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Author GM Stinson		*Page* 1 *of* 9
Subject Predicted operating parameters of the beam line 2A switching magnet		

1. Introduction

In November of 2001 D. Evans[1)] measured the operating parameters of the $\pm 15°$ switching magnet for beam line 2A. This magnet is designed to feed the primary 500 MeV proton beam to either the existing (west) target of the ISAC facility or the new (east) target that is to be installed in the January 2002 shutdown.

This report presents the results of these measurements and of a calculation to determine the appropriate operating parameters for each target.

2. Measurements

The measurements made on this magnet consisted of water flow, pressure drop, temperature, and resistance characteristics at an excitation current of 550 A. In addition, a full midplane field map was obtained at an excitation current of 500 A.

For completeness, table 1 lists the properties of the magnet that were measured at the higher current. The program PLOTDATA[2)] was used to fit the $B-I$ and inverse $I-B$ data that was obtained. Figure 1 shows the $B-I$ data and the results of both linear and quadratic fits found for it. Similarly, figure 2 shows the (inverse) $I-B$ fits that were obtained from the measured data.

3. Ray trace calculations

Because only a midplane field map was obtained, we will assume in what follows that the beam remains in the horizontal plane. A computer program was written to compute the trajectory of the beam using a method described by Coggeshall and Muskat[3)]. This technique is outlined in the following.

3.1 Outline of the calculation

We consider a positive particle entering a non-uniform magnetic field. A Cartesian coordinate system is set up with its origin at the point of entry into the field. The positive z-axis is directed upward and the positive x-axis directed to the right. The positive y-axis points in the direction of particle motion and forms the remainder of the right-handed reference frame. We ignore field variation in the z direction and assume the particle remains in the x-y plane. For generality, we assume that the particle enters the field such that its velocity vector makes an angle θ with the positive y-axis. Then in our case we have at the point of entry into the field

$$\boldsymbol{v} = v_x\,\boldsymbol{i} + v_y\,\boldsymbol{j} \tag{1}$$

and

$$\boldsymbol{B} = B_z\boldsymbol{k} \tag{2}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are unit vectors along the x-, y-, and z-axes respectively. Then for a particle of charge q the equation of motion becomes

$$\boldsymbol{F} = q\,\boldsymbol{v}\times\boldsymbol{B} = q\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ v_x & v_y & 0 \\ 0 & 0 & B_z \end{vmatrix} = q[v_yB_z\,\boldsymbol{i} - v_xB_z\,\boldsymbol{j}]\ . \tag{3}$$

We now set

$$B_z = B_z(y) = B_0\, h(y)\ , \tag{4}$$

that is, we assume that the field is a function of y only. In fact, the field is a function of both x and y, but for any value of y there is a unique value of x defining the field at the point (x, y). We then write

$$\begin{array}{lllll} F_x & = & m\,\ddot{x} & = & q\,v_y\,B_0\,h(y) & = & q\,B_0\,h(y)\,\dot{y} \\ F_y & = & m\,\ddot{y} & = & -q\,v_x\,B_0\,h(y) & = & -q\,B_0\,h(y)\,\dot{x} \end{array}$$

Thus we have

$$\left.\begin{array}{lll} \ddot{x} & = & k\,h(y)\,\dot{y} \\ \\ \ddot{y} & = & -k\,h(y)\,\dot{x} \end{array}\right\} \qquad k = \frac{qB_0}{m} \tag{5}$$

Defining

$$f(y) = \frac{k}{v}\int h(y)\,dy = \frac{q\,B_0}{m\,v}\int h(y)\,dy = \frac{1}{\rho}\int h(y)\,dy\ ,$$

we find from equation (5) above

$$\int_0^t d(\dot{x}) \quad = \quad k\int_0^t h(y)\,\dot{y}\,dt = k\int_0^y h(y)\,dy\ ,$$

that is

$$\dot{x}(t) - \dot{x}(0) = \dot{x}(t) - v_x = f\,v \tag{6}$$

$$\boxed{\dot{x}(\mathrm{t}) = v_x + v\,f = v(f + v_x/v)}$$

But $\dot{x}^2 + \dot{y}^2 = v^2$ so that

$$\dot{y} = \sqrt{v^2 - \dot{x}^2} = \sqrt{v^2 - [v_x + v\,f]^2} = v\sqrt{1 - \left[f + \frac{v_x}{v}\right]^2}\ . \tag{7}$$

$$\boxed{\dot{y} = v\sqrt{1 - \left[f + \frac{v_x}{v}\right]^2}}$$

where the positive sign is used because $v_y = \dot{y}$ is in the positive y direction. Further, because

$$\frac{\dot{x}}{\dot{y}} = \frac{dx}{dy}\ , \tag{8}$$

we have

$$\frac{dx}{dy} = \frac{\dot{x}}{\dot{y}} = \frac{f + v_x/v}{\sqrt{1 - (f + v_x/v)^2}}$$

or

$$dx = \frac{\dot{x}}{\dot{y}} = \frac{f + v_x/v}{\sqrt{1 - (f + v_x/v)^2}}\,dy\ . \tag{9}$$

Thus we have

$$\boxed{x = \int_0^y \frac{f + v_x/v}{\sqrt{1 - (f + v_x/v)^2}}\,dy}$$

In the special case of a *uniform* field, $h(y) = 1$, $f(y) = \dfrac{1}{\rho}\displaystyle\int_0^y dy = \dfrac{y}{\rho}$ and we have for $\theta = 0$ (and $v_x = 0$)

$$
\begin{aligned}
x = \int_0^Y \frac{h(y)}{\sqrt{1-(h(y))^2}}\,dy &= \int_0^Y \frac{y}{\rho}\,\frac{dy}{\sqrt{1-(y/\rho)^2}} \\
&= \rho\int_0^T \frac{t}{\sqrt{1-t^2}}\,dt \\
&= -\rho\,\sqrt{1-t^2}\,\Big|_0^T \\
&= -\rho\,\sqrt{1-\frac{y^2}{\rho^2}}\,\Bigg|_0^Y \\
&= -\rho\left\{\sqrt{1-\frac{Y^2}{\rho^2}}-1\right\} \qquad (10)
\end{aligned}
$$

in which we have made the substitutions $t = y/\rho$ and $dt = dy/\rho$. From the above it follows that

$$(x - \rho)^2 = \rho^2\left[1-\frac{y^2}{\rho^2}\right] = \rho^2 - y^2$$

or

$$\boxed{(x - \rho)^2 + y^2 = \rho^2 \;,}$$

that is, the trajectory of the particle is a circle of radius ρ.

3.2 Calculations and results

As noted above, a program was written to trace the path of a particle through the measured field map using the method outlined above. The field map was measured at a current of 500 A and produced a field of 10.183 kG at the (nominal) geometric center of the dipole. This (maximum) field value was itself scaled to produce an overall deflection of $\pm 15°$ at the exit of the measured field grid. It was found that the required deflection was found for at a field of 10.066 kG. This corresponds to an excitation current of 494.7 A using the fitted expression for the $I - B$ data.

Magnetic measurements were on a grid 16 inches wide by 60 inches long with a grid spacing of 0.5 inch in each direction. The grid was centered on the geometric center of the magnet. To interpolate between grid points, a bilinear interpolation routine was used. This routine was adapted for FORTRAN from the technique given in ref[4)].

The steps along the beam direction were fixed at 0.1 inch (0.254 cm). Initial values for the field were taken to be those values measured along the centerline of the magnet. Using the values of (y) positions and of the (interpolated fields, values of the horizontal (x) deflection were calculated. These calculated values of x and y positions were then used to calculate another set of fields from which new x displacements were obtained. It was found that four (4) iterations were required before the ratio of the new and old values of x differed from unity by less than one part in 10^5.

Figure 3 shows the computed trajectories for a 500 MeV particle for beam delivery to each the two target locations. The outline of the yoke and the pole of the magnet is also indicated, although their locations are only approximate. Note that in this (and the following figures) the positive x-axis is plotted to the left (looking downstream) and that its direction is opposite to that used in the calculation. Figure 4 is a similar plot in which the calculated trajectories are overlaid on a contour plot of the measured field. The contour intervals are in kG. The pole and yoke of the magnet are indicated by the dotted lines.

Figure 5 is identical to figure 4 except that the horizontal deflections are shown on a enlarged scale.

Figure 6 shows a comparison of the field calculated along the trajectory and that measured along the geometric centerline of the magnet. Because it has been assumed that a particle enters normal to the upstream edge of the dipole, the particle will encounter fields similar to those on the centerline until it gets sufficiently off axis that different fields are encountered. This is clearly shown in this figure.

From the calculated data we may deduce an effective length by integrating the field along the trajectory. We find

$$\int B\, d\ell \quad = \quad 9.520466 \text{ kG-m}$$

and

$$\frac{\int B\, d\ell}{B_{0,0}} \quad = \quad \frac{9.520466 \text{ kG-m}}{10.06570 \text{ kG}} \; = \; 0.945833 \text{ m} = 37.238 \text{ inches},$$

where the integral is computed along the particle trajectory. This compares well with the estimate of 37.20 inches given in ref[5)].

4. Discussion

This report presents a study of the characteristics to be expected from the $\pm 15^{\circ}$ switching dipole that is to be installed in beam line 2A in the spring of 2002. The study was undertaken primarily to find the operating current at which the dipole should be run. To that end it is suggested that an operating current of approximately 495 A is required at a beam energy of 500 MeV.

References

1. D. Evans, *Private communication*, TRIUMF, November, 2001.
2. J. L. Chuma, *PLOTDATA Command and Reference Manual*, TRIUMF Report TRI-CD-87-03b, June, 1991.
3. N. D. Coggeshall and M. Muskat, *Phys. Rev.*, **66** (1944) 187.
4. William H. Press, Saul A. Teukolsky, William T. Vettering, and Brian P. Flannery, *Numerical Recipes in C*, 2nd edition, *pp* 123ff, Cambridge University Press, Cambridge, U. K., 1999.
5. George S. Clark, *Concept Design of the 2A 15 degree Switching Dipole*, TRIUMF report TRI-DN-00-18, TRIUMF, June, 2000.

Table 1

Measured characteristics at 550 A

Insulation resistance	1.GΩ at 1000 V
Water flow	3.0 IGPM
Pressure at inlet	90. psi
Pressure at outlet	28. psi
Pressure drop across magnet	72. psi
Temperature at inlet	22.0°C
Temperature at outlet	41.2°C
Temperature rise at 550 A	19.2°C
Voltage required (hot)	36.5 V
Magnet resistance (hot)	0.066 Ω

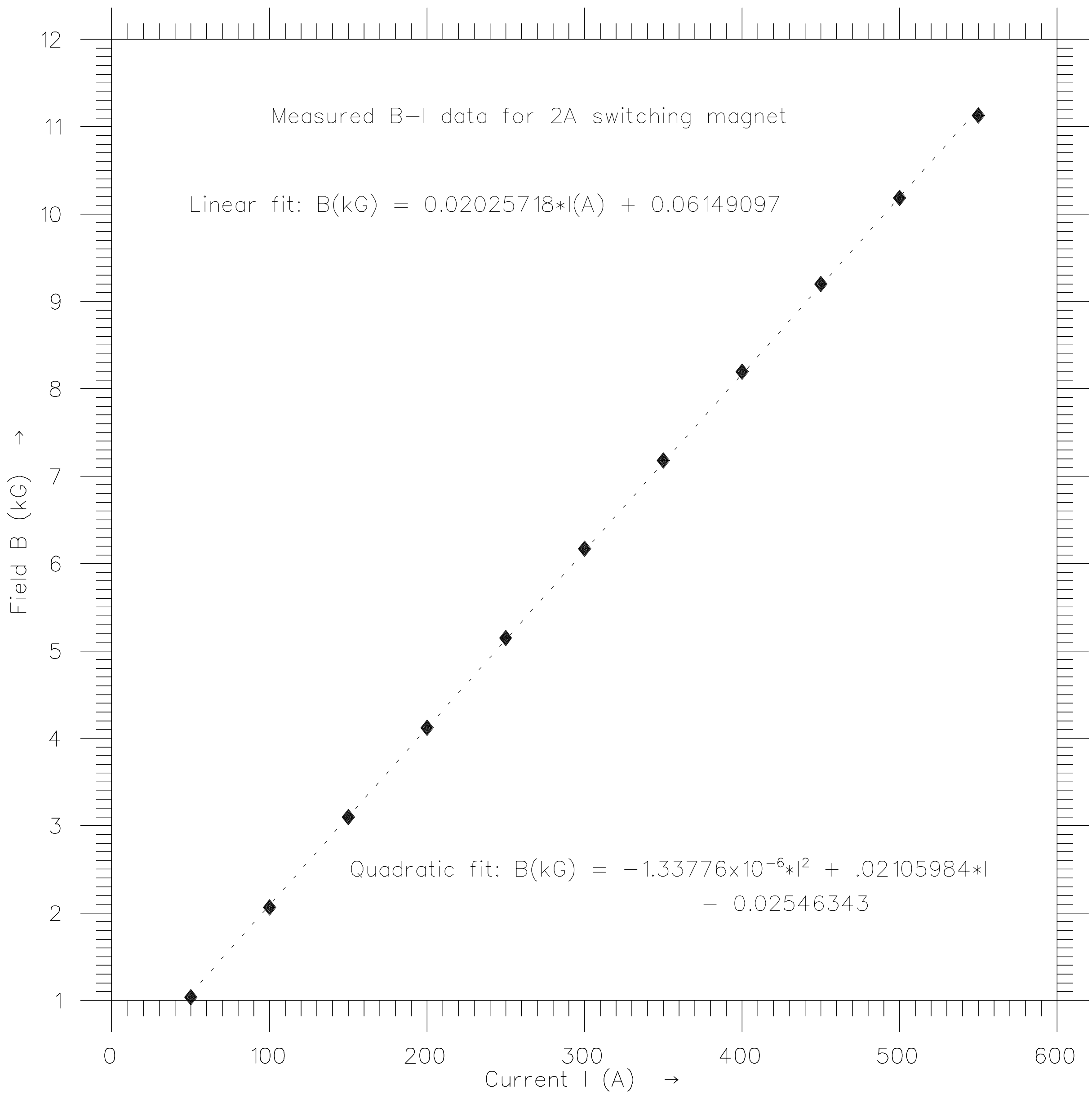

Fig. 1. The measured and fitted $B - I$ characteristics of the beam line 2A switching magnet.

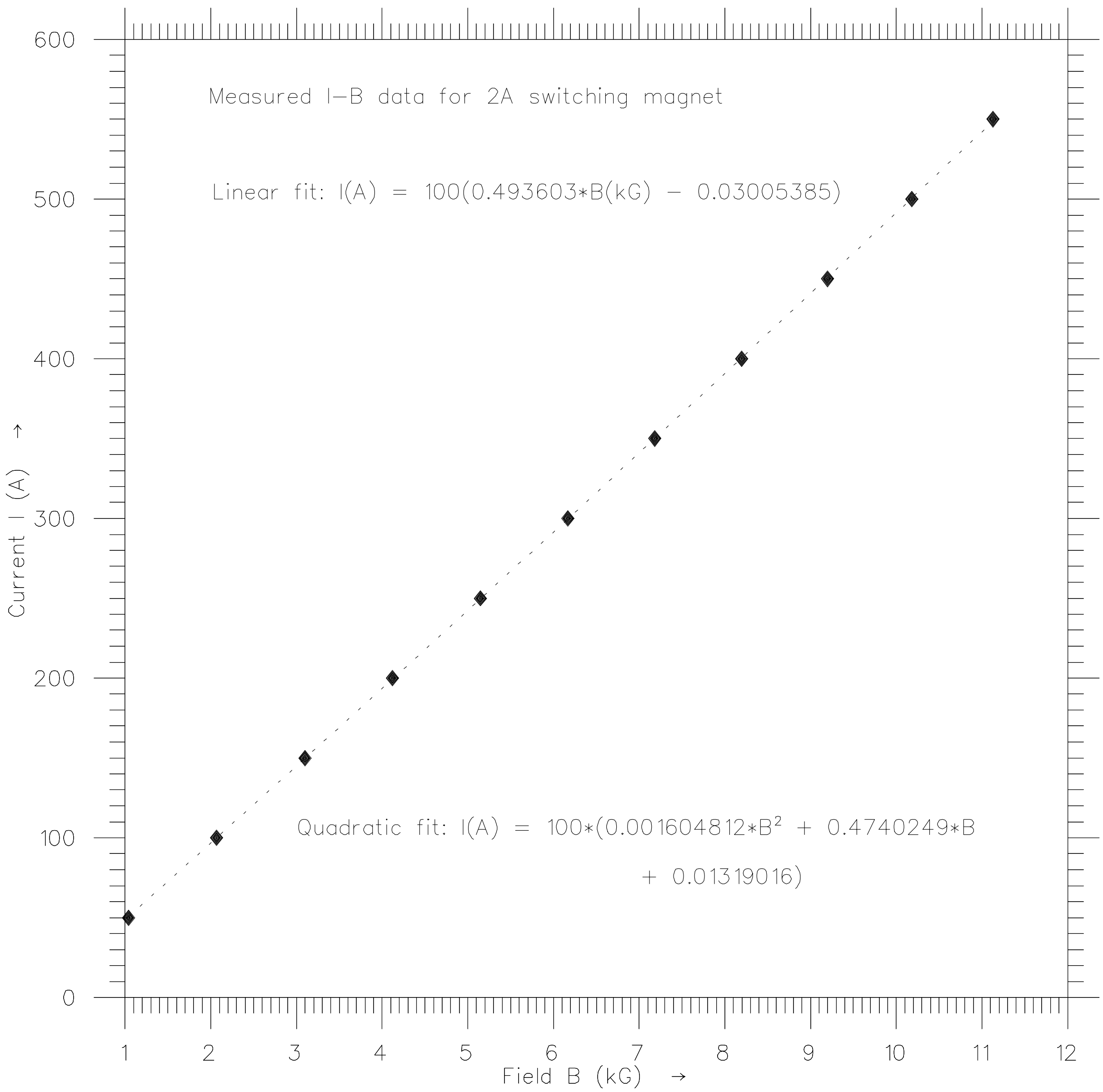

Fig. 2. The measured and fitted $I - B$ characteristics of the beam line 2A switching magnet.

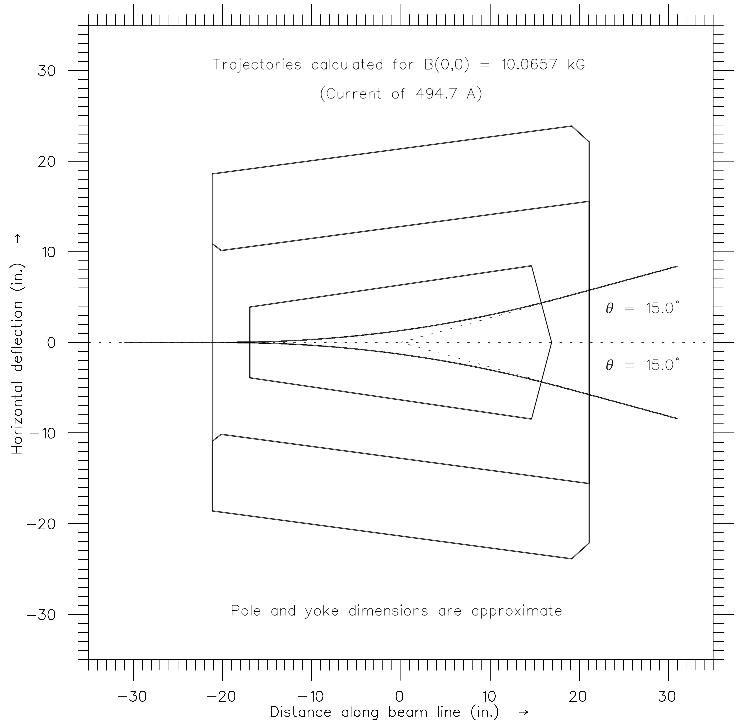

Fig. 3. The computed trajectories in the beam line 2A switching magnet.

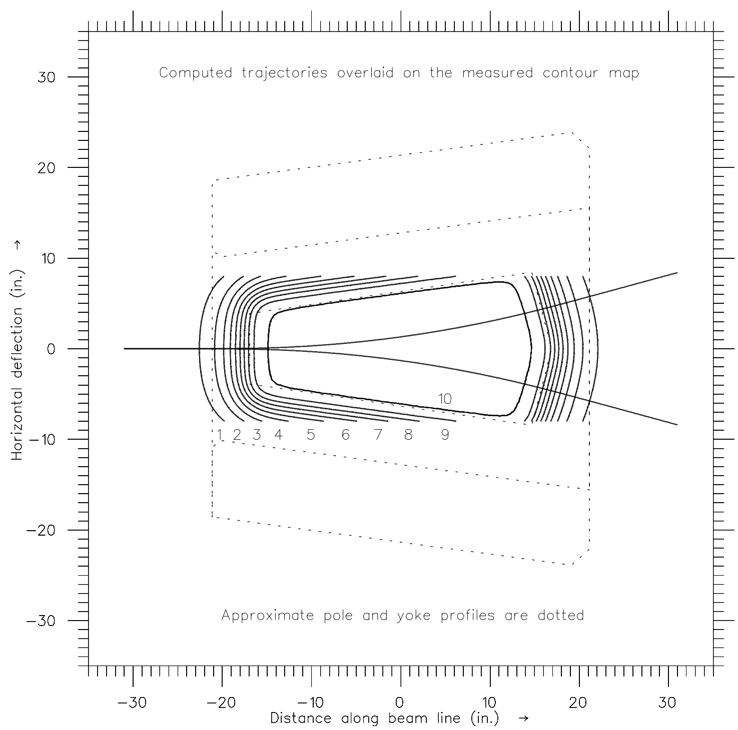

Fig. 4. The computed trajectories and measured field contours of the beam line 2A switching magnet.

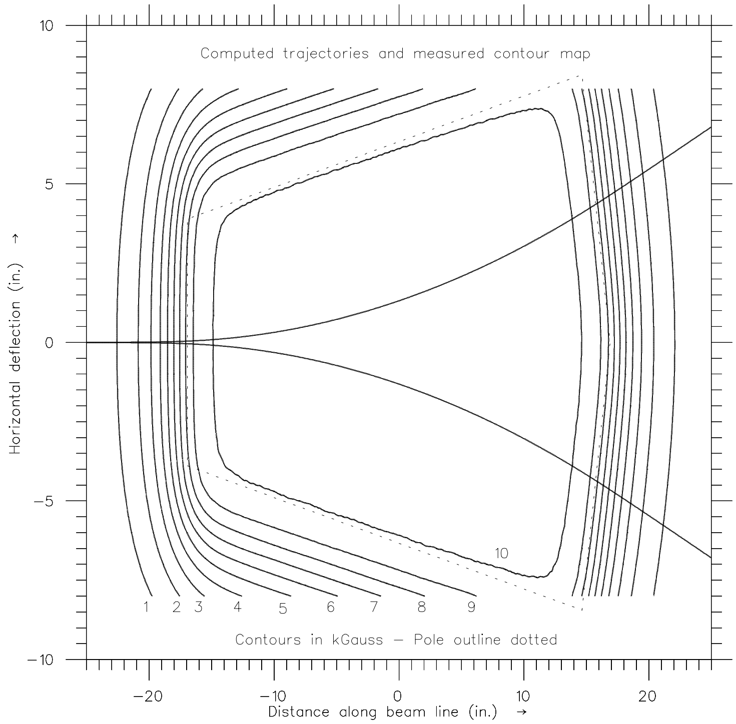

Fig. 5. The computed trajectories and contour plot of the beam line 2A switching magnet.

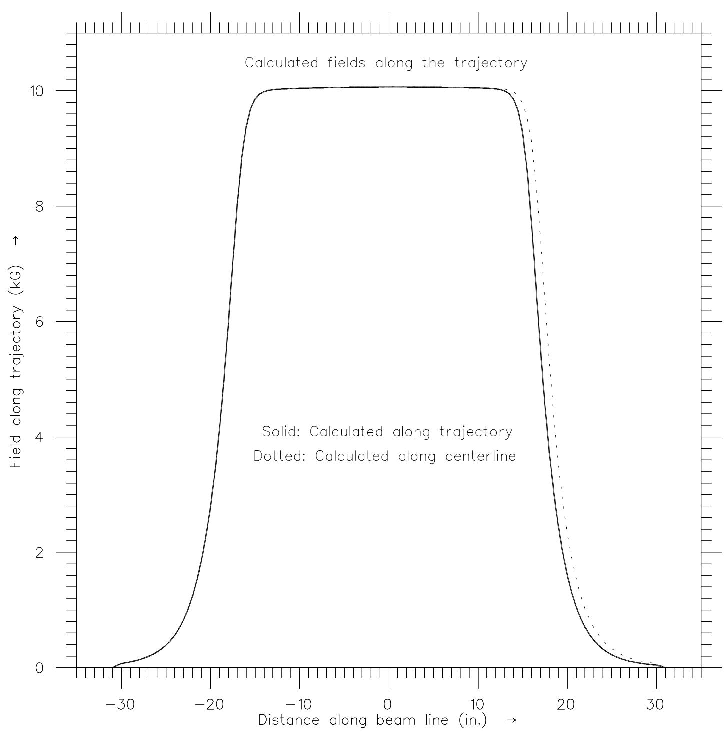

Fig. 6. The difference between the fields along the computed trajectories and that along the centerline of the 2A switching magnet.