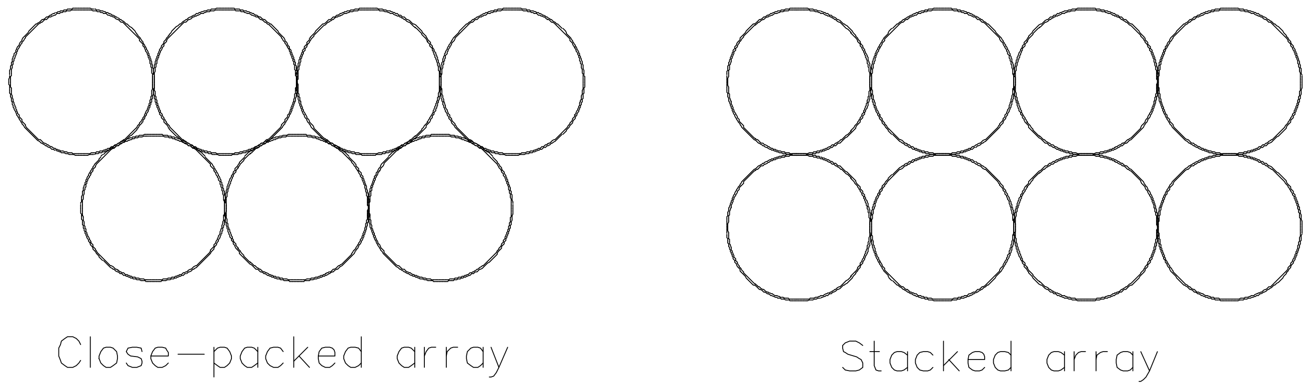

Fig. 3. Illustration of close-packed and stacked arrays.

Table 2

Calculation of minimum depth of a stator slot for 144 turns in total

Close-packed array				
Wire gauge	Turns per 2 layers	Number of 2 layers	Height per 2 layers (in.)	Overall height (in.)
16	9	16	0.103	1.65
18	11	13	0.084	1.09

Stacked array				
Wire gauge	Turns per layer	Number of layers	Height per layer (in.)	Overall height (in.)
16	5	29	0.055	1.60
18	6	24	0.045	1.08

Clearly, for a given wire size the winding method makes little difference insofar as the depth of the stator slots is concerned. However, the use of 16 AWG wire would require a slot approximately 0.5 inch deeper than would the use of the 18 AWG conductor.

4.3 Choice and length of conductor

Given our assumption of a magnet whose dimensions are twice that of ref[4)], a slot depth of 1.5625 inches is indicated. Further, given that the coils of that magnet were constructed from 23 AWG conductor, we shall assume in what follows that the TRIUMF magnet can be built with a slot depth of 1.5 inches and a coil constructed from 18 AWG wire.

To estimate the length of conductor required we proceed as follows. We define R_{mean} as the average radius of a slot; $R_{mean} = (2.125 + 1.5/2)$ inches $= 2.875$ inches. Then the distance separating the R_{mean} of a slot at an angle θ from the corresponding point of its return slot at an angle of $\pi - \theta$ is $d = 2\,R_{mean}\cos(\theta)$. We then assume a length of conductor equal to $\pi\, d$ is required to complete the end connections of all turns in that slot. Thus we take the length of one complete turn in a slot at angle θ, l_{turn}, to be

$$\begin{aligned} l_{turn} &= 2[\text{Iron length}] + 2\,\pi[R_{mean}\cos(\theta)] \\ &= 2[(9.0\text{ in.}) + \pi(2.875\text{ in.})\cos(\theta)] \\ &\approx 18(1+\cos(\theta))\text{ inches.} \end{aligned}$$

The following table shows a calculation of the length of conductor required for *one-half* of one coil.

Table 3

Calculation of the required length of conductor

Slot angle (°)	Number of Conductors	l_{turn} (in.)	Length per slot (in.)	A-t per slot
5	102	35.9	3,665	459
15	98	35.4	3,468	441
25	92	34.3	3,157	414
35	83	32.7	2,718	373.5
45	72	30.7	2,212	324
55	58	28.3	1,643	261
65	43	25.6	1,101	193.5
75	26	22.7	589	117
85	9	19.6	176	40.5
Total			18,729	

Thus the total length of conductor per magnet is 2(18,729 inches) = 37,458 inches = 3,122 ft. We suspect that this estimate is low because additional length will probably be required at the ends of the coil. This we account for by increasing the calculated length by approximately 5% to 3,300 feet. Because we have two identical coils, one rotated 90° with respect to the other, the total length of conductor required for an (x, y) steering magnet will be approximately 6,600 feet.

5. Electrical parameters

Ref[9)] lists 18 AWG annealed copper wire as having a resistance of 6.385 Ω at 20°C and a weight of 4.917 lb, both being values per 1000 feet of conductor. Similar data for 16 AWG wire are given as 4.016 Ω/1000 ft and 7.818 lb/1000 ft.

The weight of a coil constructed from 18 AWG conductor will be approximately 16.2 lbs and its resistance at 20°C will be approximately 21 Ω. A coil constructed of 16 AWG conductor will have a weight of approximately 26 lb and a resistance at 20°C of approximately 13.3 Ω. If we assume a (canonical) 30°C rise in temperature during operation, the hot resistance of the coils will be

$$R_{hot} = R_{20°C}\,[1 + (\text{Temp. coeff}/°\text{C})\Delta T(°C)] = 1.1179\,R_{20°C}$$

At a current of $I_{peak} = 4.5$ A or $I_{RMS} = 3.182$ A, the resistive power loss is

$$P = I_{RMS}^2 R_{hot} = \left[\frac{4.5}{\sqrt{2}}\right]^2 R_{hot}\text{ W} = 10.125\,R_{hot}\text{ W}\,.$$

Resistance and resistive power loss for each conductor is summarized in the table on the following page. The calculated resistive power losses imply that the yoke will require cooling windings.

Table 4

Resistance and resistive power losses for each conductor size

	Conductor size	
	16 AWG	18 AWG
Weight per coil	25.800 lb	16.226 lb
Resistance at 20°C	13.253 Ω	21.483 Ω
Hot resistance	14.816 Ω	24.016 Ω
Resistive power loss	150 W	243 W

6. Iron

As in ref[1] we assume that the laminations for the magnet will be punched from 26 gauge steel that is 0.47mm thick. The finished diameter of the magnet is to be 10 inches. If we allow 1 inch punching allowance on all sides, each lamination will be punched from a piece of iron 12 inches square. Thus the gross area of a lamination is 144 square inches. Assuming a lamination factor of 0.95 and a magnet length of 9 inches, the number of laminations per magnet, N_l will be

$$N_l = \frac{(9 \text{ in.})(25.4 \text{ mm/in.})(0.95)}{0.47 \text{ mm}} = 462$$

and, at a density of 7.6 gm/cc, the gross weight of each lamination, W_l, is

$$W_l = [(12 \text{ in.})(2.54 \text{ cm/in.})]^2 (0.047\text{cm})(7.6 \text{ gm/cc}) = 331.85 \text{ gm} = 0.732 \text{ lb.}$$

Thus the gross weight of laminations for the magnet, W_{gross}, is

$$W_{gross} = (462 \text{ laminations})(0.732 \text{ lb/lamination}) \approx 340 \text{ lb.}$$

The net weight of a lamination is that of an annulus 0.47 mm thick with an outer radius of 5 inches and an inner radius of $R_{mean} = 2.875$ inches. Thus the net area of a lamination is

$$A_{l,net} = \pi(R^2_{outer} - R^2_{mean}) = (2.54)^2 \pi (5^2 - 2.875^2) = 339.2 \text{ cm}^2$$

and its net weight is

$$W_{l,net} = A_{net}(0.047 \text{ cm})(7.6 \text{ gm/cc}) = 121.15 \text{ gm} = 0.267 \text{ lb.}$$

Thus the net weight of steel per magnet is

$$W_{net} = (0.267 \text{ lb/lamination})(462 \text{ laminations}) \approx 125 \text{ lb.}$$

7. POISSON simulation

To check the above calculations, a POISSON[7] run was made for the proposed magnet. Because of the symmetry of the problem, only one-quarter of the dipole is required. The geometry used is shown in figure 4. The turn pattern used and the Ampère-turns per slot are those given in the table 3 on page 10. The command file and the data input file that were used for this POISSON run are listed in tables 5 and 5 respectively.

Figure 5 is POISSON graphical output showing the computed field lines. The computed field at the origin

$[(x,y) = (0,0)]$ is $B_x = 0$ and $B_y = -596.98$ G. The latter is approximately 10% higher than the value of $B_y = 551$ G determined in §3. We conclude that this POISSON calculation is within reasonable (10%) agreement with that anticipated.

Figure 6 shows a contour plot of the field in the gap that was generated using PLOTDATA [8)] together with the data from the POISSON calculation. From this figure it is seen that the field over a region with radius 1.25 inches is predicted to vary from 595.25 G to 598 G or (596.65 ± 1.38) G. This variation of $\pm 0.23\%$ is also in reasonable agreement with that calculated from the numerical integration.

Figure 7 shows a contour plot of the field predicted in the yoke by POISSON. That field is seen to be quite low—of the order of 2 kG. However, for this calculation the yoke was assumed to be made from 1010 steel whereas the actual yoke would be constructed from transformer steel. It is therefore expected that the field in the yoke will be higher than 2 kG but not high enough to saturate the steel.

7.1 Magnet inductance

POISSON calculates the stored energy in the magnet. In this case it is predicted to be 5.4906 J/m for one-quarter of the magnet. Thus for a full magnet of length 9 inches, the stored energy is predicted to be

$$\text{Stored energy} = 4\,\frac{(9\text{ inches})(2.54\text{ cm/inch})}{(100\text{ cm/m})}\,5.4906\text{ J/m} = 5.021\text{ J}\ .$$

Because the stored energy is related to the excitation current I and magnet inductance L by

$$\text{Stored energy} = \frac{1}{2}\,L\,I^2\ ,$$

we the estimate the inductance of the magnet to be

$$L = 2\,\frac{5.021}{(4.5)^2} = 0.4959\text{ H} \approx 500\text{ mH}\ ,$$

and the inductive reactance of the magnet at 60 Hz is

$$X_L = 2\pi(60\text{ Hz})(0.4959\text{ H}) = 186.94\ \Omega\ .$$

7.2 AC parameters

Assuming that the resistances and inductance calculated above are correct, we have $R_{hot,16AWG} = 14.82\ \Omega$, $R_{hot,18AWG} = 24.02\ \Omega$, and $L = 496$ mH. Then the impedance of the magnet, assuming that it may be treated as a resistance and inductance in series, is

$$|Z| = \sqrt{R^2_{hot,16AWG} + X^2_L} = \sqrt{(14.82)^2 + (186.94)^2} = 187.52\ \Omega\ .$$

for 16 AWG conductor and

$$|Z| = \sqrt{R^2_{hot,16AWG} + X^2_L} = \sqrt{(24.02)^2 + (186.94)^2} = 188.47\ \Omega\ .$$

for 18 AWG conductor. It is seen that under these assumptions the magnet may be considered as a pure inductance. The phase angle is

$$\phi = \tan^{-1}(X_L/R_{hot}) = \begin{cases} 85.47^\circ \text{ for the 16 AWG conductor} \\ 82.68^\circ \text{ for the 18 AWG conductor} \end{cases}$$

The required RMS voltage V_{RMS} at a peak current I_{peak} of 4.5 A or $I_{RMS} = 4.5/\sqrt{2}$ A $= 3.182$ A is

$$V_{RMS} = |Z|I_{RMS} = \begin{cases} (3.182\text{ A})(187.54\ \Omega) = 596.8\text{ V for the 16 AWG conductor.} \\ (3.182\text{ A})(188.49\ \Omega) = 599.8\text{ V. for the 18 AWG conductor.} \end{cases}$$

Thus a RMS voltage of approximately 600 V is required for either conductor. The RMS voltage drop across the resistive portion of the load is

$$V_{R,RMS} = I_{RMS}R_{hot} = \begin{cases} (3.182\text{ A})(14.82\ \Omega) = 47.16\text{ V for the 16 AWG conductor.} \\ (3.182\text{ A})(24.02\ \Omega) = 76.43\text{ V for the 18 AWG conductor.} \end{cases}$$

The RMS voltage across the inductive portion of the load is

$$V_{L,RMS} = I_{RMS}X_L = (3.182\text{ A})(186.94\ \Omega) = 594.8\text{ V for the either AWG conductor.}$$

Thus the active power is

$$P = I_{RMS}V_{R,RMS} = \begin{cases} (3.182\text{ A})(47.16\text{ V}) = 150.1\text{ W for the 16 AWG conductor.} \\ (3.182\text{ A})(76.43\text{ V}) = 243.2\text{ W for the 18 AWG conductor.} \end{cases}$$

The reactive power is

$$Q = I_{RMS}V_{L,RMS} = (3.182\text{ A})(594.8\text{ V}) = 1.893\text{ kvar for the either conductor.}$$

The apparent power is

$$S = V_{RMS}I_{RMS} = \begin{cases} (596.8\text{ V})(3.182\text{ A}) = 1.899\text{ kVA for the 16 AWG conductor.} \\ (599.8\text{ V})(3.182\text{ A}) = 1.909\text{ kVA for the 18 AWG conductor.} \end{cases}$$

The power factor is the

$$\cos\phi = P/S = \begin{cases} (150.1\text{ W})/(1{,}899\text{ VA}) = 0.079\text{ for the 16 AWG conductor.} \\ (243.2\text{ W})/(1{,}909\text{ VA}) = 0.127\text{ for the 18 AWG conductor.} \end{cases}$$

This low power factor implies an inefficient transfer of energy to the magnet. The power factor may be improved by adding capacitance in series or in parallel with the steerer. We consider each of these, although a capacitance in parallel with the magnet would be the more probable solution.

7.2.1 Addition of a series capacitance

The magnitude of the impedance $|Z_{series}|$ and phase angle ϕ_{series} of a circuit in which a resistance R, an inductance L, and a capacitance C_{series} are connected in series are given by[10)]

$$|Z_{series}| = \sqrt{R^2 + (X_L - X_{C_{series}})^2} = \sqrt{R^2 + (\omega L - 1/\omega C_{series})^2}$$

and

$$\phi_{series} = \tan^{-1}\left[\frac{X_L - X_{C_{series}}}{R}\right] = \tan^{-1}\left[\frac{\omega L - 1/\omega C_{series}}{R}\right]$$

with $X_L = \omega L$ and $X_{C_{series}} = 1/(\omega C_{series})$. Clearly, if $X_L = X_{C_{series}}$ then $|Z_{series}|$ is minimum and $\phi_{series} = 0$—that is, the circuit behaves as a pure resistance. For this to occur at a given (angular) frequency ω_0 requires

$$\omega_0 L = 1/(\omega_0 C) \qquad \text{or} \qquad \omega_0^2 L C_{series} = 1.$$

At a frequency of 60 Hz, the required capacitance is

$$C_{series} = \frac{1}{\omega_0^2 L} = \frac{1}{[2\pi(60)]^2(0.500)} = 14.07\ \mu\text{f}.$$

$$\boxed{C_{series} = 14.1\ \mu\text{f}.}$$

Thus, with these values of R, L, and C_{series}, the power factor is unity and the magnitude of impedance is equal to the resistance of the magnet coil (in this case).

7.2.2 Addition of a parallel capacitance

The magnitude of the impedance $|Z_{para}|$ and phase angle ϕ_{para} of a circuit in which a resistance R and an inductance L in series are paralleled by a capacitance C_{para} are given by [10)]

$$|Z_{para}| = \sqrt{\frac{R^2 + X_L^2}{R^2/X_{C_{para}}^2 + (X_L/X_{C_{para}} - 1)^2}} = \sqrt{\frac{R^2 + \omega_0^2 L^2}{\omega_0^2 C_{para}^2 R^2 + (\omega_0^2 L C_{para} - 1)^2}}$$

and

$$\phi_{para} = \tan^{-1}\left[\frac{X_L(1 - X_L/X_{C_{para}}) - R^2/X_{C_{para}}}{R}\right] = \tan^{-1}\left[\frac{\omega_0[L(1 - \omega_0^2 L C_{para}) - C_{para} R^2]}{R}\right]$$

For $\phi = 0$—or a power factor of unity—we require

$$X_L(1 - X_L/X_{C_{para}}) = R^2/X_{C_{para}} \qquad \text{or} \qquad L(1 - \omega_0^2 L C_{para}) = C_{para} R^2$$

so that for our values of R and L we find at 60 Hz

$$C_{para} = \frac{L}{R^2 + \omega_0^2 L^2} = \frac{0.500}{(24.02)^2 + (2\pi(60))^2(0.500)^2} = 13.85\ \mu\text{f}.$$

$$\boxed{C_{para} = 13.9\ \mu\text{f}.}$$

We note that $C_{para} \approx C_{series}$. This should be the case, for with $R^2 \ll \omega_0^2 L^2$ (as is the case under discussion), then the above expression for C_{para} reduces to that for C_{series}.

We note in passing that, contrary to the result found for a series RLC circuit, the magnitude of the impedance is at a maximum when the power factor is unity [1)].

8. Discussion

This report presents an alternate design for an AC steering magnet for beam line 2A. Rather than a design similar to a conventional dipole magnet, the design presented here has a yoke similar to that of a motor stator.

The design parameters presented are deliberately vague because it is not known what sizes are available for conventional stators. The design will require modification once such parameters become available. Some cost saving may be realized if the (external) profile is made rectangular rather than circular as shown in figure 4.

A POISSON study of the design presented has shown that a steering magnet of this design is feasible. There remains only to quantify the yoke size and conductor dimensions.

One problem that might exist, however, is the high voltage required for operation of the magnet (as shown in §7.2).

References

1. G. M. Stinson, *A design for an AC steering magnet for beam line 2A*, TRIUMF Report TRI-DNA-01-1, TRIUMF, January, 2001.
2. P. Schmor, *Private communication*, TRIUMF, November, 2000.
3. K. H. Meß and P. Schmüser, *Superconducting Accelerator Magnets*, CAS CERN Accelerator School, Superconductivity in Particle Accelerators, CERN 89-04, CERN, March 1989.
4. R. Benaroya and W. J. Ramler, *Deflection Coil for an External Accelerator Beam*, Nuclear Instruments and Methods **10** (1961), 113.
5. *ARRL Handbook*, The American Radio Relay League, 1990.
6. Electrosonic catalog, p. 1264, Electro Sonic, Inc., Toronto.
7. M. T. Menzel and H. K. Stokes, *User's Guide for the POISSON/SUPERFISH Group of Codes*, Los Alamos National Laboratory Report LA-UR-87-115, January, 1987.
8. J. L. Chuma, *Plotdata Command Reference Manual*, TRIUMF Report TRI-CD-87-03b, TRIUMF, June, 1991.
9. *Reference Data for Radio Engineers, 5th Edition*, Howard W. Sams and Co. Inc., New York (1970).
10. *Radiotron Designer's Handbook*, F. Langford-Smith Ed., Wireless Press, Sydney, Australia (1953), p 144ff.

Table 5

The command file for the POISSON run

```
$set def scr0:[stinson]
$dele plot.ps;*
$dele out*.lis;*
$dele tape*.dat;*
$run automesh
arm45.in
$run lattice
tape73
 *9 2.540000
 s
$run poisson
tty
0
 *6 0 *32 2 *42 1 *43 101 *44 1 *45 101
 *46 6 *110 10 31 1. 90. 2.125 0.
 s
-1
$run psfplot
1 0 20 s
s
go
-1 s
$rename plot.ps    arm45.ps
$rename outaut.lis arm45.aut
$rename outlat.lis arm45.lat
$rename outpoi.lis arm45.poi
$dele tape*.dat;*
```

Table 6

The data file for the POISSON run

```
2A steering magnet - Armature type - 1st quadrant only
$reg   nreg=11,npoint=5,mat=1
 xmin=0.000000,xmax=5.2000,dx=0.05000
,ymin=0.000000,ymax=5.2000,dy=0.05000
$end
$po  x=0.00000,y=0.00000 $end
$po  x=5.20000,y=0.00000 $end
$po  x=5.20000,y=5.20000 $end
$po  x=0.00000,y=5.20000 $end
$po  x=0.00000,y=0.00000 $end
$reg npoint=41, mat=2 $end
$po  x=2.125000,y=0.000000 $end
$po  x=2.124516,y=0.045336 $end
$po  x=3.620713,y=0.176237 $end
$po  x=3.596310,y=0.455171 $end
```

Table 6 (Continued)

```
$po  x=2.100113,y=0.324271 $end
$po  x=2.084368,y=0.413566 $end
$po  x=3.535104,y=0.802289 $end
$po  x=3.462634,y=1.072749 $end
$po  x=2.011898,y=0.684025 $end
$po  x=1.980886,y=0.769230 $end
$po  x=3.342081,y=1.403965 $end
$po  x=3.223748,y=1.657731 $end
$po  x=1.862553,y=1.022996 $end
$po  x=1.817217,y=1.101521 $end
$po  x=3.047512,y=1.962982 $end
$po  x=2.886910,y=2.192345 $end
$po  x=1.656616,y=1.330883 $end
$po  x=1.598332,y=1.400342 $end
$po  x=2.660345,y=2.462355 $end
$po  x=2.462355,y=2.660345 $end
$po  x=1.400342,y=1.598332 $end
$po  x=1.330883,y=1.656616 $end
$po  x=2.192345,y=2.886910 $end
$po  x=1.962982,y=3.047511 $end
$po  x=1.101521,y=1.817217 $end
$po  x=1.022996,y=1.862553 $end
$po  x=1.657731,y=3.223748 $end
$po  x=1.403965,y=3.342081 $end
$po  x=0.769230,y=1.980886 $end
$po  x=0.684025,y=2.011898 $end
$po  x=1.072749,y=3.462635 $end
$po  x=0.802289,y=3.535104 $end
$po  x=0.413566,y=2.084368 $end
$po  x=0.324271,y=2.100113 $end
$po  x=0.455171,y=3.596310 $end
$po  x=0.176237,y=3.620713 $end
$po  x=0.045336,y=2.124516 $end
$po  x=0.000000,y=2.125000 $end
$po  x=0.000000,y=5.000000 $end
$po  x=5.000000,y=0.000000,nt=2 $end
$po  x=2.125000,y=0.000000 $end
$reg  npoint=5,mat=1,cur=459. $end
$po  x=2.124279,y=0.055354 $end
$po  x=3.620213,y=0.186231 $end
$po  x=3.597553,y=0.445242 $end
$po  x=2.101619,y=0.314364 $end
$po  x=2.124279,y=0.055354 $end
$reg  npoint=5,mat=1,cur=441. $end
$po  x=2.082394,y=0.423390 $end
$po  x=3.532876,y=0.812045 $end
$po  x=3.465582,y=1.063186 $end
$po  x=2.015101,y=0.674531 $end
$po  x=2.082394,y=0.423390 $end
$reg  npoint=5,mat=1,cur=414. $end
$po  x=1.977237,y=0.778562 $end
$po  x=3.338193,y=1.413186 $end
```

Table 6 (Continued)

```
$po  x=3.228312,y=1.648826 $end
$po  x=1.867356,y=1.014202 $end
$po  x=1.977237,y=0.778562 $end
$reg  npoint=5,mat=1,cur=373.5 $end
$po  x=1.812003,y=1.110077 $end
$po  x=3.042081,y=1.971387 $end
$po  x=2.892951,y=2.184367 $end
$po  x=1.662873,y=1.323057 $end
$po  x=1.812003,y=1.110077 $end
$reg  npoint=5,mat=1,cur=324. $end
$po  x=1.591711,y=1.407863 $end
$po  x=2.653537,y=2.469690 $end
$po  x=2.469689,y=2.653537 $end
$po  x=1.407864,y=1.591711 $end
$po  x=1.591711,y=1.407863 $end
$reg  npoint=5,mat=1,cur=261. $end
$po  x=1.323057,y=1.662873 $end
$po  x=2.184367,y=2.892951 $end
$po  x=1.971387,y=3.042081 $end
$po  x=1.110077,y=1.812003 $end
$po  x=1.323057,y=1.662873 $end
$reg  npoint=5,mat=1,cur=193.5 $end
$po  x=1.014202,y=1.867356 $end
$po  x=1.648826,y=3.228312 $end
$po  x=1.413186,y=3.338193 $end
$po  x=0.778562,y=1.977237 $end
$po  x=1.014202,y=1.867356 $end
$reg  npoint=5,mat=1,cur=117. $end
$po  x=0.674531,y=2.015101 $end
$po  x=1.063186,y=3.465583 $end
$po  x=0.812045,y=3.532875 $end
$po  x=0.423390,y=2.082394 $end
$po  x=0.674531,y=2.015101 $end
$reg  npoint=5,mat=1,cur=40.5 $end
$po  x=0.314364,y=2.101618 $end
$po  x=0.445242,y=3.597553 $end
$po  x=0.186231,y=3.620213 $end
$po  x=0.055354,y=2.124279 $end
$po  x=0.314364,y=2.101618 $end
```

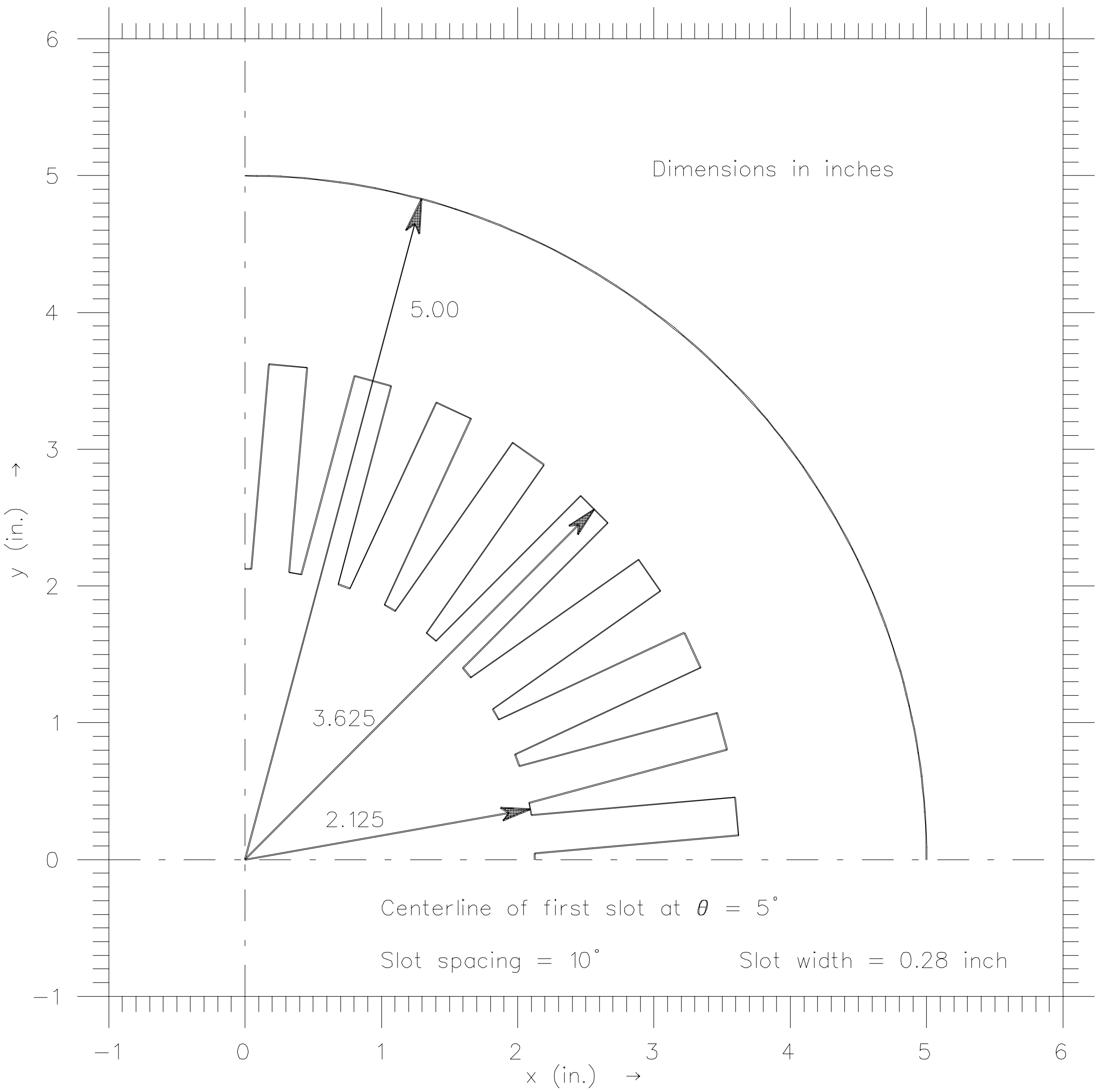

Fig. 4. The geometry used in the POISSON calculation.

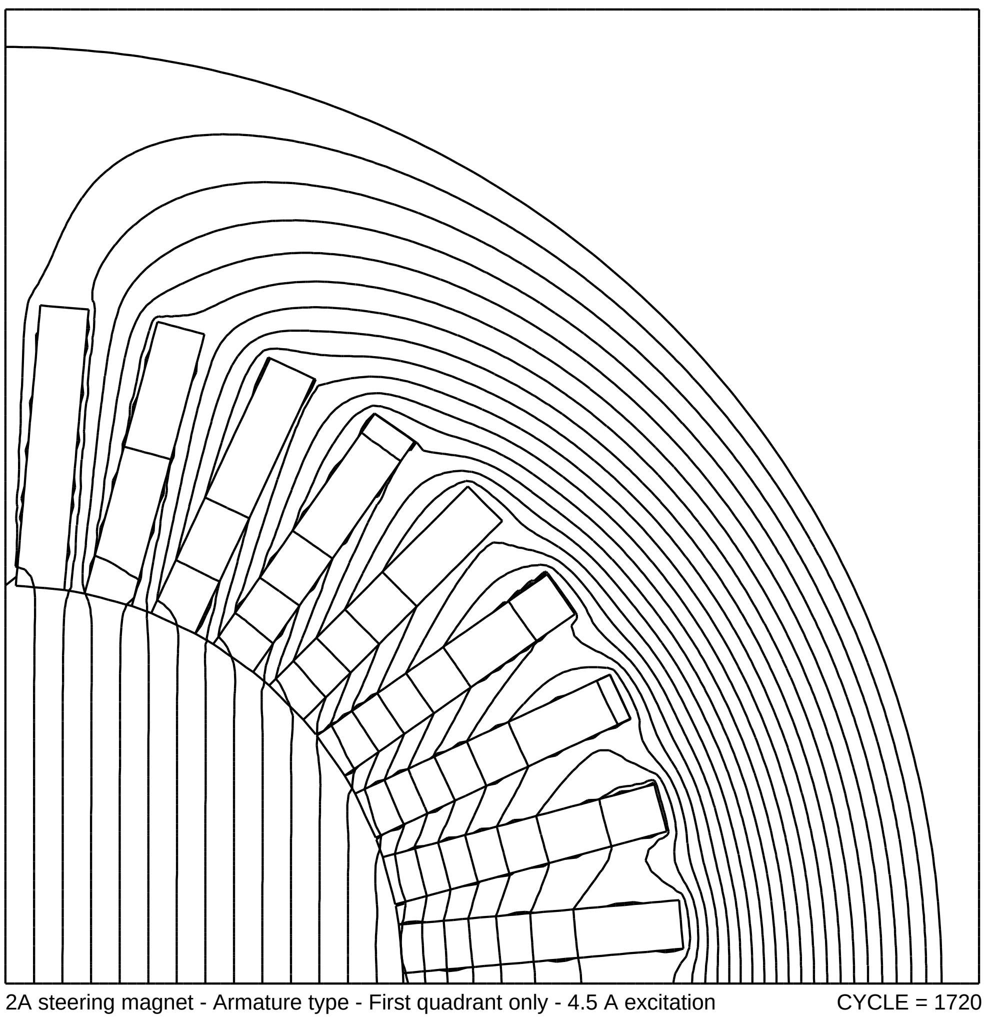

Fig. 5. The field lines predicted by POISSON.

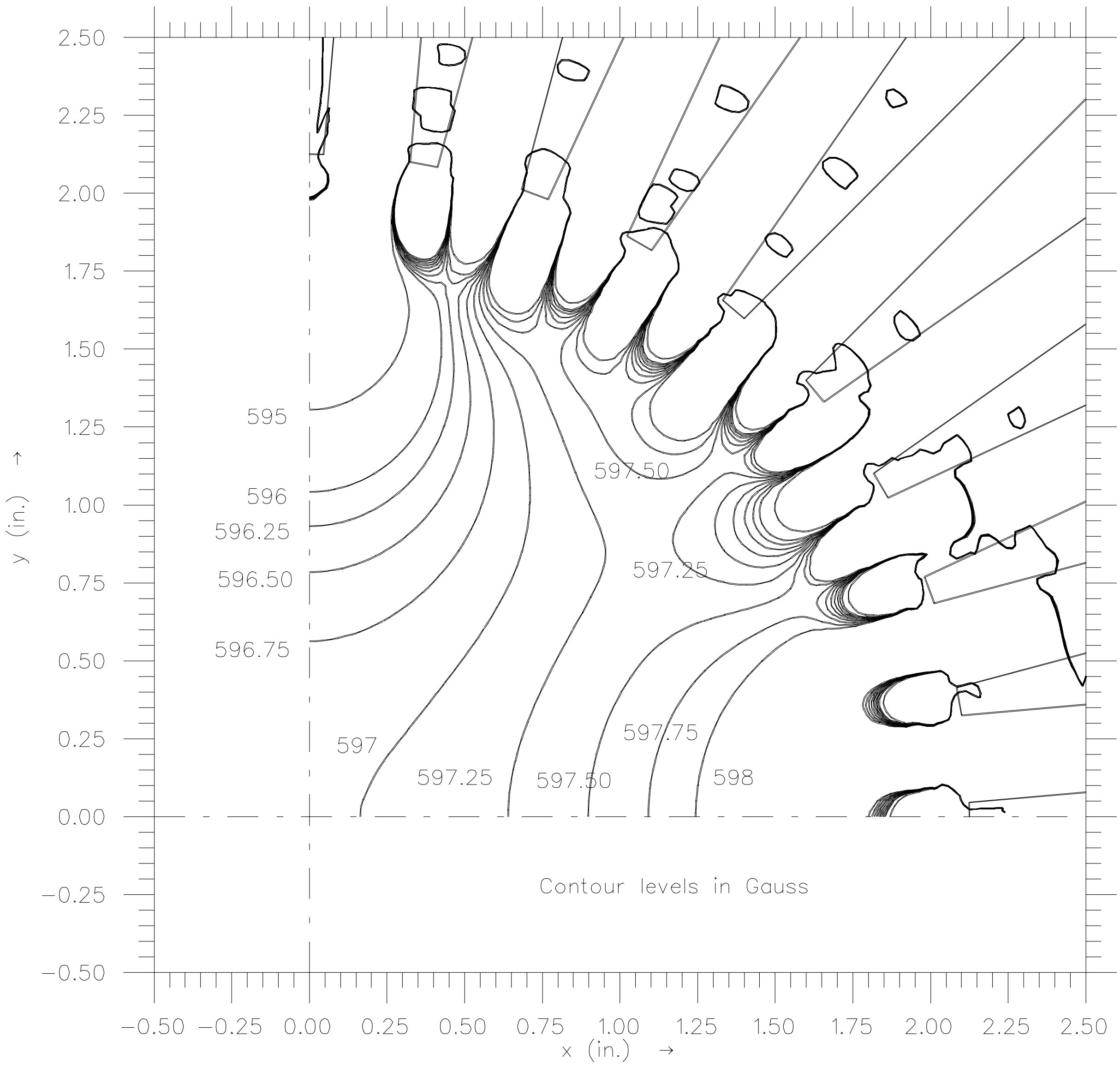

Fig. 6. A contour plot of the field predicted in the gap by POISSON.

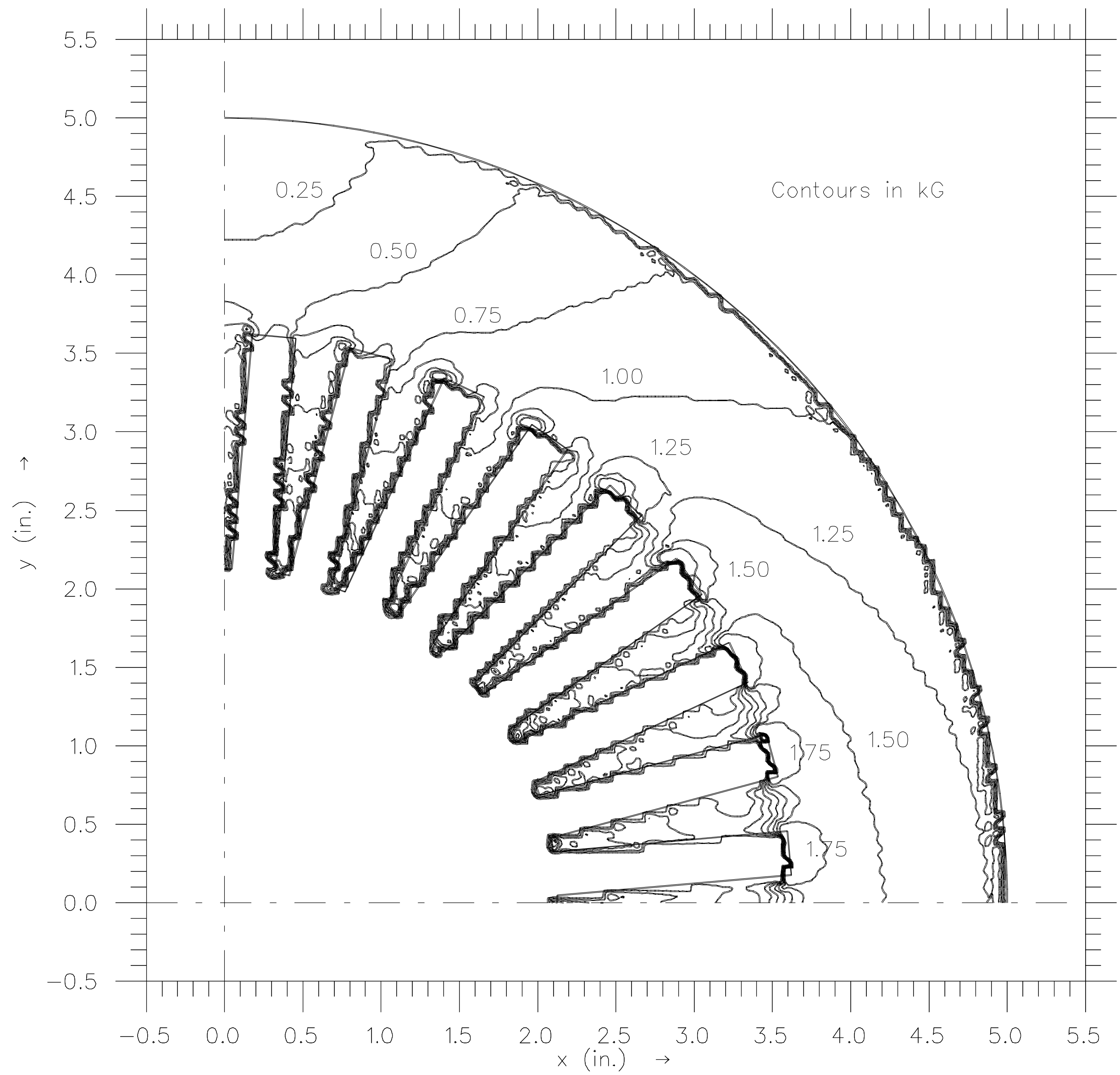

Fig. 7. A contour plot of the field predicted in the yoke by POISSON.