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Subject Conceptual design for a solenoid for the LEBT of the electron accelerator		

1. Introduction

Five solenoids are required for the low energy beam transport (LEBT) beam line of proposed 50 MeV beam line. Specified parameters of these solenoids require a bore radius of 1 cm, a length of 10 cm and a field of 0.02 T (200 G). This note provides a conceptual design for these elements.

2. Basic calculations

From basic considerations the axial field of a solenoid is given by

$$B = \mu_0 I \left[\frac{N}{L} \right] ,$$

in which B is the axial field in Tesla, I is the current in amperes flowing through the windings, N is the number of turns in the coil, and L is the axial length of the coil in metres. Rewriting the above and using a field of 0.025 T for safety we find

$$\begin{aligned} N I &= \frac{B L}{\mu_0} \\ &= \frac{(0.025)(0.10)}{4\pi \times 10^{-7}} \\ &= 1,989 \, \text{A-t} \sim 2,000 \, \text{A-t} . \end{aligned}$$

We then use the following design parameters.

Axial field B	=	0.025 T
Axial length L	=	4.000 in.
$N I$	=	2,000 A-t

3. Solenoid design

We shall assume that the coil is wound from 0.1620-inch square, hollow copper conductor. Anaconda's data for this conductor are given as follows.

OD	0.16200 in.	=	4.11480 mm
ID	0.09000 in.	=	2.28600 mm
Copper area	0.01934 in.^{2}	=	12.47739 mm^2
Cooling area	0.00636 in.^{2}	=	4.10451 mm^2
Mass	0.07473 lb/ft	=	0.11121 kg/m
Resistance at 20°C	421.10 $\mu\Omega$/ft	=	1381.56 $\mu\Omega$/m
k (British units)	0.06220		

We further assume that this conductor is coated with a 0.011-inch layer of double Dacron glass (DDG) insulation to a (total) thickness of 0.011-inch and that a spiral winding of 0.007-inch-thick fiberglass tape is applied as the coil is wound. Thus the overall outer dimension of the conductor d is

$$d = 0.1620 + 0.011 + 2(0.007) = 0.187 \, \text{inch} .$$

In further calculations we take the wrapped conductor to be 0.20-inch square.

$$\boxed{\text{Outer dimension of wrapped conductor} = 0.20\,\text{inch.}}$$

Thus over a length of 4.00 inches we can wind

$$n = \frac{4.000}{0.20} = 20 \text{ turns} .$$

so that choosing a current of $I = 50$ A requires two layers of 20 turns each to meet the required $N\,I$ value. Thus the coil thickness will be 0.040 inch. At a current of 50 A the current density J is

$$J = \frac{50\,\text{A}}{0.01934\,\text{in.}^2} \sim 2,600\,\text{A/in.}^2 ,$$

which is well below a conservative current density of 3,000 A/in.^{2}. Thus the coil parameters are

Operating current	50 A
Number of layers	2
Number of turns per layer	20
Total number of turns	40

4. Solenoid dimensions

We now assume that the (stainless steel) beam tube has a diameter of 1.00 inch and a wall thickness of 1/32 inch (0.031 inch). We further assume that the finished coil is encapsulated in epoxy such that the coating is 0.125 inch larger than coil in all dimensions. Thus the inner edge of the coil lies a distance

$$r_{coil,inner} = 0.500 + 0.031 + 0.125 = 0.656 \sim 0.65\,\text{inch}$$

from the axis of the solenoid and its outer edge lies

$$r_{coil,outer} = 0.65 + 2(0.20) = 1.05\,\text{inch}$$

from its axis. We will also assume that the coil is surrounded by an iron cylinder of inner radius

$$r_{iron,inner} = 1.05 + 0.125 = 1.175 \sim 1.20\,\text{inch} ,$$

a length of

$$L_{iron} = 4.000 + 2(0.125) = 4.25\,\text{inch} ,$$

and a wall thickness of 0.25 inch. Thus the outer radius of the iron cylinder is

$$r_{iron,outer} = 1.20 + 0.25 \sim 1.45\,\text{inch} .$$

At each end of the iron cylinder we also assume that there is a field clamp 0.25 inch thick having an inner radius of

$$r_{clamp,inner} = 0.65 - 0.125 = 0.525 \sim 0.53\,\text{inch}$$

and an outer radius equal to that of the iron cylinder

$$r_{clamp,outer} = \; = 1.45\,\text{inch} .$$

5. Copper requirements

The mean radius of the inner coil is

$$r_{coil,inner,mean} = 0.65 + 0.10 = 0.75\,\text{inch}$$

and that of the outer coil is

$$r_{coil,outer,mean} \;=\; 1.05 - 0.10 \;=\; 0.95\,\text{inch}\ .$$

Thus the length of copper required per coil is

$$L_{coil} \;=\; 20[2\pi(0.75 + 0.95)] \sim 215\,\text{inches} \sim 18\,\text{feet}\ .$$

Allowing 2 ft for leads and given a weight of 0.07473 lb/ft, approximately 1.5 pounds of conductor are required per solenoid. Thus for five solenoids approximately 7.5 pounds of conductor are required.

We should have this amount on hand.

6. Power requirements

At 20°C, the resistance of the coil is:

$$R_{20^\circ C} \;=\; (421.10{\times}10^{-6}\Omega/\text{ft}){\times}(20\ \text{ft}) \;=\; 0.008422\,\Omega\ .$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C, and an outlet water temperature of 70°C (thus allowing a 40°C coolant temperature rise). Then the mean coil temperature will be 50°C. With a 30°C rise above ambient of the coil we then have:

$$\begin{aligned} R_{hot} &= R_{20^\circ C}[1 + (\text{Temperature coefficient}/^\circ\text{C})dT(^\circ\text{C})] \\ &= 0.008422[1 + 0.00393(30)] \\ &= 0.00942\,\Omega\ \text{per coil.} \end{aligned}$$

Thus at a current of 50 A we obtain

$$\text{Voltage per coil} = 0.47\ \text{Volts}$$

Therefore, allowing for lead loss, we choose a power supply that has

$$\boxed{\begin{array}{lcl} \text{I} & = & 50.0\ \text{A maximum} \\ \text{V} & = & 1.0\ \text{V minimum} \\ \text{P} & = & 0.05\ \text{kW minimum} \end{array}}$$

We note that because of the low value of the calculated power dissipation it is highly unlikely that there would be the temperature increase of the coolant that has been used in this calculation. However, because the coil resistance increases with temperature, the power supply specification above should be conservative.

7. Cooling requirements

In these calculations we use the British system of units. The power required per coil is:

$$\text{Power per coil} \;=\; I^2R_{hot} \;=\; (50)(50)(0.00942) \sim 0.025\ \text{kW}\ .$$

The required flow rate is given by:

$$v(\text{ft/sec}) \;=\; \frac{2.19}{\Delta T} \times \frac{P(\text{kW})}{\text{Cooling area (in.}^2)} \;=\; 4.78099{\times}P(\text{kW})$$

for $\Delta T = 72^\circ\text{F} = 40^\circ\text{C}$ and $A_{H_2O} = 0.00636\,\text{in.}^2 = 4.105\,\text{mm}^2$. Choosing $v = 2.50\,\text{ft/sec}$ to define the maximum power dissipation per water circuit we have

$$P_{max} \;=\; \frac{(2.50)(72)(0.00636)}{2.19} \;=\; 0.523\,\text{kW/water circuit}\ .$$

Because the estimated power generated is approximately a factor of ten lower than this, a single cooling circuit only is required. Thus we take

$$\boxed{\text{Number of cooling circuits per solenoid} = 1\ .}$$

This requires a flow rate of $v = 0.24$ ft/sec per water circuit. The volume of flow required per circuit is

$$\text{Volume/circuit} \;=\; 2.6(v(\text{ft/min}))(\text{Cooling area(in.}^2)) \;=\; 2.6(0.240)(0.00636) \;=\; 0.004\,\text{IGPM}.$$

Thus

$$\boxed{\text{Volume per solenoid} \quad = \quad 0.004 \text{ IGPM} \quad = \quad 0.020\ \ell/\text{min} \quad = \quad 0.005 \text{ USGPM}}$$

8. Pressure Drop

The pressure drop is given by

$$dP \;=\; k\,v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case, with $k = 0.0622$ we obtain:

$$dP \;=\; (0.0622)(0.270)^{1.79} \;=\; 0.006 \text{ psi/ft} \;=\; 0.020 \text{ psi/m},$$

and the total pressure drop across one solenoid is:

$$\boxed{\text{Pressure drop per solenoid} = 0.12 \text{ psi}\ .}$$