1. Introduction

The transfer beamline from the ISAC1 DTLs to the medium-β section of the ISAC2 accelerator complex requires four identical 59.56° rectangular dipoles. Their design is complicated somewhat by the tight spatial constraints in both the ISAC1 HEBT beamline and the ISAC2 complex. In the former it is desired that the Prague magnet and its object slits remain in their present locations; in the latter space is restricted between the low-β and medium-β portions of the accelerators.

A previous report [1] presented a study of an H-frame design for the dipoles. This report presents a study of an arc-shaped design for the magnets—that is, magnets that have curved yokes and poles that are shaped to conform with the central trajectory of the beam.

2. Design parameters

We repeat here the basic design parameters of the ISAC1 to ISAC2 transfer beamline that are given in ref [2]. That report gives the following specifications.

Beam parameters:	Maximum beam height y_{max}	1.5 cm (0.5906 in.)
	Maximum beam width x_{max}	2.0 cm (0.7874 in.)
Dipole parameters:	Bend angle of dipole θ	59.56°
	Radius of curvature ρ_0	0.920 m (36.220 in.)
	Maximum $(B\rho)_0$	1.2 T-m
	Maximum dipole field B_0	1.3 T
	Magnet air gap g	5.0 cm (1.9685 in.)

3. Derived iron parameters

We have chosen to design the dipole for a maximum field of 13.5 kG (1.35 T) and an air gap of 2.0 inches (5.08 cm). Combining these and the values given above we obtain the derived parameters listed below.

Maximum *proton* momentum	=	$[B_0(\mathrm{T})\rho_0(\mathrm{m})]/3.3356$	=	0.37235 GeV/c
Maximum *proton* energy			=	71.182 MeV
Effective length along arc of trajectory	=	$\rho_0\theta$	=	0.95636 m (37.652 in.)
Straight-line effective length	=	$2\rho_0\sin(\theta/2)$	=	0.91388 m (35.979 in.)

3.1 Iron length

We take the iron length to be the straight-line effective length minus the full aperture. Thus

$$\text{Iron length} = 0.91388 \text{ m} - 0.0508 \text{ m} = 0.8631 \text{ m} = 33.980 \text{ in.}$$

We take

$$\boxed{\text{Iron length} = L_{iron} = 0.8636 \text{ m} = 34.000 \text{ inches.}}$$

3.2 Ampere turns

The required Ampere-turns per pole are calculated from the relation

$$\text{NI per pole} = \frac{1}{2}\left[1.1\,\frac{B_0(\mathrm{T})\,g(\mathrm{m})}{\mu_0}\right] = \frac{1}{2}\left[1.1\,\frac{(1.35)(0.0508)}{4\pi\times10^{-7}}\right] = 30,020 \text{ A-t}$$

where we have allowed for a flux leakage of 10%. Anticipating the POISSON results that follow we take

$$\boxed{\text{NI per pole} = 28{,}500 \text{ A-t.}}$$

3.3 Pole width

It is in this aspect of the design that an arc-shaped dipole differs from an H-frame magnet. In the latter, the deviation of the central trajectory from a line joining the entrance and exit points is found from

$$\Delta = \rho_0[1 - \cos(\theta/2)]$$

and amounts to 4.783 inches (0.1215 m) in our case. To obtain a reasonable field uniformity, the pole-width must have this dimension factored into it. In the former, this is not necessary because the pole is symmetric about the beam. Consequently, the width of he pole—and thus of the magnet—will be reduced by a comparable amount.

As in ref [1] on each side of the pole we assume a shim of 0.25 inch at 45° plus a 0.625 inch flat plus a 0.625 inch at 45° taper. From that point the pole tapers 0.50 inch to the pole root. Thus each pole side has a shim 1.5 inches wide to which is added an additional 0.5 inch taper. The nominal pole width is then

$$\begin{aligned} \text{Pole width} &= 2(\text{Shim} + \text{Taper} + \text{Gap}) + \text{Maximum beam width} \\ &= 2(1.50 \text{ in.} + 0.50 \text{ in.} + 2.00 \text{ in.}) + 0.7874 \text{ in.} \\ &= 8.787 \text{ inches} = 0.22320 \text{ m.} \end{aligned}$$

We will require a field uniformity of better than 0.1% over the area traversed by the beam. The required uniformity, again anticipating the POISSON results below, is obtained if we choose

$$\boxed{\text{Minimum base width of pole} = 11.00 \text{ inches} = 0.2794 \text{ m.}}$$

This is considerably smaller than the 18 inches that was deemed necessary for the H-frame dipoles of ref [1]. However, field uniformity is not the only factor in the determination of the width of the pole. Another factor is amount of saturation, if any, in the pole itself. Consequently, for reasons that will become evident, we consider here pole (base) widths of 11 inches, 12.5 inches, and 14 inches.

We note that the pole-side shims and tapers constitute a total width of 4 inches. Thus the pole dimensions above imply that the width of the flat portion of the pole is 7 inches, 8.5 inches, and 10 inches, respectively, for the cases that will be considered. In what follows, magnets with pole-flats of these widths will be referred to as "7-inch flat", "8.5 inch flat", and "10-inch flat", respectively.

4. Coil design

In §3.2 above we determined that the required Ampere=turns per pole were 28,500. From this we have the following table of Amperes versus turns.

I (Amperes)	100	200	300	400	500	600	700	800	900	1000
N (Turns)	285	142	95	71	57	47	41	36	32	28

We choose

$$\boxed{\begin{array}{lcl} \text{I} &=& \text{maximum current} \quad = \quad 500 \quad \text{Amperes} \\ \text{N} &=& \text{number of turns per coil} \quad = \quad 56 \end{array}}$$

in a coil configuration 7 turns wide and 8 layers high.

We assume a current density of 3000 A/in.^{2} = 4.65 A/mm^2 and calculate the required conductor area from

$$\text{Conductor area} = \frac{500\ \text{A}}{3000\text{A/in.}^2} = 0.17\ \text{in.}^2 = 108\ \text{mm}^2.$$

This is satisfied within 10% by Anaconda 0.460 and 0.516 inch-square conductors. Their parameters are listed below.

OD (in.)	0.460	0.516
ID (in.)	0.255	0.287
Copper area (in.^{2})	0.1529	0.1940
Cooling area (in.^{2})	0.05107	0.06469
Mass (lb/ft)	0.5910	0.7495
Resistance at 20°C ($\mu\Omega$/ft)	53.25	41.99
k (British units)	0.01760	0.01520

The details of the determination of the dimensions of the coils are given in ref[1] and will not be repeated here. They are summarized below. For the *width* of the coil we take

	0.460 square	0.516 square
Maximum coil width	3.500 inches	3.900 inches
Nominal coil width	3.400 inches	3.800 inches

and for the *height* of the coil we take

	0.460 square	0.516 square
Maximum coil height	4.100 inches	4.500 inches
Nominal coil height	4.000 inches	4.400 inches

These dimensions are based on the assumptions of each conductor having a half-lapped insulation of tape with a thickness of 0.007 inch, a turn-to-turn separation of 0.010 inch, and a keystoning allowance of 0.010 inch.

4.1 Copper requirements

Calculation of the amount of copper required for the coils differs from that for an H-frame magnet because of the coils of an arc-shaped magnet are curved. We begin by taking the conductor dimension D_{nom} to be

$$D_{nom} = \text{Nominal dimension} + 4(\text{Insulation thickness}) + \text{Turn separation},$$

assume a pole-coil gap of $G = 0.375$ inches, and assume that the pole corners are rounded with a radius R_{pole} of

$$R_{pole} = 4\,D_{nom} - G\ .$$

Further, we define W_{slot}, the width of the coil slot, as

$$W_{slot} = \text{Maximum coil width} + 2(\text{Pole-coil gap})\ .$$

For the two conductors that we are considering we then have

	0.460 square	0.516 square
D_{nom} (inch)	0.498	0.554
R_{pole} (inches)	1.617	1.841
W_{slot} (inches)	4.250	4.650

The geometry of the magnet is defined by the radii R_0, R_1, R_2, R_3, R_4, and R_5 that are defined by

R_0 = Radius of curvature of inner yoke and is to be determined.
R_1 = Inner radius of curvature of inner coil slot $= \rho_0 - W_{pole}/2 - W_{slot}$,
R_2 = Outer radius of curvature of inner coil slot $= \rho_0 - W_{pole}/2$,
R_3 = Inner radius of curvature of outer coil slot $= \rho_0 + W_{pole}/2$,
R_4 = Outer radius of curvature of outer coil slot $= \rho_0 + W_{pole}/2 + W_{slot}$,
R_5 = Radius of curvature of outer yoke and is to be determined.

The values of R_1 through R_4 may now be determined; we find the following values as a function of pole width and conductor. All radii are given in inches and have been rounded up or down to produce 'reasonable' numbers.

	0.460 square			0.516 square		
	7-inch flat	8.5-inch flat	10-inch flat	7-inch flat	8.5-inch flat	10-inch flat
R_1	26.40	25.70	24.90	26.00	25.30	24.50
R_2	30.75	30.00	29.25	30.75	30.00	29.25
R_3	41.75	42.50	43.25	41.75	42.50	43.25
R_4	46.00	46.75	47.50	46.40	47.15	47.90

Consider first that portion of a coil in the outer coil slot. The (outer) radius of curvature of the nth conductor of a layer is

$$\rho_{n,outer} = R_3 + G + nD_{nom} + 4\,t_{ins}$$

with the term $4\,t_{ins}$ representing the thickness of the ground insulation. The angle subtended between the center of the magnet and the nth turn at the end of the magnet is

$$\theta_{n,outer} = \sin^{-1}\left[\frac{L_{iron}}{2\,\rho_{n,outer}}\right] ,$$

and the length of this portion of the turn is

$$\ell_{n,outer} = 2\,\rho_{n,outer}\,\theta_{n,outer} .$$

Similarly, the (inner) radius of curvature of the nth conductor of a layer in the inner coil slot is

$$\rho_{n,inner} = R_2 - G - nD_{nom} - 4\,t_{ins} ,$$

and the length of this portion of the nth turn is

$$\ell_{n,inner} = 2\,\rho_{n,inner}\,\theta_{n,inner}$$

with

$$\theta_{n,inner} = \sin^{-1}\left[\frac{L_{iron}}{2\,\rho_{n,inner}}\right] .$$

As in ref[1)] we take the (outer) radius of curvature at the ends of the pole to be

$$r_n = R_{pole} + G + n\,D_{nom} + 4\,t_{ins} ,$$

and we approximate the straight length of the coil at its end by

$$\ell_{straight} = W_{pole} - 2\,R_{pole} .$$

Thus we estimate the length of the nth turn of a layer to be

$$
\begin{aligned}
\ell_n &= 2[\rho_{n,inner}\,\theta_{n,inner} - R_{pole}] + 2[\rho_{n,outer}\,\theta_{n,outer} - R_{pole}] + 2\,\ell_{straight} + 2\,\pi\,r_n \\
&= 2[\rho_{n,inner}\,\theta_{n,inner} - R_{pole}] + 2[\rho_{n,outer}\,\theta_{n,outer} - R_{pole}] + 2[W_{pole} - 2\,R_{pole}] + 2\,\pi\,r_n \\
&= 2[\rho_{n,inner}\,\theta_{n,inner} + \rho_{n,outer}\,\theta_{n,outer} + W_{pole} + (\pi - 4)R_{pole} + \pi(G + n\,D_{nom} + 4\,t_{ins})]
\end{aligned}
$$

and the total length of an N-turn layer becomes $L_N = \sum_{n=1}^{N} \ell_n$.

In our case with $N = 7$ and inserting the appropriate values into the above we find the following estimated lengths of copper required for the coils. All lengths are given in inches.

	0.460 square			0.516 square		
	7-inch flat	8.5-inch flat	10-inch flat	7-inch flat	8.5-inch flat	10-inch flat
ℓ_1	96.03	99.13	102.25	96.00	99.11	102.23
ℓ_2	99.22	102.34	105.47	99.56	102.67	105.81
ℓ_3	102.43	105.55	108.69	103.12	106.25	109.40
ℓ_4	105.64	108.77	111.93	106.70	109.84	113.00
ℓ_5	108.85	112.00	115.18	110.28	113.44	116.62
ℓ_6	112.08	115.24	118.43	113.88	117.05	120.26
ℓ_7	115.31	118.50	121.71	117.48	120.68	123.91
L_7	739.56	761.53	783.65	747.02	769.04	791.21

Thus, depending on the pole-width and the conductor, the estimated length per 7-turn layer may be taken as (770 ± 20) inches.

(An optional approach is to note that the mean turn lies at a distance of approximately $(4\,D_n + G)$ from the pole sides and ends. The lengths of the curved sections of the winding are calculated as above using the angle $\theta_{n,j}$ obtained from

$$
\theta_{4,j} = \sin^{-1}\left[\frac{(L_{iron}/2) + (4\,D_n + G)}{\rho_{4,j}}\right] .
$$

Assuming a straight-line connection of length d between the inner and outer curved sections at each end of the dipole, the length of the mean turn may be estimated from

$$
L_{mean} = 2[\,\rho_{4,inner}\,\theta_{4,inner} + \rho_{4,outer}\,\theta_{4,outer} + d\,] .
$$

When this is done for the case of the 8.5 inch flat and the 0.460 inch square conductor we obtain a value for ℓ_4 of 111 inches. The length of the mean turn becomes $L_{mean} = 777$ inches, in good agreement with the value of 762 inches tabulated above.)

In subsequent calculations we will round this up to a value of 840 inches or 70 feet regardless of the pole/conductor configuration. Thus

$$
\text{Length of 7-turn layer } L_7 = 70 \text{ ft (21.3 m)} .
$$

and the length per coil becomes

$$
\text{Length of copper per coil} = 560 \text{ ft (171 m)}.
$$

Because 2 coils are required per dipole, then for either conductor we have

Total length per dipole	1120 ft	(341 m)
Allow 10% for winding losses	112 ft	(34 m)
Total	1232 ft	(375 m)

Then, for the 0.460-inch square conductor (of mass 0.5910 lb/ft) or for the 0.516-inch square conductor (of mass 0.7495 lb/ft) , we should order

	0.460-inch square		0.516-inch square	
A total length of	1250 ft	(381 m)	1250 ft	(381 m)
A total mass of	740 lb	(336 kg)	940 lb	(425 kg)

5. Power requirements

At 20°C, the resistance of the coil is:

$$R_{20^\circ C} = \text{(Resistance per foot)(Coil length)}$$

so that

$$\begin{aligned} R_{20^\circ C} &= (53.25\times10^{-6}\Omega/\text{ft})\times(560 \text{ ft}) = 0.0298\ \Omega \text{ [0.460-inch conductor]}. \\ &= (41.99\times10^{-6}\Omega/\text{ft})\times(560 \text{ ft}) = 0.0235\ \Omega \text{ [0.516-inch conductor]}. \end{aligned}$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C, and an outlet water temperature of 70°C (thus allowing a 40°C coolant temperature rise). Then the mean coil temperature will be 50°C. With a 30°C rise above ambient of the coil we then have:

$$\begin{aligned} R_{hot} &= R_{20^\circ C}[1 + (\text{Temperature coefficient}/^\circ\text{C})dT(^\circ\text{C})] \\ &= \begin{cases} 0.0298[1 + 0.00393(30)] = 0.0333\ \Omega \text{ per coil [0.460-inch conductor]}. \\ 0.0235[1 + 0.00393(30)] = 0.0263\ \Omega \text{ per coil [0.516-inch conductor]}. \end{cases} \end{aligned}$$

Thus at a current of 500 A we require a voltage of 16.7 V per coil of the smaller conductor and of 13.2 V per coil of the larger conductor. Therefore, allowing for lead loss, we choose power supplies that have

	0.460 square	0.516 square
I (A maximum)	500.0	500.0
V (V total)	40.0	35.0
P (kW total)	20.0	17.5

6. Cooling requirements

In these calculations we use the British system of units. The power required per coil is:

$$\begin{aligned} \text{Power per coil} &= I^2 R_{hot} \\ &= \begin{cases} (500)(500)(0.03334) = 8.334 \text{ kW [0.460 conductor]}. \\ (500)(500)(0.02816) = 6.572 \text{ kW [0.516 conductor]}. \end{cases} \end{aligned}$$

The required flow rate is given by:

$$v(\text{ft/sec}) = \frac{2.19}{\Delta T} \times \frac{P(\text{kW})}{\text{Cooling area (in.}^2)} .$$

For $\Delta T = 72^\circ\text{F} = 40^\circ\text{C}$ and choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit, we have

$$P_{max} = \begin{cases} \frac{(2.50)(72)(0.05107)}{2.19} = 4.198 \text{ kW/water circuit of 0.460 conductor}. \\ \frac{(2.50)(72)(0.06469)}{2.19} = 5.317 \text{ kW/water circuit of 0.516 conductor}. \end{cases}$$

From these we calculate the number of cooling circuits per coil (excluding lead loss) as

		0.460 square	0.516 square
P	Total power per coil	8.334 kW	6.572 kW
Number of circuits	P / P_{max}	1.99	1.24

Thus we take

$$\text{Number of cooling circuits per coil} = \begin{cases} 2 \text{ [0.460-inch conductor]} \\ 2 \text{ [0.516-inch conductor]} \end{cases}$$

This requires a flow rate of v = 2.48 ft/sec per water circuit of a coil of the smaller conductor and one of 1.55 ft/sec per water circuit of a coil of the larger conductor. The volume of flow required per circuit is found from

$$\begin{aligned} \text{Volume/circuit} &= 2.6(v(\text{ft/min}))(\text{Cooling area}(\text{in.}^2)) \text{ IGPM} \\ &= \begin{cases} 2.6(2.482)(0.05107) = 0.330 \text{ IGPM [0.460 conductor].} \\ 2.6(1.545)(0.06469) = 0.260 \text{ IGPM [0.516 conductor].} \end{cases} \end{aligned}$$

Thus the calculated flow requirements for the two types of coils are as follows.

	0.460 conductor			0.460 conductor		
	IGPM	ℓ/min	USGPM	IGPM	ℓ/min	USGPM
Volume per cooling circuit	0.330	1.500	0.396	0.260	1.182	0.312
Volume per coil	0.660	3.000	0.793	0.520	2.364	0.625
Volume per magnet	1.320	6.002	1.585	1.040	4.729	1.249

7. Pressure Drop

The pressure drop is given by

$$dP = k\,v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case, with k = 0.0176 for the 0.460 conductor and k = 0.0152 for the 0.516 conductor we obtain:

$$dP = \begin{cases} (0.0176)(2.482)^{1.79} = 0.08958 \text{ psi/ft [0.460 conductor],} \\ (0.03745)(1.655)^{1.79} = 0.037 \text{ psi/ft [0.516 conductor],} \end{cases}$$

and the total pressure drop across one cooling circuit is

$$\boxed{\text{Pressure drop per cooling circuit} = \begin{cases} 25.1 \text{ psi [0.460 conductor.]} \\ 10.5 \text{ psi [0.516 conductor.]} \end{cases}}$$

8. Iron dimensions

We have taken the width of the coil slot as

$$\begin{aligned} \text{Coil-slot width} &= \text{Maximum coil width} + 2(\text{Pole-coil gap}) \\ &= \begin{cases} 3.50 \text{ inches} + 2(0.375 \text{ inch}) = 4.25 \text{ inches [0.460 conductor].} \\ 3.90 \text{ inches} + 2(0.375 \text{ inch}) = 4.65 \text{ inches [0.516 conductor].} \end{cases} \end{aligned}$$

We assume that the field rises from zero at the outer edge of the coil slot to $0.6B_0$ at the pole flat and is constant at B_0 across the pole flat. In this context the *pole flat* is taken as the outer edge of the flat portion of the pole shim and the *coil slot* as the width of the coil slot plus the 0.5 inch and 0.625 inch tapers of the pole shim. We further assume that the flux divides equally between the side yokes. Setting

B_y and t as the yoke flux and thickness, respectively, we then may calculate the flux in each yoke by equating it to one-half of that through the gap region. Thus

$$B_y[t\,L_{iron}] \;=\; \frac{1}{2}\times\frac{2[W_{Coilslot}L_{Coilslot}][0.6B_0]}{2}+\frac{1}{2}\times[W_{poleflat}L_{poleflat}]B_0$$

where the W_i and L_i are the width and length, respectively, of each component in the expression. For an H-frame dipole all lengths are equal and thus divide out leaving the expression

$$B_y t \;=\; \frac{1}{2}\times\frac{2[W_{Coilslot}][0.6B_0]}{2}+\frac{1}{2}\times[W_{poleflat}]B_0$$

that was used in ref[1]. However, this is not true for an arc-shaped magnet; each of the areas must be computed individually.

For our arc-shaped magnet all areas are generated by circles; their equations, in an $x-y$ coordinate system, are

$$x^2+y^2 \;=\; R^2\ .$$

Thus the area between two section of radii R_i and R_j $(R_i > R_j)$ over the interval $x_1 \le x \le x_2$ is found from

$$\int_{x_1}^{x_2}\left[\sqrt{R_i^2-x^2}-\sqrt{R_j^2-x^2}\right]dx\ ,$$

and we are given

$$\int\sqrt{a^2-x^2}\,dx \;=\; \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{|a|}\right)\right]\ .$$

In our case, integration is over the interval $x_1 = -L_{iron}/2$ and $x_2 = L_{iron}/2$ so that the area in question becomes

$$\begin{aligned}
A_{i,j} &= 2\int_{0}^{x_2}\left[\sqrt{R_i^2-x^2}-\sqrt{R_j^2-x^2}\right]dx \\
&= \left[\frac{L_{iron}}{2}\sqrt{R_i^2-\left(\frac{L_{iron}}{2}\right)^2}+R_i^2\sin^{-1}\left(\frac{L_{iron}}{2R_i}\right)\right] \\
&\quad -\left[\frac{L_{iron}}{2}\sqrt{R_j^2-\left(\frac{L_{iron}}{2}\right)^2}+R_j^2\sin^{-1}\left(\frac{L_{iron}}{2R_j}\right)\right]\ .
\end{aligned}$$

The area of the inner coil slot is found by integrating between radii of $R_j = R_1$ and $R_i = R_{2a} = R_2 - W_{pole}/2+1.125$, that of the pole flat by integrating between $R_j = R_{2a}$ and $R_i = R_{3a} = R_3+W_{pole}/2-1.125$, and that of the outer coil slot by integrating between $R_j = R_{3a}$ and $R_i = R_4$. The numerically subscripted radii are those defined in §4.1 and will be measured in units of inches. Using these values we find the areas $A_{i,j}$ in.^{2}, the flux through them, $F_{i,j} = B_{average}A_{i,j}$ kG-in.^{2}, and total flux through the gap region, F_{total} kG-in.^{2}.

The results of such calculations are, given in table 1. In these calculations we have assumed a constant field of $B_0 = 13.5$ kG across the pole flat and an average field of $0.3B_0$ over the coil-slot regions.

8.1 Inner and outer dimensions of the dipole

The inner and outer dimensions of the dipole are determined by our assumption that the flux through the gap region of the magnet is divided equally between its inner and outer yokes. To keep a reasonable comparison of these designs with the H-frame design of ref[1)], we again choose the yoke flux to be of the order of $B_y = 13$ kG.

We find the outer radius of the yoke R_5 by proceeding as follows. From table 1 we know the (total) flux through the gap region F_{gap} and we have have settled on a value for the field in yoke B_y. We then use the above equations to find an area, $A_{5,4}$ in the notation above—and hence a radius R_5—such that

$$B_y A_{5,4} = \frac{F_{gap}}{2}$$

As an example, consider the 8.5-inch flat design with a 0.460 inch square conductor. From table 1 we have $F_{gap} = 6464$ kG-in.^{2} and with $B_y = 13$ kG we find the area required for an equal division of flux to be

$$A_{req} = \frac{F_{gap}}{2B_y} = 274 \text{ in.}^2$$

where we have added 10% to allow for flux leakage. With $R_5 = 54.75$ inches we find $A_{5,4} = 278$ in.^{2}. The thickness of the outer yoke is then $W_{outer} = R_5 - R_4 = 8.00$ inches.

The inner dimension is found by first calculating A_{seg}, the area of the segment formed by the radius R_1 and its intersections with the iron boundaries. This we subtract from the required area and then assume a rectangular area is added to provide the required area. This rectangle has the length W_{iron} and width w such that

$$w\, W_{iron} = A_{5,4} - A_{seg}$$

The area of the segment is given by

$$A_{seg} = \frac{R_1^2}{2}(\theta - \sin\theta)$$

with the angle θ given by

$$\theta = 2\sin^{-1}\left[\frac{L_{iron}}{2\,R_1}\right]$$

Again using the 8.5-inch flat design with a 0.460 inch square conductor as an example, we find

$$\begin{aligned}
\theta &= 2\sin^{-1}\left[\frac{34.0}{2(25.7)}\right] = 82.83^\circ\ , \\
A_{seg} &= \frac{(34.0)^2}{2}(1.4456 - 0.9922) = 149.74 \text{ in.}^2, \\
w &= \frac{A_{5,4} - A_{seg}}{W_{iron}} = \frac{278 - 150}{34} = 3.76 \text{ in.}
\end{aligned}$$

We take $w = 4.00$ inches. The *total* width of the inner yoke at the *center* of the dipole is

$$W_{inner} = w + R_1[1 - \cos(\theta/2)] = 10.43 \text{ in.}$$

Consequently, for this case we take

Outer radius of dipole R_5	=	54.75 inches
Thickness of outer yoke W_{outer}	=	8.00 inches
Thickness of inner yoke at dipole center W_{inner}	=	10.50 inches

Similar calculations for the other cases are listed in table 2. Of course, the calculations resulting in the data given in tables 1 and 2 are subject to verification by a program such as POISSON. The results of such runs are given in the next section.

9. POISSON calculations

The program POISSON[3)] was used to test the validity of the above calculations. However, the program was run only for the case of a coil constructed from the 0.460-inch square conductor, the assumption being that results would be similar for the case of the larger conductor. The output from the POISSON runs were analyzed using the programs PLOTDATA[4)] and PHYSICA[5)].

All of the POISSON calculations were made at an excitation of 28,500 A-t. This produces a field in the gap of approximately 13.7 kG. A mesh width 0.20 inch in the horizontal plane and one of 0.125 inch in the vertical plane were used in all runs. Runs were made for each of the three pole widths, and a different yoke thickness was used for each pole width. Specifically, for the 7 inch flat case the yoke width was taken as 7 inches, for the 8.5 inch case the yoke width was taken as 9.5 inches, and for the 10 inch flat case the yoke width was taken as 10 inches. Although the differences in these yoke thicknesses are small, it was felt advisable to see if any significant differences occurred.

Because this magnet has curved yoke and poles, it must also be remembered that the figures to be shown are to be construed as transverse cuts through the *center* of the dipole.

POISSON prediction of the field in the yoke of the 7 inch flat case are shown in the contour map of figure 1. Those for the 8.5 inch flat and 10 inch flat are shown in figures 2 and 3 respectively. From these figures it is seen that as the pole and yoke widths increase the field distribution in the yoke approaches the design value of 13 kG. However, the field distribution in the pole becomes more concentrated as the pole narrows. In all cases, the pole shims show saturation. This is more apparent in figures 4a, 4b, and 4c that show the predicted field distribution in the poles themselves.

Figure 4a, that for the 7 inch flat, shows that most of the pole is predicted to run between 16.5 kG and 17 kG. It also shows that the pole shims are saturated. Figure 4b, that for the 8.5 inch flat, indicates that a most of the pole runs runs between 16 kG and 16.5 kG with the pole sides running between 16.5 kG and 17 kG. Again saturation is seen in the pole shims. Figure 4c, that for the 10 inch flat, indicates that virtually all of the pole runs between 16 kG and 16.5 kG. Only a relatively small portion of the pole is predicted to run in excess of 16.5 kG. However, saturation is still shown in the pole shims. t must be remembered that these values of field in the pole and yoke have been obtained with a field in the gap fo approximately 13.7 kG—a value 5% higher than the maximum field required in operation. Thus it is expected that the fields in the poles would be reduced by a similar amount. However, a reduction of this magnitude will not completely eliminate the saturation in the pole shims.

Figures 5, 6, and 7 show the fields predicted in the gaps of the three cases. for the 7 inch flat case figure 5 indicates that the field could be expected to be (13.644 ± 0.001) kG over a region roughly ± 1 inch about the geometric center-line of the magnet. This uniformity is maintained over virtually the entire gap of the magnet. Figure 6 for the 8.5 inch flat shows a field of (13.692 ± 0.001) kG, and for the 10 inch flat case figure 7 shows a field of (13.702 ± 0.001) kG, both over similar areas. Given that the maximum beam width is expected to be 2 cm, any of these values would be acceptable. However, we again note that these values have been obtained at the midpoint of the dipole.

There is definitely further design to be done on these arc-shaped dipoles. For example, the inner yoke is too thin at the entry and exit of the dipoles. It is expected that the thickness of the rectangular inner yoke will have to be increased. However, we can estimate the weight of a magnet designed along these lines. For this we consider the intermediate, 8.5 inch flat case with coils made from the 0.416-inch square conductor and for the 10 inch pole with coils of the same conductor. For these cases that were considered in the POISSON run we have the following dimensions.

	8.5 inch flat	10 inch flat
R_1 (in.)	25.50	24.90
R_2 (in.)	30.00	29.25
R_3 (in.)	42.50	43.25
R_4 (in.)	46.75	47.50
R_5 (in.)	56.25	57.50
w (in.)	4.75	4.50

Using these values, assuming that the poles are rectangular blocks 5 inches high and pole widths of W_{pole} = 12.5 inches and 14 inches respectively, and taking the density of iron to be 0.2833 lb/in.^{3} we find the following.

Element	8.5 inch pole				10 inch pole			
	Area (in.^{2})	Height (in.)	Volume (in.^{3})	Weight (lb)	Area (in.^{2})	Height (in.)	Volume (in.^{3})	Weight (lb)
Top yoke	1,390	9.5	13,205	3,741	1,461	10.0	14,610	4,140
Bottom yoke	1,390	9.5	13,205	3,741	1,461	10.0	14,610	4,140
Top pole	443	5.0	2,215	628	497	5.0	2,485	705
Bottom pole	443	5.0	2,215	628	497	5.0	2,485	705
Outer spacer	330	10.0	3,300	935	346	10.0	3,460	980
Inner spacer	312	10.0	3,120	885	310	10.0	3,100	878
Total				10,558				11,548

To make allowance for further dimensional changes we would then estimate the weight of the 8.5 inch flat dipole to be approximately 10,750 lb and that of the 10 inch flat dipole to be 11,750 lb. These are to be compared to an estimated weight of 13,100 lb for the H-frame dipole of ref [1] made with coils made from the same conductor.

A comparison (of the outer dimensions only) of the above designs and that of ref [1] is shown in figures 8a for the 8.5 inch case and 8b for the 10 inch case. In each of these figures the arc-shaped magnet is the solid outline and the H-frame magnet is the dashed outline. The curved dashed line represents the central trajectory and runs from *effective* edge to *effective* edge; the central horizontal dashed line is the centerline of the H-magnet. The magnet itself has been shifted so that the central radius of curvature is common to both magnets and the geometric centerline of the H-frame dipole is equidistant from the maximum and minimum excursions of the trajectory.

It is seen from figure 8a that the outer dimension of the 8.5 inch flat design is (roughly) 2 inches narrower at its center and 5 inches at its entrance and exit than is the H-frame design. Because there is a spatial constraint at the exit of the dipole, this arc-shaped magnet should be considered. On the other hand, it may be concluded from figure 8b concluded that there is little to gain by using an arc-shaped magnet.

Figure 9 gives the dimensions of the dipole with the 8.5 inch pole flat that was used in the POISSON calculations.

10. Discussion

This report has presented the results of a study of an arc-shaped dipole for the ISAC1 to ISAC2 transfer beamline. It is to be stressed that the design presented here is incomplete; further work is required. However, this document is should provide a basis for such study.

The designs presented in this and ref[1] have each been based on the assumption of the requirement of a field uniformity of much better than 0.1% over the region in which the beam travels. In the case of the H-frame dipole of the earlier work this was accomplished by using a pole width of 18 inches. In contrast, the pole widths of the designs of this report are in some cases significantly lower. This, however, comes at the cost of an increased flux through the poles of the dipoles. This increase is sufficient to drive the pole shims to saturation in each of the designs considered here. POISSON studies indicate that despite such saturation field quality in the magnet gap is not expected to be sacrificed. In fact, the field uniformity of the designs presented in this report is superior to that found for the H-frame design.

Other than some additional fabrication costs, the main argument against using an arc-shaped design for the dipoles is the predicted saturation of the pole shims. The field distribution and saturation in the poles is dictated by the pole width. As it is increased the field distribution improves and the amount of saturation decreases. The widest (base) pole width considered in this report was 14 inches. Were that increased 4 inches results similar to those reported in ref[1] would be expected. In fact, in the terminology of this report, the H-frame dipole of ref[1] would be termed the '14 inch flat' case.

The deciding factors that will determine what type of dipole to use are then the requirements of field uniformity and of spatial constraints.

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4. J. L. Chuma, *PLOTDATA Command and Reference Manual*, TRIUMF Report TRI-CD-87-03b, June, 1991.

5. J. L. Chuma, *PHYSICA User's Guide*, TRIUMF Report TRI-CD-98-01, February, 1998.

Table 1

Calculation of the areas and fluxes through the inner and outer coil slots and the pole flat
(Notation used here is given in §8.1 of the text)

	0.460 square			0.516 square		
	7-inch flat	8.5-inch flat	10-inch flat	7-inch flat	8.5-inch flat	10-inch flat
$A_{2a,1}$ (in.^{2})	198.94	197.93	200.74	213.74	212.82	215.74
$A_{3a,2a}$ (in.^{2})	309.89	363.12	416.40	309.89	363.12	416.40
$A_{4,3a}$ (in.^{2})	187.82	187.64	187.46	201.75	201.56	201.37
$F_{2a,1}$ (kG-in.^{2})	805.68	801.62	813.00	865.63	861.91	873.74
$F_{3a,2a}$ (kG-in.^{2})	4183.47	4902.07	5621.38	4183.47	4902.07	5621.38
$F_{4,3a}$ (kG-in.^{2})	760.69	759.94	759.23	817.09	816.30	815.55
F_{total} (kG-in.^{2})	5749.85	6463.62	7193.60	5866.19	6580.28	7310.67

Table 2

Calculation of the areas and fluxes through the inner and yokes
(Notation used here is given in §8.1 of the text)

	0.460 square			0.516 square		
	7-inch flat	8.5-inch flat	10-inch flat	7-inch flat	8.5-inch flat	10-inch flat
A_{req} (in.^{2})	243.26	273.46	304.34	248.19	278.40	309.30
$A_{5,4}$ (in.^{2})	251.63	277.40	311.78	255.00	289.41	315.14
R_5 (in.)	53.25	54.75	56.50	53.75	55.50	57.00
W_{outer} (in.)	7.25	8.00	9.00	7.35	8.35	9.10
θ (°)	80.17	82.83	86.12	81.66	84.43	87.88
A_{seg} (in.^{2})	144.25	149.74	156.64	147.33	153.10	160.39
A_{inner} (in.^{2})	254.75	285.74	318.14	257.83	289.10	321.89
w (in.)	3.25	4.00	4.75	3.25	4.00	4.75
W_{inner} (in.)	9.45	10.43	11.45	9.60	10.60	11.60

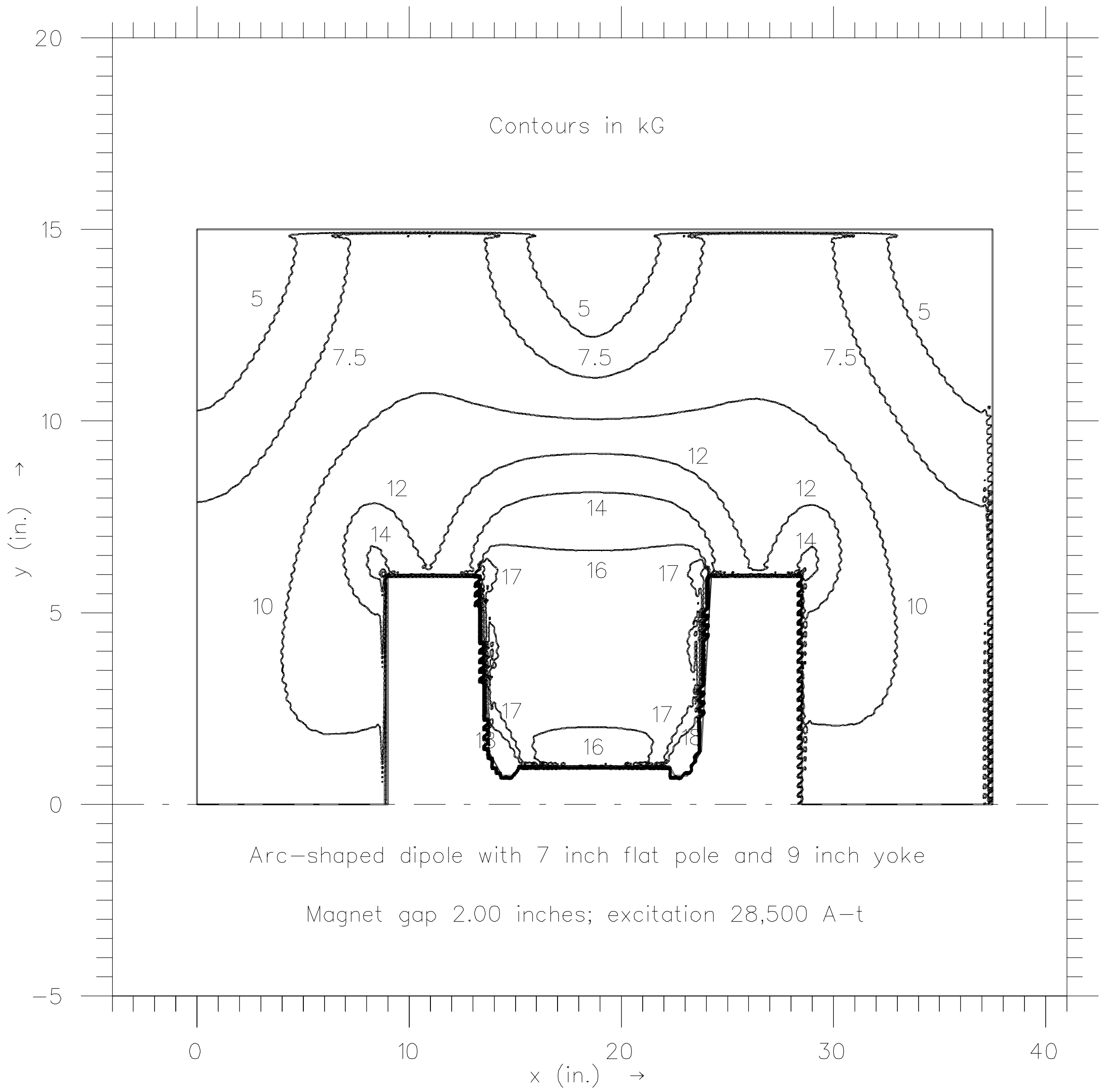

Fig. 1. The POISSON prediction of the field in the yoke of the dipole with a 7 inch flat pole.

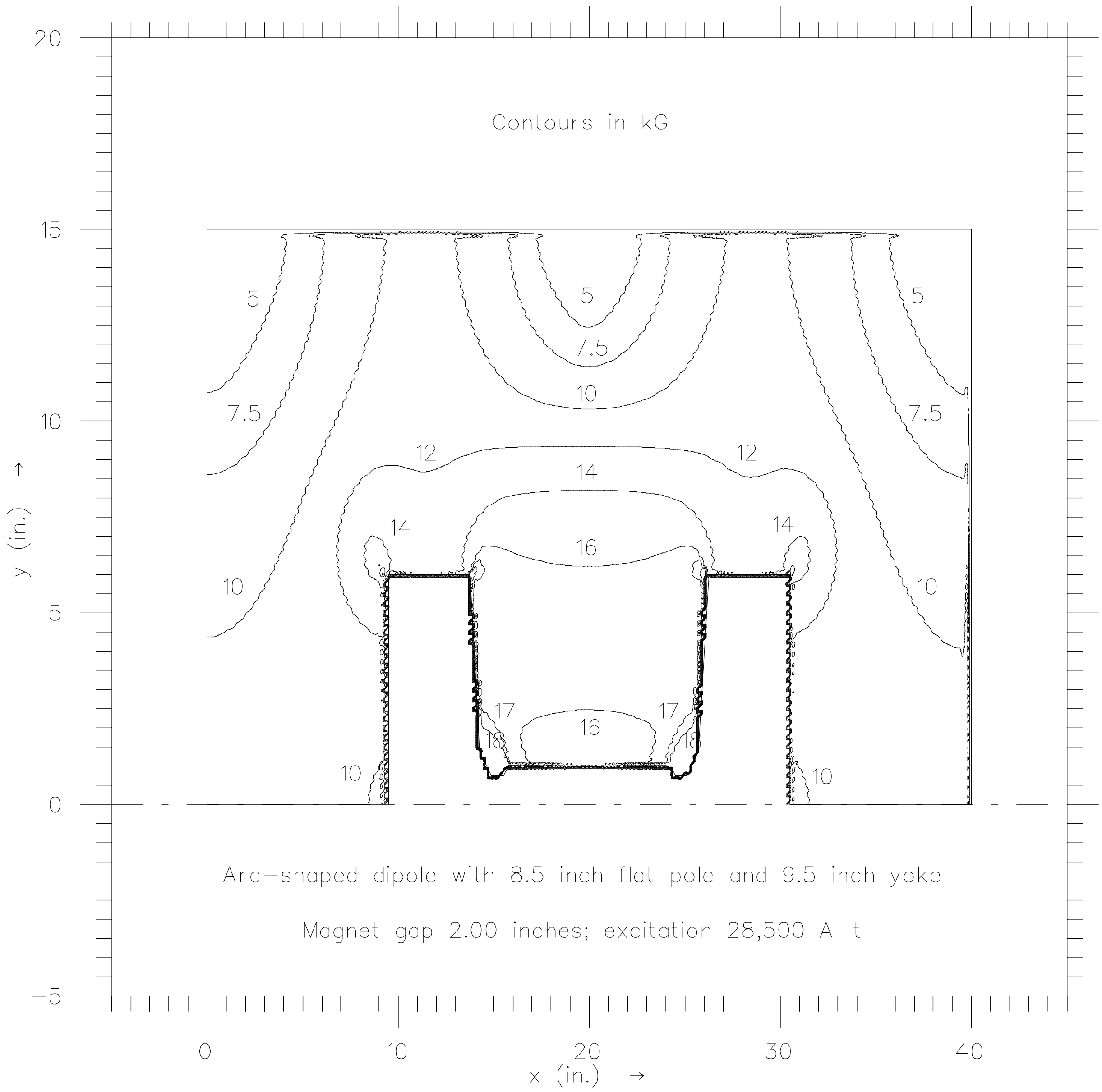

Fig. 2. The POISSON prediction of the field in the yoke of the dipole with a 8.5 inch flat pole.

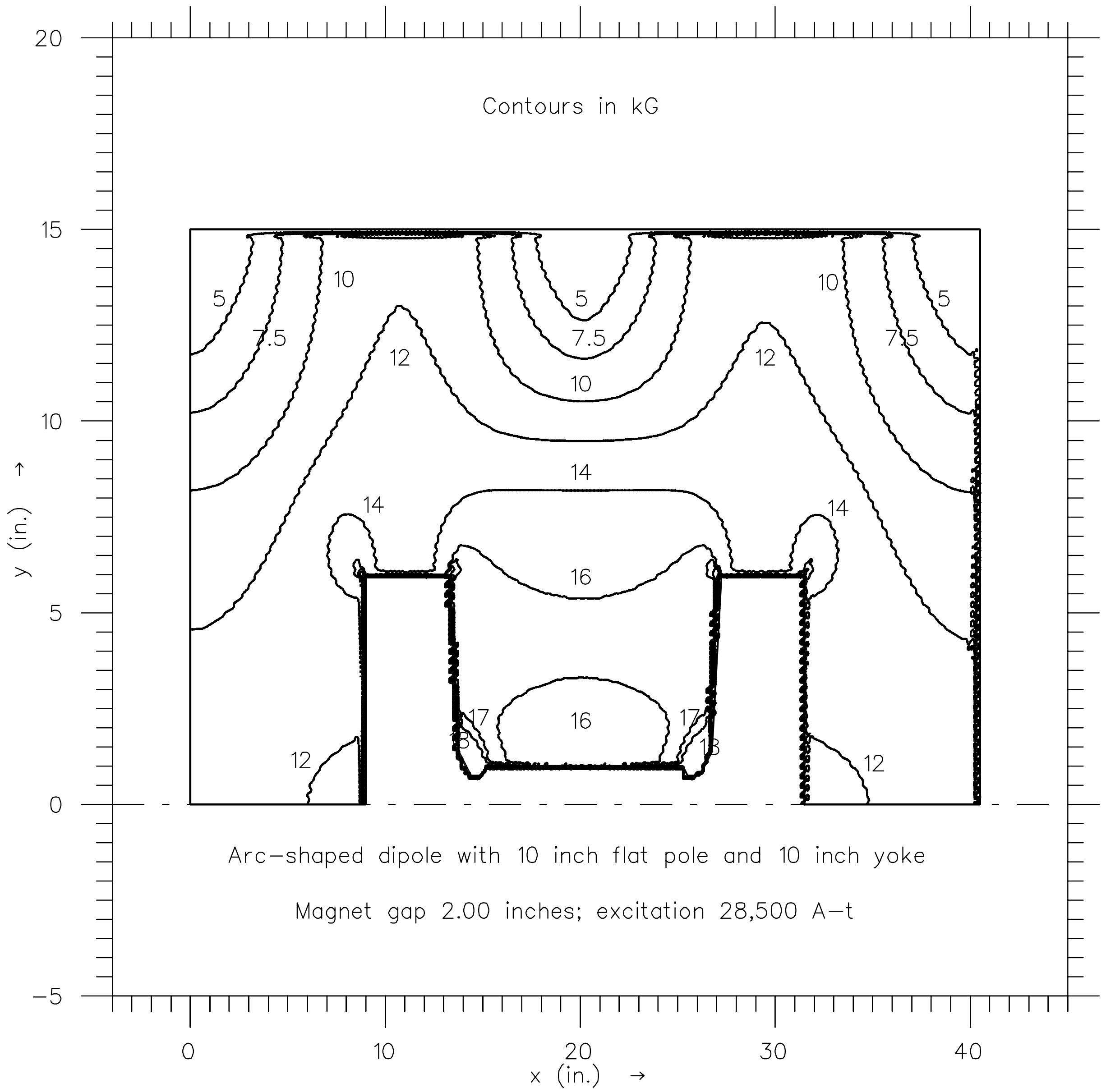

Fig. 3. The POISSON prediction of the field in the yoke of the dipole with a 10 inch flat pole.

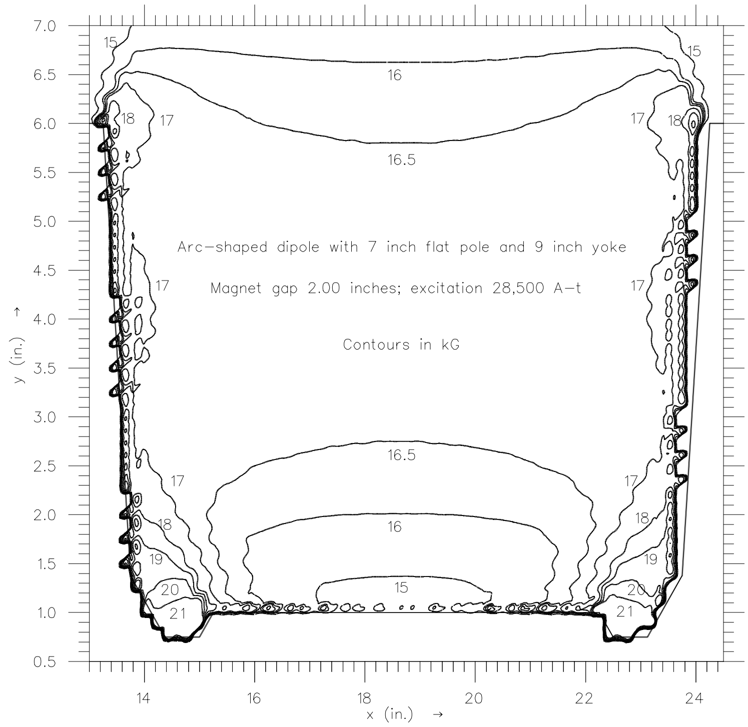

Fig. 4(a). Detailed contour plot of fields in the 7 inch flat pole.

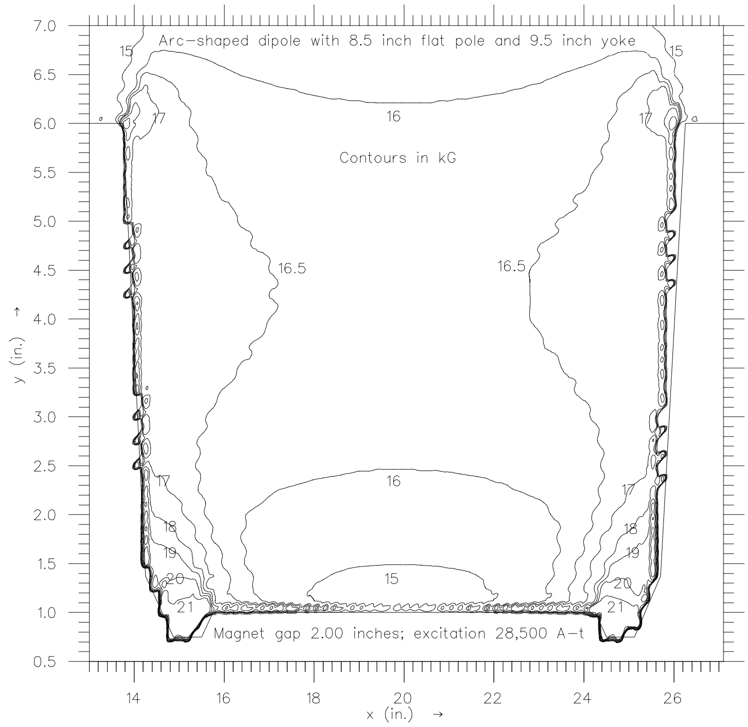

Fig. 4(b). Detailed contour plot of fields in the 8.5 inch flat pole.

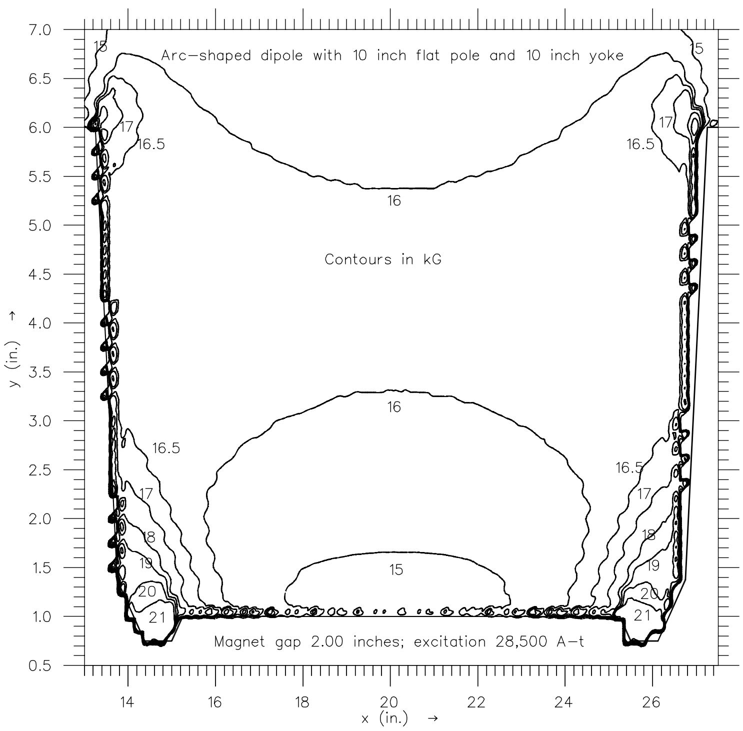

Fig. 4(c). Detailed contour plot of fields in the 10 inch flat pole.

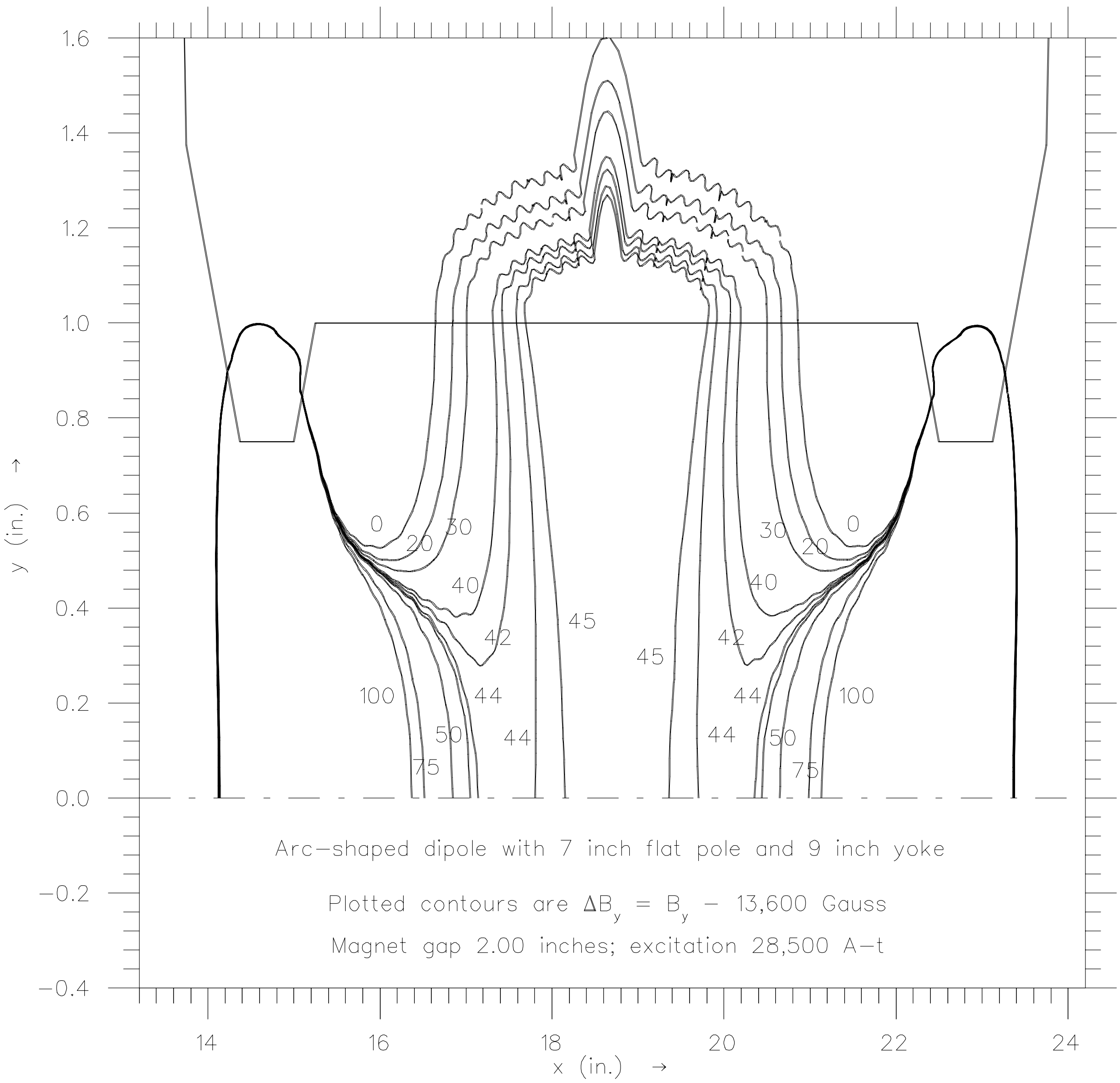

Fig. 5. The POISSON prediction of the field in the gap of the dipole with a 7 inch flat pole.

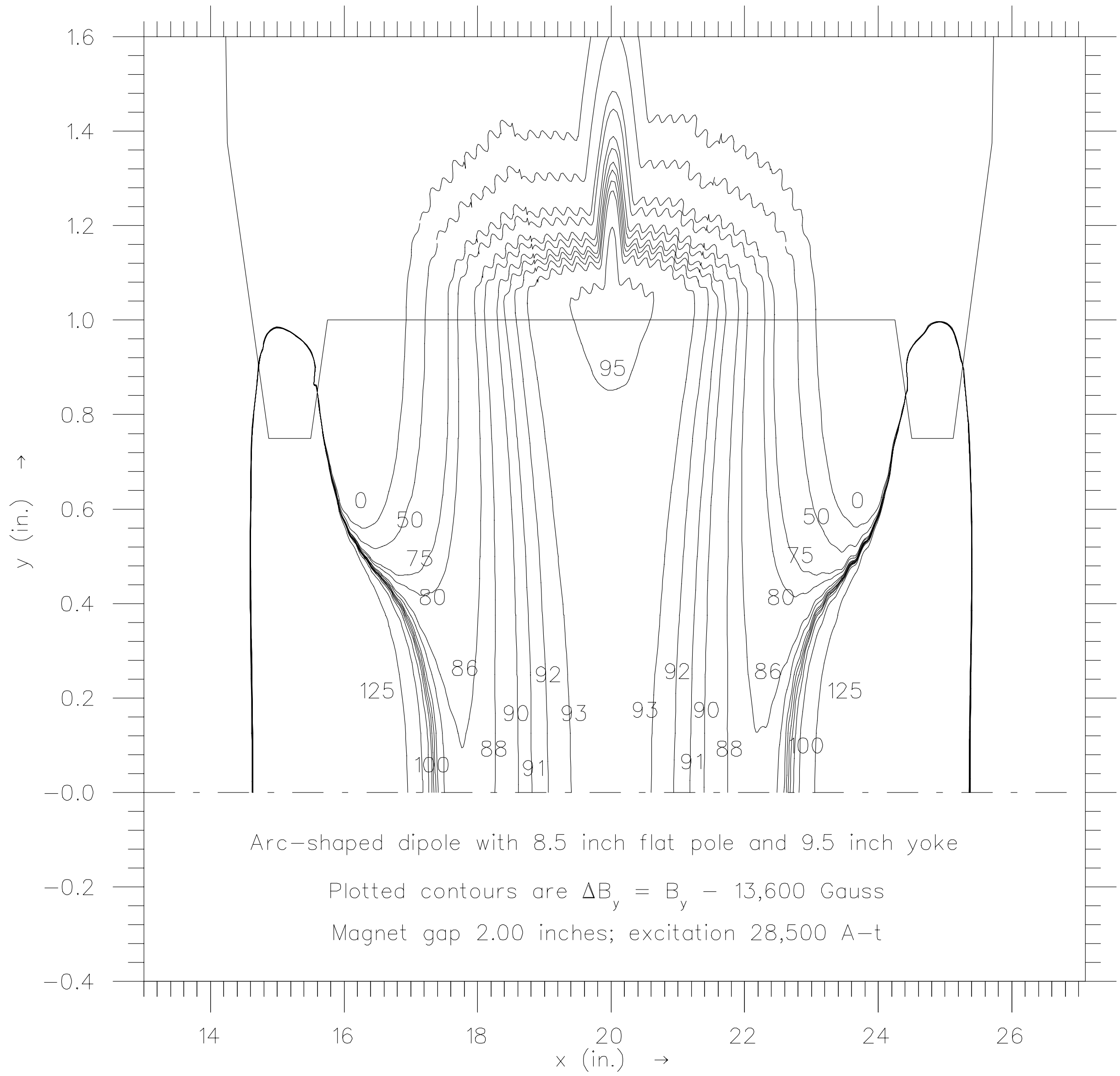

Fig. 6. The POISSON prediction of the field in the gap of the dipole with a 8.5 inch flat pole.

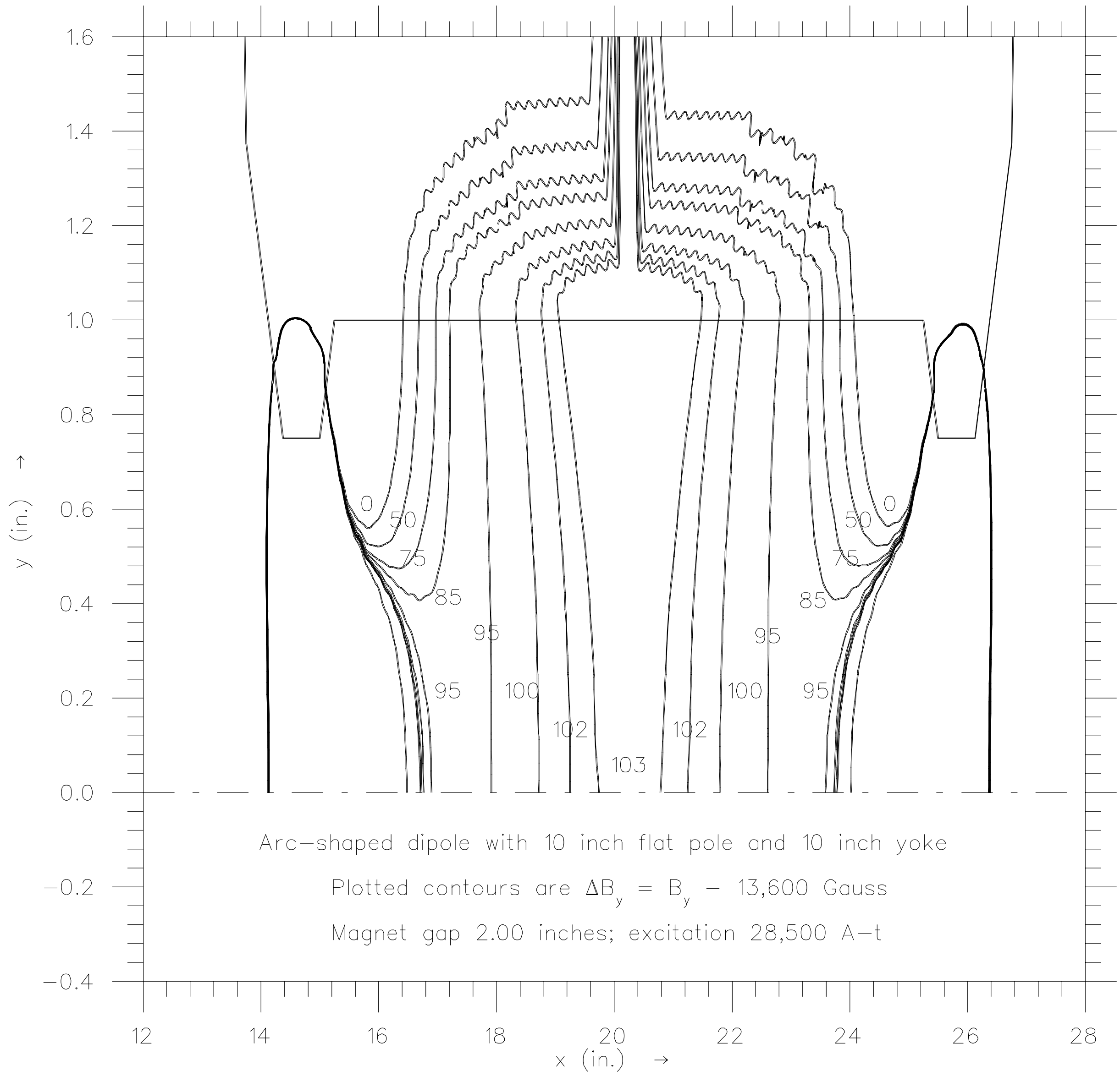

Fig. 7. The POISSON prediction of the field in the gap of the dipole with a 10 inch flat pole.

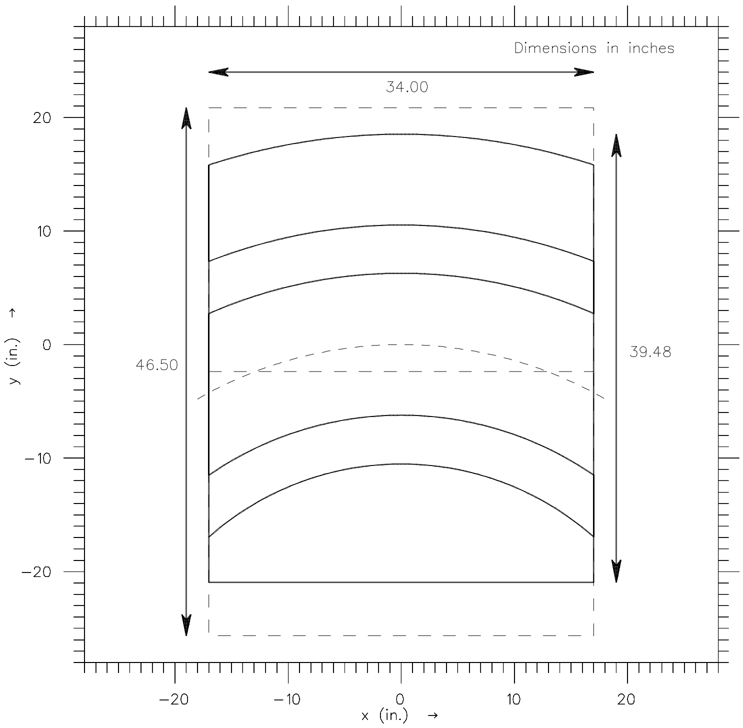

Fig. 8(a). Comparative dimensions of the 8.5 inch flat design with the H-frame design of ref[1].

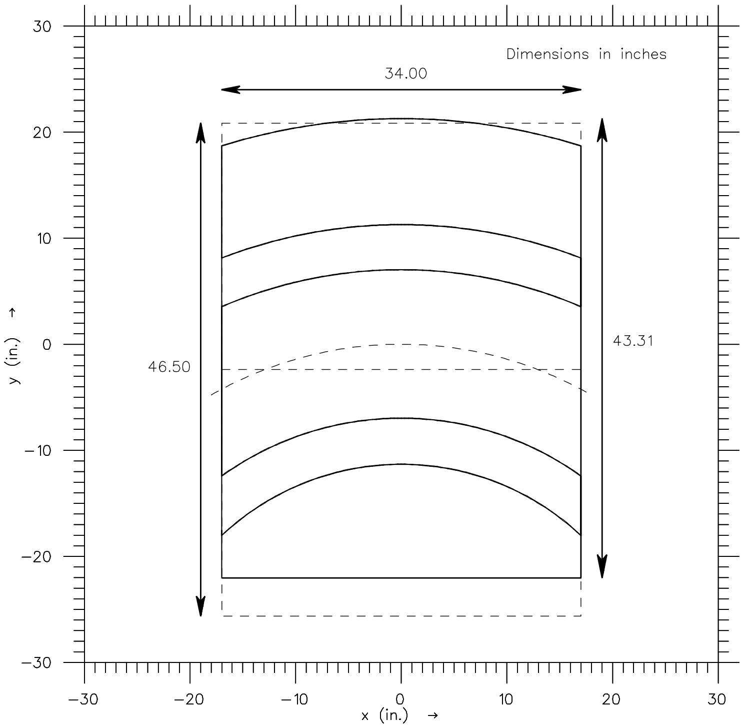

Fig. 8(b). Comparative dimensions of the 10 inch flat design with the H-frame design of ref[1].

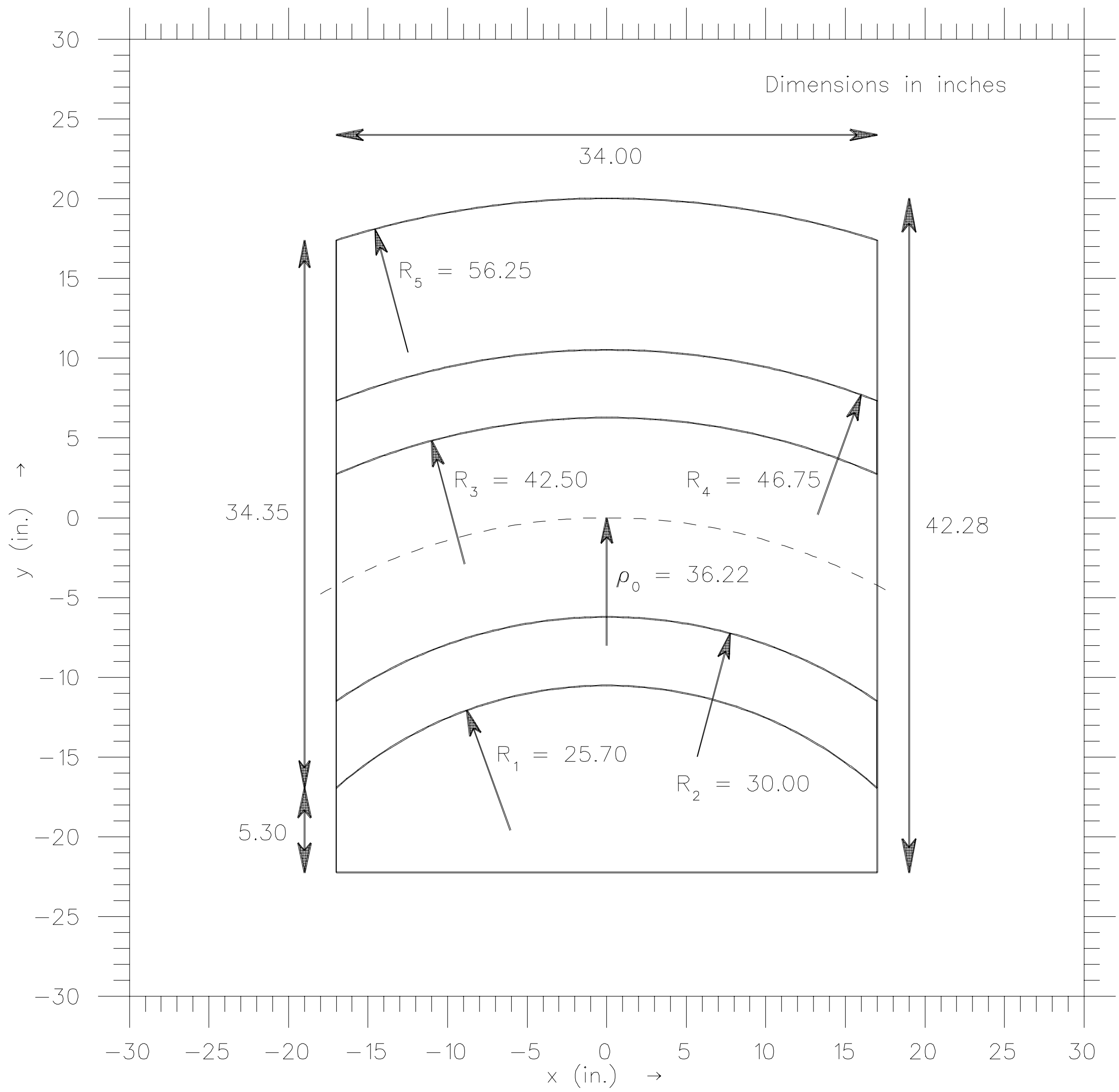

Fig. 9. Dimensions of the arc-shaped dipole with an 8.5inch pole flat.