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| *Author* GM Stinson | | *Page* 1 *of* 32 |
| *Subject* A six-quadrupole twister for beam line 4B | | |

## 1. Introduction

A previous reportrefsup1 presented a design for a five-quadrupole twister for beam line 4B. Subsequent to the time of issue of that report, Dr. K. L. Brown suggested during a visit to TRIUMF that one should be able to use a six-quadrupole twister. An advantage of his design is that the six-quadrupole twister is a unit section. Consequently, were a six-quadrupole twister set up at TRIUMF, the beam conditions set up at 4BT1 would be reproduced at the 4BT2 target location.

As was noted in ref[1)], a five-quadrupole, unit-section twister design is possible. However, because of space and experimental constraints, the five-quadrupole twister was designed so that its transfer matrix was identical to that of a thin lens—that is the transfer matrix takes the form

$$\begin{bmatrix} x_2 \\ \theta_2 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & R_{16} \\ -\frac{1}{f} & 1 & R_{26} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \\ \delta_1 \end{bmatrix}$$

In the above matrix equation the subscripts '1' and '2' would refer to the $x$, $\theta$, and $\delta$ parameters at target locations 4BT1 and 4BT2 respectively. The magnitude of the term $-1/f$ is of the order of 4. It is clear that the angular divergence at 4BT2 is coupled to the spatial dispersion at 4BT1. Because the (spatial) dispersion at 4BT1 is of the order of 10 cm/%, one can see that the angular dispersion at 4BT2 is quite large.

This report presents a design for a six-quadrupole twister for beam line 4B.

## 2. Theory of a six-quadrupole twister

The analysis presented in this section follows that of Dr. Brown.

Suppose that the transfer matrix $\boldsymbol{R}$ from the point $A$ to the point $B$ is represented as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}(x) & 0 \\ 0 & \boldsymbol{R}(y) \end{bmatrix}$$

in which $\boldsymbol{R}(x)$ and $\boldsymbol{R}(y)$ represent the (uncoupled) transfer matrices for the horizontal and vertical planes respectively. If one rotates the coordinate system by an angle $\theta$ at the entrance and at the exit of the section, the transfer matrix becomes

$$\boldsymbol{T} = \boldsymbol{R}(\theta) \cdot \boldsymbol{R} \cdot \boldsymbol{R}(\theta) ,$$

where the matrices $\boldsymbol{R}(\theta)$ are the rotation matrices given by

$$\boldsymbol{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} .$$

Thus the matrix $\boldsymbol{T}$ becomes

$$\boldsymbol{T} \;=\; \begin{bmatrix} c^2\boldsymbol{R}(x) \,-\, s^2\boldsymbol{R}(y) & sc[\boldsymbol{R}(x) \,+\, \boldsymbol{R}(y)] \\ -sc\boldsymbol{R}(x) \,+\, \boldsymbol{R}(y)] & c^2\boldsymbol{R}(x) \,-\, s^2\boldsymbol{R}(y) \end{bmatrix}$$

in which $s$ and $c$ have been written for $\sin(\theta)$ and $\cos(\theta)$ respectively. One can see immediately from the above expression that if

$$\boldsymbol{R}(y) \;=\; -\,\boldsymbol{R}(x)$$

the matrix $\boldsymbol{T}$ becomes

$$\boldsymbol{T} \;=\; \begin{bmatrix} (c^2+s^2)\boldsymbol{R}(x) & 0 \\ 0 & -(c^2+s^2)\boldsymbol{R}(x) \end{bmatrix} \;=\; \begin{bmatrix} \boldsymbol{R}(x) & 0 \\ 0 & -\boldsymbol{R}(x) \end{bmatrix}.$$

If one now rotates the coordinate system through an angle $(-2\theta)$—that is, one returns to the original reference frame—the transfer matrix from point $A$ to the point $B$ becomes

$$\boldsymbol{T} \;=\; \begin{bmatrix} \cos(-2\theta)\boldsymbol{R}(x) & -\sin(-2\theta)\boldsymbol{R}(x) \\ -\sin(-2\theta)\boldsymbol{R}(x) & \cos(-2\theta)\boldsymbol{R}(x) \end{bmatrix} \;=\; \begin{bmatrix} \cos(2\theta)\boldsymbol{R}(x) & -\sin(2\theta)\boldsymbol{R}(x) \\ \sin(2\theta)\boldsymbol{R}(x) & \cos(2\theta)\boldsymbol{R}(x) \end{bmatrix}.$$

In particular, if $\theta = 45°$, one has

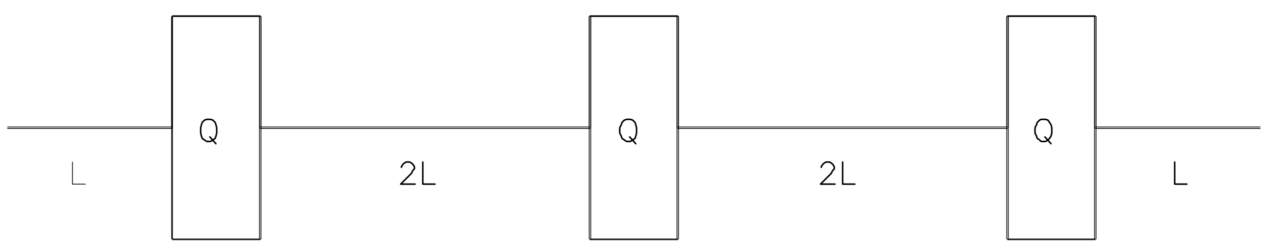
$$\boldsymbol{T} \;=\; \begin{bmatrix} 0 & \boldsymbol{R}(x) \\ \boldsymbol{R}(x) & 0 \end{bmatrix} \;=\; \begin{bmatrix} 0 & \boldsymbol{R}(x) \\ -\boldsymbol{R}(y) & 0 \end{bmatrix}.$$

This relationship shows that any transport system that is designed such that $\boldsymbol{R}(y) = -\boldsymbol{R}(x)$ will, under a rotation of 45°, interchange the $x$ and the $y$ coordinates. If, in addition,

$$\boldsymbol{R}(y) \;=\; -\,\boldsymbol{I} \qquad \text{and} \qquad \boldsymbol{R}(x) \;=\; +\,\boldsymbol{I}$$

then the system will be a unit section.

Brown has designed a simple six-quadrupole, unit-section twister that may be depicted as follows.




In the above diagram, $Q$ represents a quadrupole doublet. Each doublet is identical. This system could then be powered from two power supplies, one driving those quadrupoles focussing horizontally and the other driving those focussing vertically.

Another important aspect of a unit section is that if it is broken at any point and the two sections interchanged, the resulting section is still a init module. Thus the above system could be redrawn as follows.


That this is so is apparent if one writes the transfer matrix as

$$\boldsymbol{T} = \boldsymbol{A} \cdot \boldsymbol{B}$$

in which the matrix $\boldsymbol{B}$ represents that for a drift length $L$ and the matrix $\boldsymbol{A}$ is that for the remainder of the system. Because $\boldsymbol{T}$ is a unit matrix, then

$$\boldsymbol{B} = \boldsymbol{A}^{-1}$$

so that

$$\boldsymbol{B} \cdot \boldsymbol{A} = \boldsymbol{A}^{-1} \cdot \boldsymbol{A} = \boldsymbol{I} \ ,$$

indicating that the beam transport system represented by the matrix $\boldsymbol{A}$ followed by that represented by the matrix $\boldsymbol{B}$ is also a unit section. This property is used in the design of the twister section presented here.

**3. A six-quadrupole twister for beam line 4B**

The design of a twister for beam line 4B is constrained by the requirements that target locations 4BT1 and 4BT2 are to be left intact and that standard TRIUMF quadrupoles are to be used. A six-quadrupole twister meeting these requirements is shown schematically in figure 1. The system shown is an asymmetric twister; the entrance drift length is 0.7 m longer than that at its exit. As was indicated above, this system was obtained by rearranging a symmetric system. TRANSPORT parameters at 500 MeV for the twister are given in table 1.

In order that the specifications noted above be met, it was necessary to redesign the six-quadrupole twister described in §2. The twister described by the parameters listed in table 1 might best be described as a 'split' five-quadrupole twister. By this it is meant that the central quadrupole of the five-quadrupole twister has been split into two individual quadrupoles separated by a drift space. The unit is adjusted such that the longest drifts of the section are at the entrance and exit of the twister. Further, it was necessary to fix the pole-tip fields of the twister quadrupoles TQ3 and TQ4 at 8 kG. The drift spaces between quadrupoles TQ2 and TQ3, TQ3 and TQ4, and TQ4 and TQ5, were allowed to vary subject to the condition that the overall length of the twister be maintained at 10.31 m. In the resulting symmetric twister, the drift lengths labelled 'DIN' and 'DEX' are each 2.40963 m long. By adding 0.7 m to the drift DIN and subtracting a like amount from that labelled DEX, the parameters given in table 1 are obtained. Note that the section could be powered by three power supplies.

**4. Standard operation of beam line 4B**

In what shall be defined as 'standard' beam line 4B operation the (spatial) dispersion at the 4BT2 target is $-10$ cm/%. Dispersion may be in either of the horizontal or vertical planes. Table 2 lists TRANSPORT input for the complete line at 500 MeV; table 3 gives element settings for other energies. Table 4 gives the predicted beam sizes and overall transfer-matrix elements at 4BT2 for the various energies.

Figures 2–5 show beam profiles along the beam line for these cases. As in ref[1], quadrupoles 4BQ6 and 4BQ7 are adjusted so as to focus from the 4BT2 target to the beam dump. Required beam conditions are set up at the target location 4BT1 and are reimaged at the 4BT2 location.

The listings of tables 2 and 3 are for the case of vertical dispersion at target 4BT2. As was indicated in ref[1], horizontal dispersion at 4BT2 is attained by rotating the twister to its 'normal' orientation. Similarly, if a solenoid is installed upstream of 4BT1, vertical and horizontal phase spaces may be decoupled by rotating the twister array some 8° from the orientation that would normally be used. See ref[1] for a discussion of this effect.

**5. An alternate operating mode for beam line 4B**

For some proposed experiments [$(p, 2p)$ and $(p, \pi)$, for example] spatial dispersions of the order of $-5$ cm/% and a beam-line $D/M$ ratio of $-10$ cm/% are both required at some energies. This mode of operation is termed the 'alternate' operating mode for the beam line. It was found not possible to meet these specifications with the present arrangement of the beam line. In order to meet the criteria it was necessary to add one quadrupole downstream of the (existing) quadrupole 4BQ5.

Figure 6 shows the proposed (new) configuration for beam line 4-B. Quadrupole 4BQ6 has been added and the quadrupoles downstream of the 4BT2 location have been relabelled 4BQ7 and 4BQ8. The notation for the twister quadrupoles, TQx, has been maintained.

Listed in table 5 is the TRANSPORT input at 500 MeV for a dispersion of $-5$ cm/% and a beam-line $D/M$ ratio of $-10$ cm/% at the target 4BT2. Element settings for other energies are given in table 6. Figures 7–10 show the beam profiles for the various energies. Table 7 gives the beam sizes and overall transfer matrices at 4BT2 in this operating mode.

Operation of the beam line upstream of target 4BT1 differs considerably from current operation. In order to obtain the required dispersion and $D/M$ ratio at 4BT2, it is necessary to tune the triplet 4BQ4/5/6 in a V-H-V configuration. The result is that the beam size in quadrupole 4BQ5 becomes large in the horizontal plane.

At the time of the writing of this report, REVMOC studies of beam spill had been made only at an energy of 500 MeV. This predicted a beam loss of approximately 0.1% in quadrupole 4BQ5. A spill of this magnitude could cause experimental problems. The spill problem is under investigation but it may be that this quadrupole should be replaced with one with an eight-inch bore in order to meet the experimental requirements.

**6. The alternate mode with $R_{36}$ = -10 cm/%**

If it is assumed that the beam line is to be modified as indicated above, it would be reasonable to find a mode of operation that would reproduce those parameters of the standard operating mode indicated above. Table 8 lists TRANSPORT input at 500 MeV that will give $-10$ cm/% vertical dispersion at 4BT2 were the beam line arranged as in figure 6. TRANSPORT data for other energies are given in table 9 and beam sizes and overall transfer matrices at the 4BT2 target are given in table 9. Figures 11–14 show the predicted beam profiles along the beam line at the various energies.

It will be noted that the horizontal beam size at 4BT2 varies considerably with energy. In developing these tunes no attempt was made to minimize the horizontal spot size at 4BT2. Similarly, although the vertical angular dispersion at 4BT2 is small, no constraint was required on its magnitude.

The parameters presented here are given only to indicate that a viable TRANSPORT solution is possible with a triplet located upstream of 4BT1. Should this transport configuration be adopted, it should be possible to attain the desired beam properties at the 4BT2 target.

**References**

1. D. A. Hutcheon and G. M. Stinson, *A six-quadrupole twister for beam line 4B*, TRI-DNA-81-2, TRIUMF, November 1981.

Table 1

TRANSPORT input for a six-quadrupole twister at 500 MeV

| | | | | | | |
|---|---|---|---|---|---|---|
| 3.0 | '4BT1' | 0.00001 | | | | |
| 20.0 | 'ROT1' | 45. | | | | |
| 3.0 | 'DIN ' | 3.10963 | | | | |
| 5.00 | 'TQ1 ' | 0.4090 | | 5.12576 | | 5.08 |
| 3.0 | 'DQ12' | 0.3048 | | | | |
| 5.00 | 'TQ2 ' | 0.4090 | | −7.00250 | | 5.08 |
| 3.0 | 'DQ23' | 0.63839 | | | | |
| 5.00 | 'TQ3 ' | 0.4090 | | 8.00000 | | 5.08 |
| 3.0 | 'DQ34' | 0.57500 | | | | |
| 5.00 | 'TQ4 ' | 0.4090 | | 8.00000 | | 5.08 |
| 3.0 | 'DQ45' | 0.63839 | | | | |
| 5.00 | 'TQ5 ' | 0.4090 | | −7.00250 | | 5.08 |
| 3.0 | 'DQ56' | 0.3048 | | | | |
| 5.00 | 'TQ6 ' | 0.4090 | | 5.12576 | | 5.08 |
| 3.0 | 'DEX ' | 1.70973 | | | | |
| 20.0 | 'ROT2' | 45. | | | | |
| 20.0 | 'ROT3' | −90. | | | | |
| 3.0 | '4BT2' | 0.00001 | | | | |
| −10.0 | 'FX10' | −1. | 1. | | 0. | 0.001 ; |
| −10.0 | 'FX11' | −1. | 2. | | 0. | 0.001 ; |
| −10.0 | 'FX12' | −2. | 1. | | 0. | 0.001 ; |
| −10.0 | 'FX13' | −2. | 2. | | 0. | 0.001 ; |
| −10.0 | 'FX14' | −3. | 2. | | 0. | 0.001 ; |
| −10.0 | 'FX15' | −3. | 3. | | 0. | 0.001 ; |
| −10.0 | 'FX16' | −3. | 4. | | 0. | 0.001 ; |
| −10.0 | 'FX17' | −4. | 3. | | 0. | 0.001 ; |
| −10.0 | 'FX18' | −4. | 4. | | 0. | 0.001 ; |
| −10.0 | 'FX18' | −4. | 1. | | 0. | 0.001 ; |

Table 2

500 MeV TRANSPORT input for beam line 4B with a six-quadrupole twister and $R_{36} = -10$ cm/% and $R_{31} = 1$

```
‘500 MeV – SIX QUAD TWISTER – R36 = −10 AND R46 = 0’
  0
  1.  ‘BEAM’      0.12700    1.60000    0.66900  0.55600  0.0    0.1      1.09007 ;
 17.  ‘2ND’ ;
 12.  ‘12 ’       0.         0.         0.       0.       0.    −0.963    0.
                  0.         0.         0.       0.       0.     0.       0.         0.;
  1.  ‘FOIL’      0.         0.659      0.       0.659    0.     0.       0.         0.;
 14.  ‘R1 ’     −0.06689    0.34860    0.       0.       0.     1.27800  1.00000 ;
 14.  ‘R2 ’     −2.94700    0.26320    0.       0.       0.     1.57300  2.00000 ;
 14.  ‘R3 ’       0.         0.         1.06900  0.62430  0.     0.       3.00000 ;
 14.  ‘R4 ’       0.         0.         0.25800  1.08600  0.     0.       4.00000 ;
  3.  ‘4VM1’      0.31440 ;
  3.  ‘ ’         0.17060 ;
  5.  ‘4VQ1’      0.40640    5.90899    5.08 ;
  3.  ‘ ’         0.63020 ;
  5.  ‘4VQ2’      0.40640   −4.16461    5.08 ;
  3.  ‘ ’         0.69900 ;
  3.  ‘ ’         0.75750 ;
  3.  ‘ ’         0.42950 ;
  5.  ‘4VQ3’      0.40640    1.48429    5.08 ;
  3.  ‘ ’         0.50000 ;
 18.  ‘SEX1’      0.25000    0.00001   10.16 ;
  3.  ‘4VM2’      0.44330 ;
  3.  ‘ ’         1.08230 ;
 20.  ‘ ’       180.0 ;
  2.  ‘ ’        17.5 ;
  4.  ‘M35 ’      1.56290   14.21160    0.0 ;
  2.  ‘ ’        17.5 ;
 20.  ‘ ’      −180.0 ;
  3.  ‘MID ’      0.27130 ;
  3.  ‘M2IN’      0.27130 ;
 20.  ‘ ’       180.0 ;
  2.  ‘ ’        12.5 ;
  4.  ‘M25 ’      1.56520   10.13620    0.0 ;
  2.  ‘ ’        12.5 ;
 20.  ‘ ’      −180.0 ;
  3.  ‘4BM3’      2.75130 ;
  3.  ‘SX2I’      0.50000 ;
 18.  ‘SEX2’      0.25000    0.00001   10.16 ;
  3.  ‘Q4IN’      0.42567 ;
  5.  ‘4BQ4’      0.40640    2.40435    5.08 ;
```

Table 2 (Continued)

| | | | | | |
|---|---|---|---|---|---|
| 3. | ‘ ’ | 0.34780 ; | | | |
| 5. | ‘4BQ5’ | 0.40640 | −2.58568 | 5.08 ; | |
| 3. | ‘4BM4’ | 0.40890 ; | | | |
| 3. | ‘SX3I’ | 0.50000 ; | | | |
| 18. | ‘SEX3’ | 0.25000 | 0.00001 | 10.16 ; | |
| 3. | ‘4BW1’ | 4.00480 ; | | | |
| 3. | ‘4BM5’ | 2.92300 ; | | | |
| 3. | ‘4BT1’ | 0.20000 ; | | | |
| −10. | ‘FX1 ’ | −1. | 2. | 0. | 0.001 ; |
| −10. | ‘FX2 ’ | −1. | 6. | −10. | 0.0001 ; |
| −10. | ‘FX3 ’ | −2. | 6. | 0. | 0.0001 ; |
| −10. | ‘FY1 ’ | 4. | 3. | 0. | 0.001 ; |
| 20. | ‘ROT1’ | 45.0 ; | | | |
| 3. | ‘DIN ’ | 3.10963 ; | | | |
| 5. | ‘TQ1 ’ | 0.40900 | 5.12576 | 5.08 ; | |
| 3. | ‘DQ12’ | 0.30480 ; | | | |
| 5. | ‘TQ2 ’ | 0.40900 | −7.00250 | 5.08 ; | |
| 3. | ‘DQ23’ | 0.63839 ; | | | |
| 5. | ‘TQ3 ’ | 0.40900 | 8.00000 | 5.08 ; | |
| 3. | ‘DQ34’ | 0.57500 ; | | | |
| 5. | ‘TQ4 ’ | 0.40900 | 8.00000 | 5.08 ; | |
| 3. | ‘DQ45’ | 0.63839 ; | | | |
| 5. | ‘TQ5 ’ | 0.40900 | −7.00250 | 5.08 ; | |
| 3. | ‘DQ56’ | 0.30480 ; | | | |
| 5. | ‘TQ6 ’ | 0.40900 | 5.12576 | 5.08 ; | |
| 3. | ‘DEX ’ | 1.70973 ; | | | |
| 20. | ‘ROT2’ | 45.0 ; | | | |
| 20. | ‘ROT3’ | −90.0 ; | | | |
| 3. | ‘4BT2’ | 0.00001 ; | | | |
| 3. | ‘W2T2’ | 0.66000 ; | | | |
| 3. | ‘WXIM’ | 2.50000 ; | | | |
| 3. | ‘DQ11’ | 5.27810 ; | | | |
| 3. | ‘DADD’ | 2.00000 ; | | | |
| 5. | ‘4BQ7’ | 0.50000 | −4.18500 | 10.16 ; | |
| 3. | ‘ ’ | 0.40000 ; | | | |
| 5. | ‘4BQ8’ | 0.50000 | 4.32240 | 10.16 ; | |
| 3. | ‘4BM8’ | 0.40000 ; | | | |
| 3. | ‘WALL’ | 4.67360 ; | | | |
| 3. | ‘DUMP’ | 2.22570 ; | | | |
| −10. | ‘FX20’ | −1. | 2. | 0. | 0.001 ; |
| −10. | ‘FX21’ | −3. | 4. | 0. | 0.001 ; |
| SENTINEL : | | | | | |
| SENTINEL : | | | | | |

Table 3

Beam transport settings for delivery to 4BT2

| Element | Field (kG) at Energy (MeV) 500 | 400 | 300 | 200 |
|---|---|---|---|---|
| 4VQ1 | 5.9090 | 4.2473 | 2.5548 | 2.3595 |
| 4VQ2 | −4.1646 | −3.6069 | −2.3297 | −2.8958 |
| 4VQ3 | 1.4843 | 1.4414 | 1.9928 | 1.6862 |
| 4VB1 | 14.2116 | 12.4410 | 10.5350 | 8.4018 |
| 4AB2 | 10.1362 | 8.8733 | 7.5139 | 5.9924 |
| 4BQ4 | 2.4044 | 2.0212 | 3.0114 | 0.5415 |
| 4BQ5 | −2.5857 | −2.1213 | −5.1559 | −0.4936 |
| TWQ1 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| TWQ2 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ3 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ4 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ5 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ6 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| 4BQ11 | −4.1850 | −3.6636 | −3.3339 | −2.5166 |
| 4BQ12 | 4.3224 | 3.7839 | 3.5606 | 2.5857 |

Transfer matrix from 4BT1 to 4BT2

| | | | | | |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 1.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0001 | 0.0000 | 0.0000 | 1.0000 | 0.0 | 0.0 |
| 1.0000 | −0.0003 | 0.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 |

Table 4

Beam sizes at 4BT2 for $R_{36} = -10$ cm/% and $R_{31} = 1$

| Parameter | Energy (MeV) | | | |
|---|---|---|---|---|
| | 500 | 400 | 300 | 200 |
| $\pm x$ (cm) | 0.227 | 0.219 | 0.197 | 0.397 |
| $\pm\theta$ (mr) | 1.995 | 1.521 | 0.477 | 1.562 |
| $\pm y$ (cm) | 1.003 | 1.006 | 1.008 | 1.012 |
| $\pm\phi$ (mr) | 2.944 | 2.157 | 2.145 | 2.339 |
| $R_{13}$ (cm/cm) | 0.0495 | 0.2996 | 1.1628 | 0.8356 |
| $R_{14}$ (cm/mr) | −0.2372 | −0.1215 | 1.4990 | 0.1751 |
| $R_{23}$ (mr/cm) | 3.7792 | 3.2281 | 0.9339 | −2.4887 |
| $R_{24}$ (mr/mr) | 2.0863 | 2.0289 | 2.0640 | 0.7116 |
| $R_{31}$ (cm/cm) | 0.5941 | 0.8427 | 0.9998 | 1.0001 |
| $R_{32}$ (cm/mr) | 0.0000 | 0.0000 | −0.0001 | 0.0000 |
| $R_{36}$ (cm/%) | −10.0000 | −10.0001 | −10.0003 | −9.9998 |
| $R_{41}$ (mr/cm) | 1.0397 | 1.9135 | 4.3812 | 2.8524 |
| $R_{42}$ (mr/mr) | 1.6995 | 1.2382 | 0.9514 | 0.7420 |
| $R_{46}$ (mr/%) | −0.0006 | −0.0007 | −5.4772 | −4.1325 |

Table 5

500 MeV TRANSPORT input for beam line 4B with a six-quadrupole twister
Triplet upstream of 4BT1 – $R_{36} = -5$ cm/% and $R_{31} \leq 0.5$

```
'500 MeV – 6 QUAD TWISTER – (− + −) TRIPLET UPSTREAM 4BT1'             0
  1.  'BEAM'    0.12700    1.60000   0.66900  0.55600  0.0   0.1      1.09007 ;
−17.  '2ND' ;
 12.  '12 '     0.         0.        0.       0.       0.    −0.963   0.
                0.         0.        0.       0.       0.    0.       0.        0.;
  1.  'FOIL'    0.         0.659     0.       0.659    0.    0.       0.        0.;
 14.  'R1 '     −0.06689   0.34860   0.       0.       0.    1.27800  1.00000 ;
 14.  'R2 '     −2.94700   0.26320   0.       0.       0.    1.57300  2.00000 ;
 14.  'R3 '     0.         0.        1.06900  0.62430  0.    0.       3.00000 ;
 14.  'R4 '     0.         0.        0.25800  1.08600  0.    0.       4.00000 ;
  3.  '4VM1'    0.31440 ;
  3.  ' '       0.17060 ;
  5.  '4VQ1'    0.40640    4.32132   5.08 ;
  3.  ' '       0.63020 ;
  5.  '4VQ2'    0.40640    −3.66204  5.08 ;
  3.  ' '       0.69900 ;
  3.  ' '       0.75750 ;
  3.  ' '       0.42950 ;
  5.  '4VQ3'    0.40640    1.29250   5.08 ;
  3.  ' '       0.50000 ;
 18.  'SEX1'    0.25000    0.00001   10.16 ;
  3.  '4VM2'    0.44330 ;
  3.  ' '       1.08230 ;
 20.  ' '       180.0 ;
  2.  ' '       17.5 ;
  4.  'M35 '    1.56290    14.21160  0.0 ;
  2.  ' '       17.5 ;
 20.  ' '       −180.0 ;
  3.  'MID '    0.27130 ;
  3.  'M2IN'    0.27130 ;
 20.  ' '       180.0 ;
  2.  ' '       12.5 ;
  4.  'M25 '    1.56520    10.13620  0.0 ;
  2.  ' '       12.5 ;
 20.  ' '       −180.0 ;
  3.  '4BM3'    2.75130 ;
  3.  'SX2I'    0.50000 ;
 18.  'SEX2'    0.25000    0.00001   10.16 ;
  3.  'Q4IN'    0.42567 ;
  5.  '4BQ4'    0.40640    −2.26698  5.08 ;
```

Table 5 (Continued)

| | | | | | | |
|---|---|---|---|---|---|---|
| 3. | ‘ ’ | 0.34780 ; | | | | |
| 5. | ‘4BQ5’ | 0.40640 | 3.72874 | 5.08 ; | | |
| 3. | ‘4BM4’ | 0.40890 ; | | | | |
| 5. | ‘4BQ6’ | 0.40640 | −1.77453 | 5.08 ; 3. | ‘SX3I’0. | 09100 ; |
| 18. | ‘SEX3’ | 0.25000 | 0.00001 | 10.16 ; | | |
| 3. | ‘4BW1’ | 4.00480 ; | | | | |
| 3. | ‘4BM5’ | 2.92300 ; | | | | |
| 3. | ‘4BT1’ | 0.20000 ; | | | | |
| −10. | ‘FX1 ’ | −1. | 2. | 0. | 0.001 ; | |
| −10. | ‘FX2 ’ | −1. | 6. | −5. | 0.0001 ; | |
| −10. | ‘FX3 ’ | −1. | 1. | 0.5 | 0.0501 ; | |
| −10. | ‘FY1 ’ | 3. | 3. | 0.4 | 0.0500 ; | |
| −10. | ‘FY2 ’ | 4. | 3. | 0. | 0.0010 ; | |
| 20. | ‘ROT1’ | 45.0 ; | | | | |
| 3. | ‘DIN ’ | 3.10963 ; | | | | |
| 5. | ‘TQ1 ’ | 0.40900 | 5.12576 | 5.08 ; | | |
| 3. | ‘DQ12’ | 0.30480 ; | | | | |
| 5. | ‘TQ2 ’ | 0.40900 | −7.00250 | 5.08 ; | | |
| 3. | ‘DQ23’ | 0.63839 ; | | | | |
| 5. | ‘TQ3 ’ | 0.40900 | 8.00000 | 5.08 ; | | |
| 3. | ‘DQ34’ | 0.57500 ; | | | | |
| 5. | ‘TQ4 ’ | 0.40900 | 8.00000 | 5.08 ; | | |
| 3. | ‘DQ45’ | 0.63839 ; | | | | |
| 5. | ‘TQ5 ’ | 0.40900 | −7.00250 | 5.08 ; | | |
| 3. | ‘DQ56’ | 0.30480 ; | | | | |
| 5. | ‘TQ6 ’ | 0.40900 | 5.12576 | 5.08 ; | | |
| 3. | ‘DEX ’ | 1.70973 ; | | | | |
| 20. | ‘ROT2’ | 45.0 ; | | | | |
| 20. | ‘ROT3’ | −90.0 ; | | | | |
| 3. | ‘4BT2’ | 0.00001 ; | | | | |
| 3. | ‘W2T2’ | 0.66000 ; | | | | |
| 3. | ‘WXIM’ | 2.50000 ; | | | | |
| 3. | ‘DQ11’ | 7.27810 ; | | | | |
| 5. | ‘4BQ7’ | 0.50000 | −4.18500 | 10.16 ; | | |
| 3. | ‘ ’ | 0.40000 ; | | | | |
| 5. | ‘4BQ8’ | 0.50000 | 4.32240 | 10.16 ; | | |
| 3. | ‘4BM8’ | 0.40000 ; | | | | |
| 3. | ‘WALL’ | 4.67360 ; | | | | |
| 3. | ‘DUMP’ | 2.22570 ; | | | | |
| −10. | ‘FX20’ | −1. | 2. | 0. | 0.001 ; | |
| −10. | ‘FX21’ | −3. | 4. | 0. | 0.001 ; | |
| SENTINEL : | | | | | | |
| SENTINEL : | | | | | | |

Table 6

Beam transport settings for delivery to 4BT2 – $R_{36} = -5$ cm/% and $R_{36}/R_{31} = -10$ cm/%

| Element | Field (kG) at Energy (MeV) 500 | 400 | 300 | 200 |
|---|---|---|---|---|
| 4VQ1 | 4.5087 | 3.3213 | 2.7327 | 2.2137 |
| 4VQ2 | −3.5364 | −2.7437 | −2.5815 | −2.6857 |
| 4VQ3 | 1.2267 | 1.2134 | 2.0500 | 1.6525 |
| 4VB1 | 14.2116 | 12.4410 | 10.5350 | 8.4018 |
| 4AB2 | 10.1362 | 8.8733 | 7.5139 | 5.9924 |
| 4BQ4 | −2.5060 | −2.5059 | −2.5603 | −0.5415 |
| 4BQ5 | 3.9898 | 3.9156 | 3.7404 | 1.4936 |
| 4BQ6 | −1.9993 | −2.2336 | −2.3954 | −0.8354 |
| TWQ1 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| TWQ2 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ3 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ4 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ5 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ6 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| 4BQ7 | −4.1850 | −3.6636 | −3.3339 | −2.5166 |
| 4BQ8 | 4.3224 | 3.7839 | 3.5606 | 2.5857 |

Transfer matrix from 4BT1 to 4BT2

| | | | | | |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 1.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0001 | 0.0000 | 0.0000 | 1.0000 | 0.0 | 0.0 |
| 1.0000 | −0.0003 | 0.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 |

Table 7

Beam sizes at 4BT2 for $R_{36} = -5$ cm/% and $R_{36}/R_{31} = -10$ cm/%

| Parameter | Energy (MeV) 500 | 400 | 300 | 200 |
|---|---|---|---|---|
| $\pm x$ (cm) | 0.465 | 0.495 | 0.138 | 0.247 |
| $\pm\theta$ (mr) | 0.973 | 0.705 | 0.730 | 1.475 |
| $\pm y$ (cm) | 0.504 | 0.506 | 0.507 | 0.506 |
| $\pm\phi$ (mr) | 4.285 | 3.430 | 3.920 | 4.300 |
| $R_{13}$ (cm/cm) | −0.1922 | 0.3746 | 0.5058 | 0.3710 |
| $R_{14}$ (cm/mr) | −0.6189 | −0.4396 | 0.5322 | 0.3314 |
| $R_{23}$ (mr/cm) | 1.8103 | 1.7794 | −1.1511 | −2.2062 |
| $R_{24}$ (mr/mr) | 0.6273 | 0.5813 | 0.7659 | 0.8066 |
| $R_{31}$ (cm/cm) | 0.4233 | 0.5624 | 0.5301 | 0.4641 |
| $R_{32}$ (cm/mr) | 0.0000 | −0.0001 | 0.0001 | 0.0000 |
| $R_{36}$ (cm/%) | −5.0004 | −5.0002 | −4.9999 | −5.0000 |
| $R_{41}$ (mr/cm) | 2.2550 | 2.8512 | 6.5087 | 4.7732 |
| $R_{42}$ (mr/mr) | 2.3846 | 1.8552 | 1.7951 | 1.5991 |
| $R_{46}$ (mr/%) | 11.1837 | 11.5032 | 6.8357 | 5.0221 |

Table 8

500 MeV TRANSPORT input for beam line 4B with triplet upstream of 4BT1
$R_{36} = -10$ cm/% and $R_{31} = 1$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| '500 MeV – 6 QUAD TWISTER – ($-+-$) TRIPLET UPSTREAM 4BT1' | | | | | | | | 0 | |
| 1. | 'BEAM' | 0.12700 | 1.60000 | 0.66900 | 0.55600 | 0.0 | 0.1 | 1.09007 ; | |
| −17. | '2ND' ; | | | | | | | | |
| 12. | '12 ' | 0. | 0. | 0. | 0. | 0. | −0.963 | 0. | |
| | | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0.; |
| 1. | 'FOIL' | 0. | 0.659 | 0. | 0.659 | 0. | 0. | 0. | 0.; |
| 14. | 'R1 ' | −0.06689 | 0.34860 | 0. | 0. | 0. | 1.27800 | 1.00000 ; | |
| 14. | 'R2 ' | −2.94700 | 0.26320 | 0. | 0. | 0. | 1.57300 | 2.00000 ; | |
| 14. | 'R3 ' | 0. | 0. | 1.06900 | 0.62430 | 0. | 0. | 3.00000 ; | |
| 14. | 'R4 ' | 0. | 0. | 0.25800 | 1.08600 | 0. | 0. | 4.00000 ; | |
| 3. | '4VM1' | 0.31440 ; | | | | | | | |
| 3. | ' ' | 0.17060 ; | | | | | | | |
| 5. | '4VQ1' | 0.40640 | 5.34657 | 5.08 ; | | | | | |
| 3. | ' ' | 0.63020 ; | | | | | | | |
| 5. | '4VQ2' | 0.40640 | −3.39126 | 5.08 ; | | | | | |
| 3. | ' ' | 0.69900 ; | | | | | | | |
| 3. | ' ' | 0.75750 ; | | | | | | | |
| 3. | ' ' | 0.42950 ; | | | | | | | |
| 5. | '4VQ3' | 0.40640 | 1.92332 | 5.08 ; | | | | | |
| 3. | ' ' | 0.50000 ; | | | | | | | |
| 18. | 'SEX1' | 0.25000 | 0.00001 | 10.16 ; | | | | | |
| 3. | '4VM2' | 0.44330 ; | | | | | | | |
| 3. | ' ' | 1.08230 ; | | | | | | | |
| 20. | ' ' | 180.0 ; | | | | | | | |
| 2. | ' ' | 17.5 ; | | | | | | | |
| 4. | 'M35 ' | 1.56290 | 14.21160 | 0.0 ; | | | | | |
| 2. | ' ' | 17.5 ; | | | | | | | |
| 20. | ' ' | −180.0 ; | | | | | | | |
| 3. | 'MID ' | 0.27130 ; | | | | | | | |
| 3. | 'M2IN' | 0.27130 ; | | | | | | | |
| 20. | ' ' | 180.0 ; | | | | | | | |
| 2. | ' ' | 12.5 ; | | | | | | | |
| 4. | 'M25 ' | 1.56520 | 10.13620 | 0.0 ; | | | | | |
| 2. | ' ' | 12.5 ; | | | | | | | |
| 20. | ' ' | −180.0 ; | | | | | | | |
| 3. | '4BM3' | 2.75130 ; | | | | | | | |
| 3. | 'SX2I' | 0.50000 ; | | | | | | | |
| 18. | 'SEX2' | 0.25000 | 0.00001 | 10.16 ; | | | | | |
| 3. | 'Q4IN' | 0.42567 ; | | | | | | | |
| 5. | '4BQ4' | 0.40640 | −2.41859 | 5.08 ; | | | | | |

Table 8 (Continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3. | ‘ ’ | 0.34780 ; | | | | | |
| 5. | ‘4BQ5’ | 0.40640 | 4.67849 | 5.08 ; | | | |
| 3. | ‘4BM4’ | 0.40890 ; | | | | | |
| 5. | ‘4BQ6’ | 0.40640 | −3.89725 | 5.08 ; 3. | ‘SX3I’0. | | 09100 ; |
| 18. | ‘SEX3’ | 0.25000 | 0.00001 | 10.16 ; | | | |
| 3. | ‘4BW1’ | 4.00480 ; | | | | | |
| 3. | ‘4BM5’ | 2.92300 ; | | | | | |
| 3. | ‘4BT1’ | 0.20000 ; | | | | | |
| −10. | ‘FX1 ’ | −1. | 2. | 0. | | 0.001 ; | |
| −10. | ‘FX2 ’ | −1. | 6. | −10. | | 0.0001 ; | |
| −10. | ‘FX3 ’ | −1. | 1. | 1.0 | | 0.0501 ; | |
| −10. | ‘FY1 ’ | 3. | 3. | 0.4 | | 0.0500 ; | |
| −10. | ‘FY2 ’ | 4. | 3. | 0.25 | | 0.0750 ; | |
| 20. | ‘ROT1’ | 45.0 ; | | | | | |
| 3. | ‘DIN ’ | 3.10963 ; | | | | | |
| 5. | ‘TQ1 ’ | 0.40900 | 5.12576 | 5.08 ; | | | |
| 3. | ‘DQ12’ | 0.30480 ; | | | | | |
| 5. | ‘TQ2 ’ | 0.40900 | −7.00250 | 5.08 ; | | | |
| 3. | ‘DQ23’ | 0.63839 ; | | | | | |
| 5. | ‘TQ3 ’ | 0.40900 | 8.00000 | 5.08 ; | | | |
| 3. | ‘DQ34’ | 0.57500 ; | | | | | |
| 5. | ‘TQ4 ’ | 0.40900 | 8.00000 | 5.08 ; | | | |
| 3. | ‘DQ45’ | 0.63839 ; | | | | | |
| 5. | ‘TQ5 ’ | 0.40900 | −7.00250 | 5.08 ; | | | |
| 3. | ‘DQ56’ | 0.30480 ; | | | | | |
| 5. | ‘TQ6 ’ | 0.40900 | 5.12576 | 5.08 ; | | | |
| 3. | ‘DEX ’ | 1.70973 ; | | | | | |
| 20. | ‘ROT2’ | 45.0 ; | | | | | |
| 20. | ‘ROT3’ | −90.0 ; | | | | | |
| 3. | ‘4BT2’ | 0.00001 ; | | | | | |
| 3. | ‘W2T2’ | 0.66000 ; | | | | | |
| 3. | ‘WXIM’ | 2.50000 ; | | | | | |
| 3. | ‘DQ11’ | 7.27810 ; | | | | | |
| 5. | ‘4BQ7’ | 0.50000 | −4.18500 | 10.16 ; | | | |
| 3. | ‘ ’ | 0.40000 ; | | | | | |
| 5. | ‘4BQ8’ | 0.50000 | 4.32240 | 10.16 ; | | | |
| 3. | ‘4BM8’ | 0.40000 ; | | | | | |
| 3. | ‘WALL’ | 4.67360 ; | | | | | |
| 3. | ‘DUMP’ | 2.22570 ; | | | | | |
| −10. | ‘FX20’ | −1. | 2. | 0. | | 0.001 ; | |
| −10. | ‘FX21’ | −3. | 4. | 0. | | 0.001 ; | |
| SENTINEL : | | | | | | | |
| SENTINEL : | | | | | | | |

Table 9

Beam transport settings for delivery to 4BT2 – $R_{36} = -10$ cm/% and $R_{31} = 1$

| Element | Field (kG) at Energy (MeV) 500 | 400 | 300 | 200 |
|---|---|---|---|---|
| 4VQ1 | 5.3466 | 4.3687 | 3.0074 | 2.3579 |
| 4VQ2 | −3.3913 | −3.0453 | −2.6286 | −2.4176 |
| 4VQ3 | 1.9233 | 1.7207 | 2.0500 | 1.6522 |
| 4VB1 | 14.2116 | 12.4410 | 10.5350 | 8.4018 |
| 4AB2 | 10.1362 | 8.8733 | 7.5139 | 5.9924 |
| 4BQ4 | −2.4186 | −1.9106 | −2.5603 | −0.5415 |
| 4BQ5 | 4.6785 | 3.8381 | 3.6641 | 1.2698 |
| 4BQ6 | −3.8973 | −3.0994 | −2.5659 | −0.8346 |
| TWQ1 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| TWQ2 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ3 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ4 | 8.0000 | 7.0033 | 5.9303 | 4.7295 |
| TWQ5 | −7.0047 | −6.1320 | −5.1925 | −4.1411 |
| TWQ6 | 5.1270 | 4.4883 | 3.8006 | 3.0311 |
| 4BQ7 | −4.1850 | −3.6636 | −3.3339 | −2.5166 |
| 4BQ8 | 4.3224 | 3.7839 | 3.5606 | 2.5857 |

Transfer matrix from 4BT1 to 4BT2

| | | | | | |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 1.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0001 | 0.0000 | 0.0000 | 1.0000 | 0.0 | 0.0 |
| 1.0000 | −0.0003 | 0.0000 | −0.0003 | 0.0 | 0.0 |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0000 |

Table 10

Beam sizes at 4BT2 for $R_{36} = -10$ cm/% and $R_{31} = 1$

| Parameter | Energy (MeV) 500 | 400 | 300 | 200 |
|---|---|---|---|---|
| $\pm x$ (cm) | 0.400 | 0.367 | 0.130 | 0.377 |
| $\pm\theta$ (mr) | 1.259 | 1.044 | 0.777 | 0.965 |
| $\pm y$ (cm) | 1.008 | 1.008 | 1.011 | 1.010 |
| $\pm\phi$ (mr) | 1.840 | 1.798 | 2.009 | 2.344 |
| $R_{13}$ (cm/cm) | 0.2421 | 0.6887 | 0.4696 | 0.7049 |
| $R_{14}$ (cm/mr) | −0.3200 | −0.0684 | 0.5033 | 0.4272 |
| $R_{23}$ (mr/cm) | 2.3809 | 2.5008 | −1.2996 | −0.9667 |
| $R_{24}$ (mr/mr) | 0.9629 | 1.2036 | 0.7366 | 0.8761 |
| $R_{31}$ (cm/cm) | 0.9544 | 1.0100 | 1.0346 | 0.8590 |
| $R_{32}$ (cm/mr) | 0.0000 | −0.0001 | 0.0001 | 0.0000 |
| $R_{36}$ (cm/%) | −10.0000 | −9.9999 | −9.9999 | −9.9999 |
| $R_{41}$ (mr/cm) | 0.8733 | 1.4074 | 3.9377 | 3.0943 |
| $R_{42}$ (mr/mr) | 1.0579 | 1.0332 | 0.9195 | 0.8645 |
| $R_{46}$ (mr/%) | −1.3123 | −0.0582 | −2.3155 | −3.3689 |

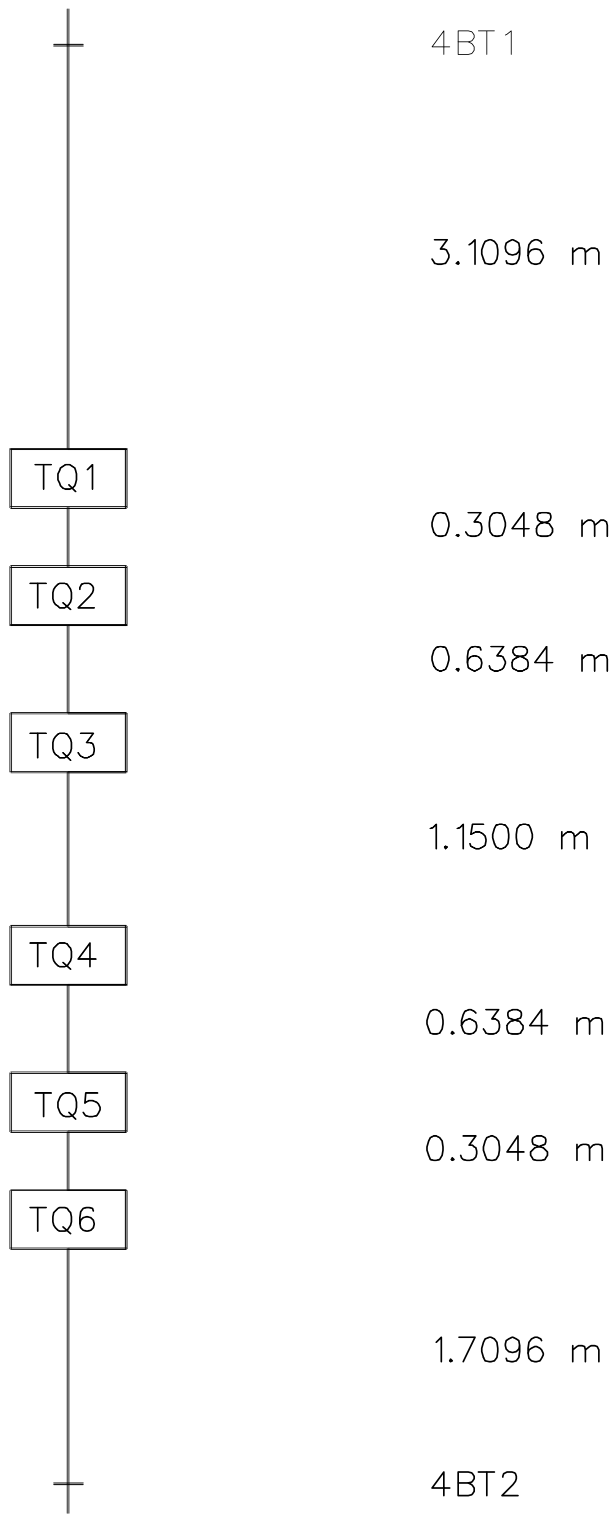



Fig. 1. Geometry of a six-quadrupole twister for beam line 4B.

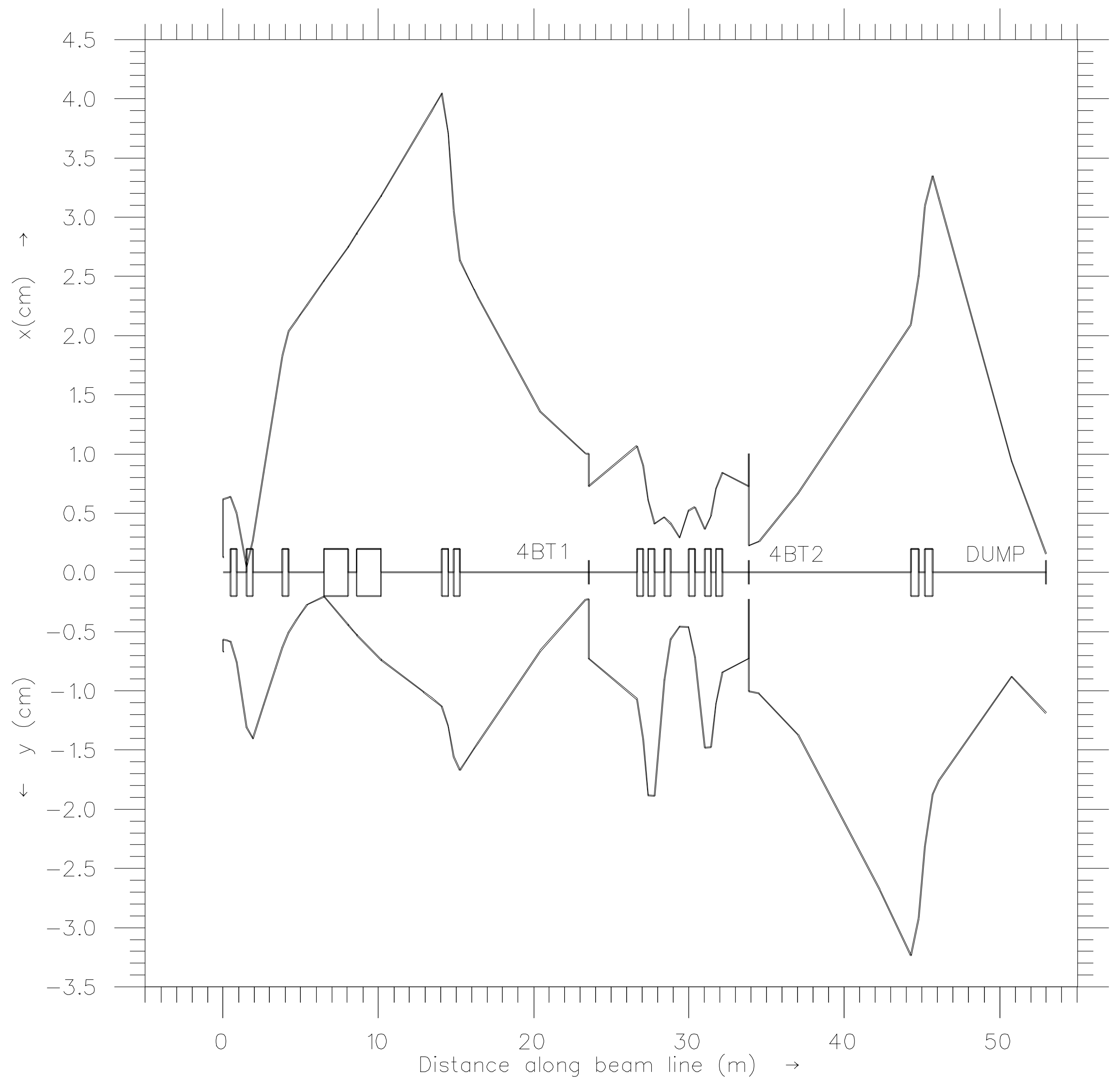



Fig. 2. 500 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

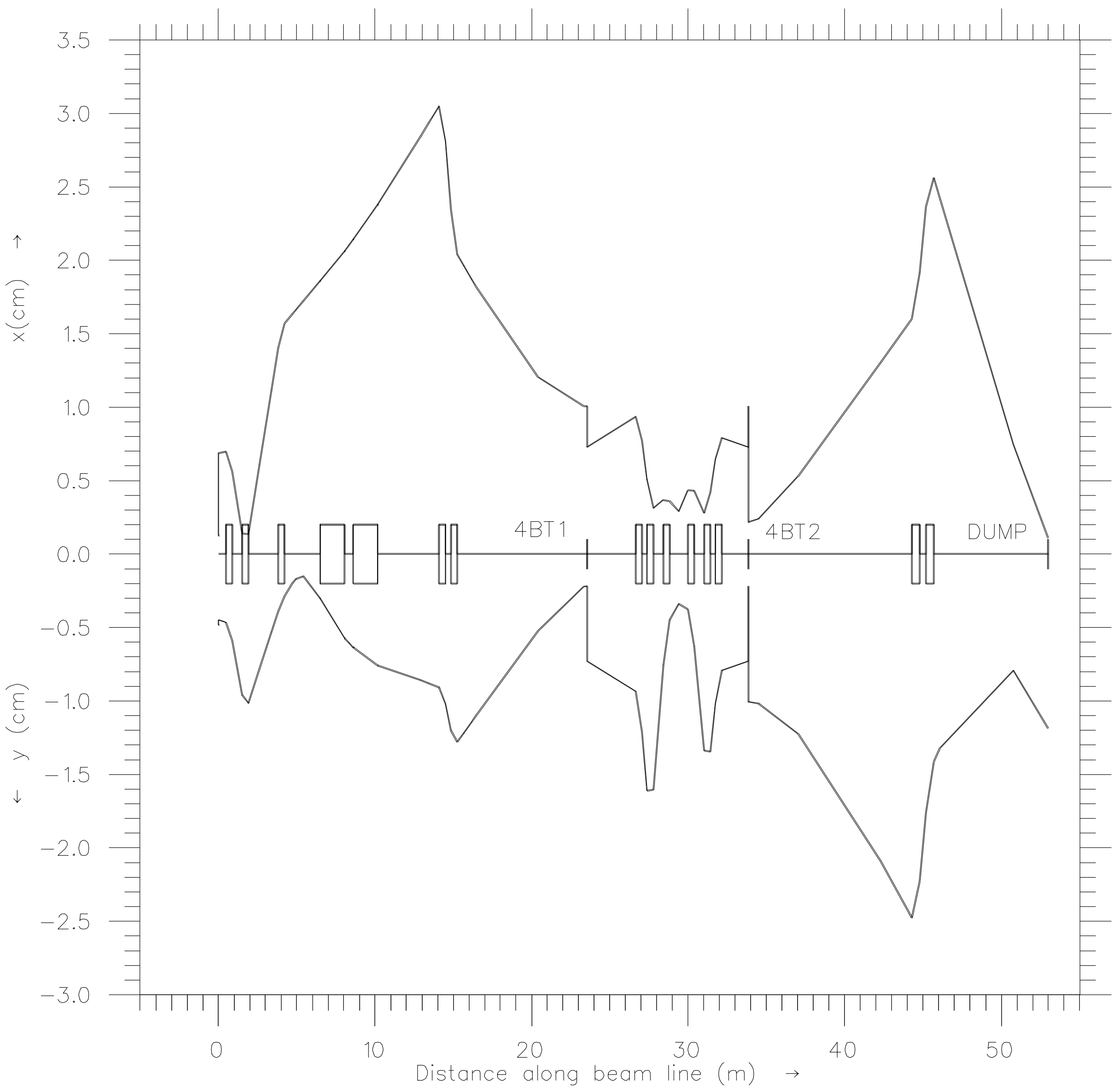



Fig. 3. 400 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

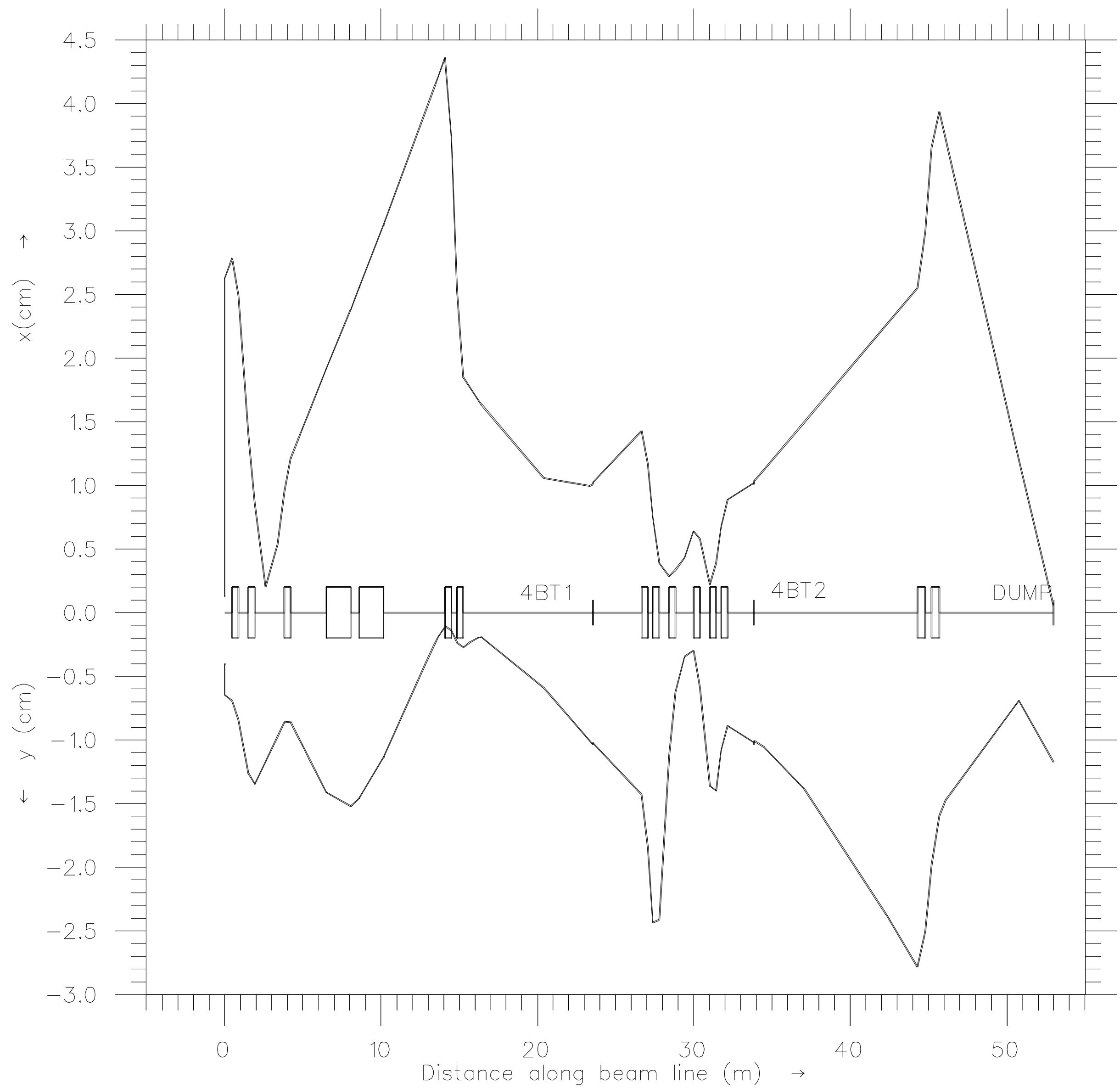



Fig. 4. 300 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

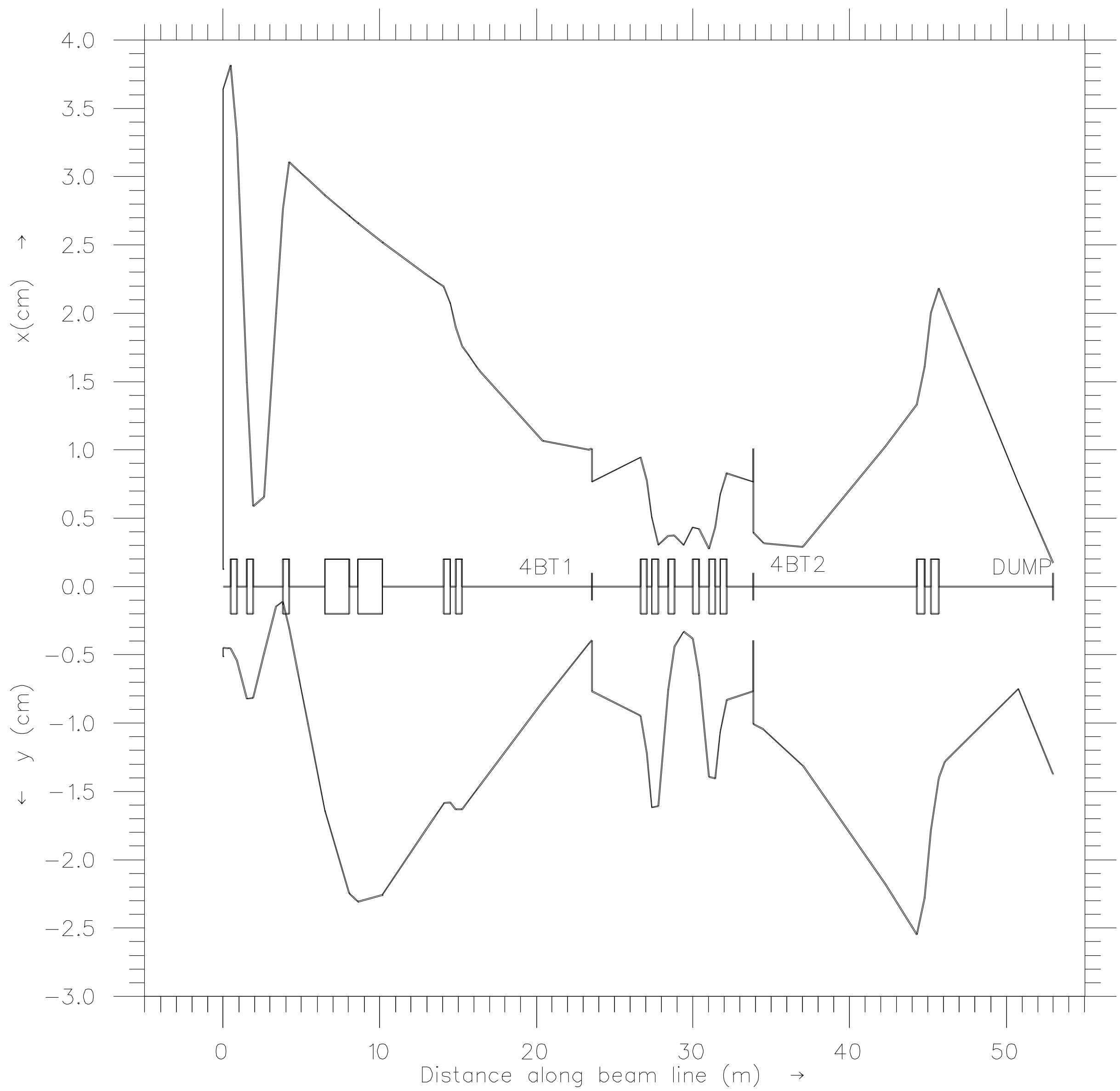



Fig. 5. 200 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

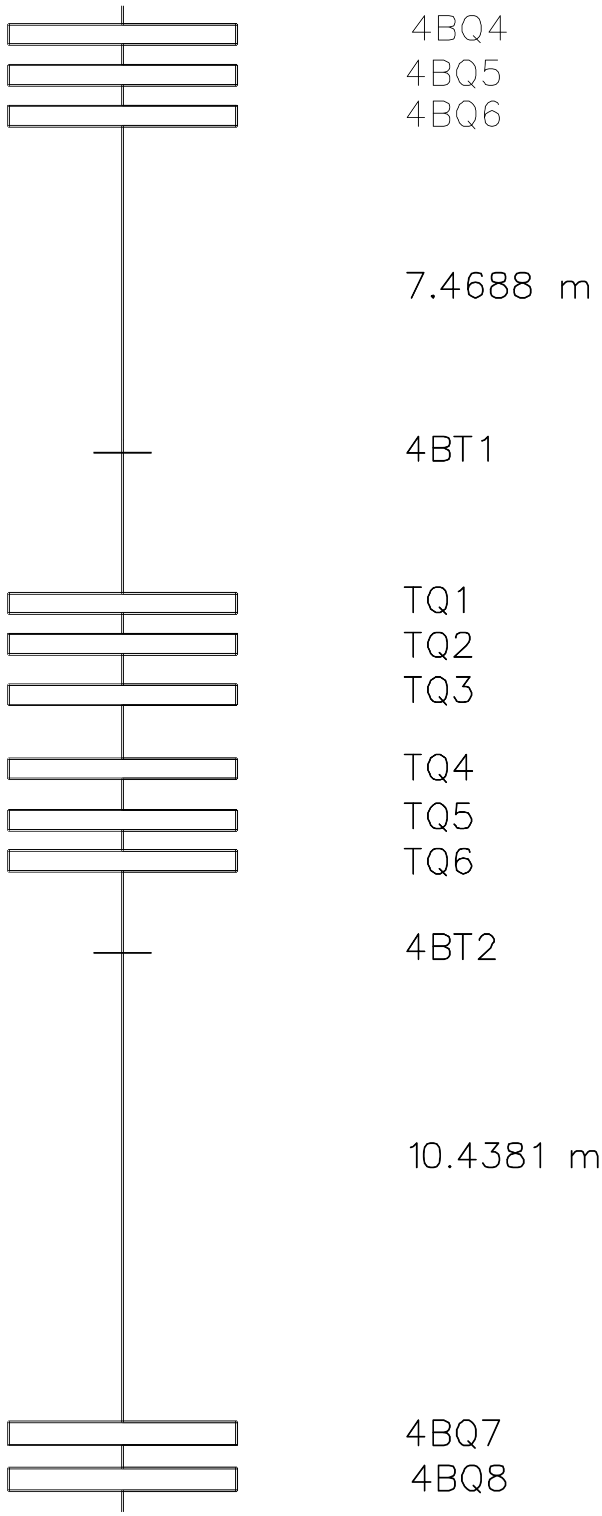



Fig. 6. An alternate configuration for beam line 4B.

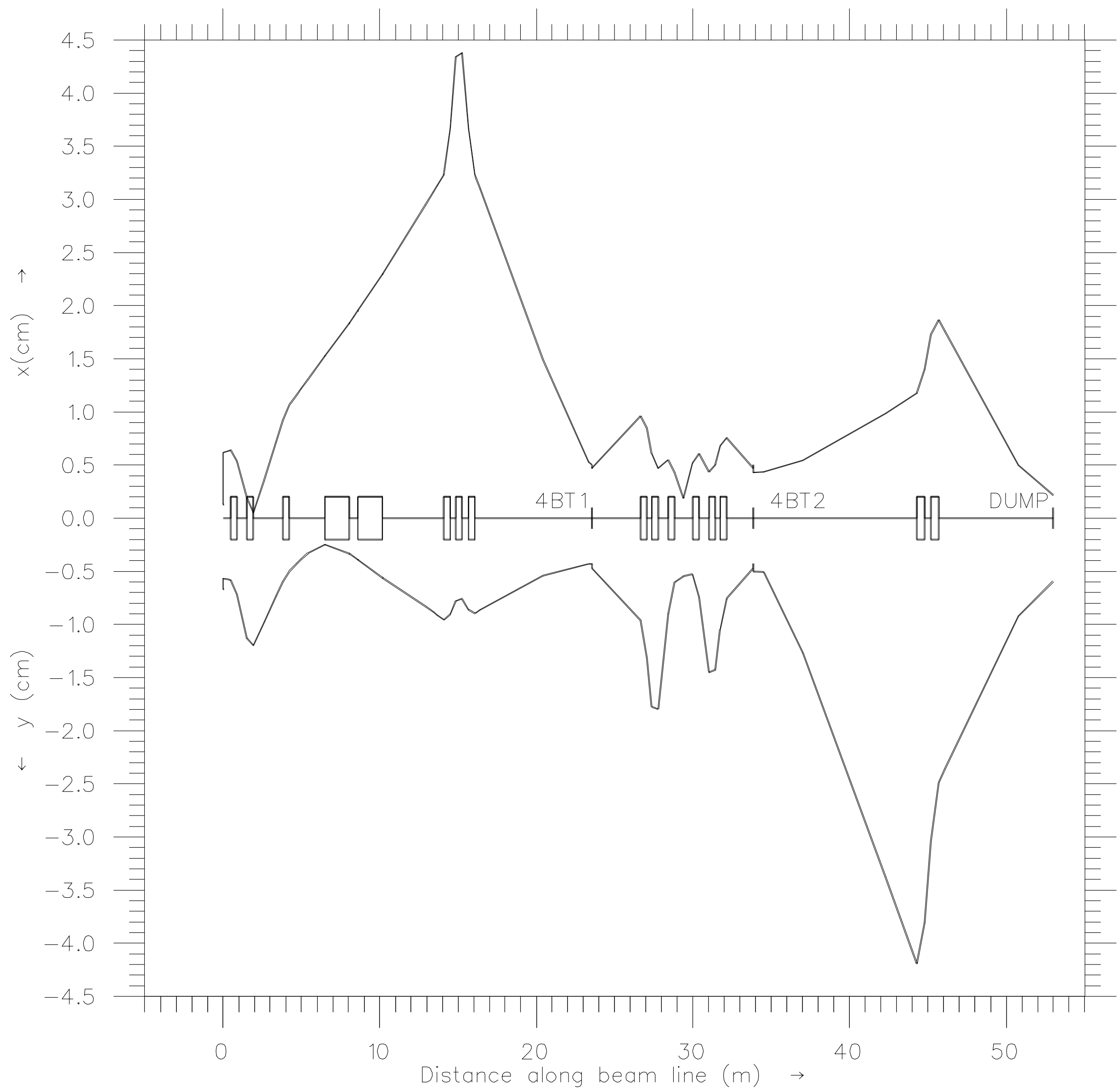



Fig. 7. 500 MeV beam profiles on beam line 4B for $R_{36} = -$ cm/% and $R_{31} \leq 0.5$

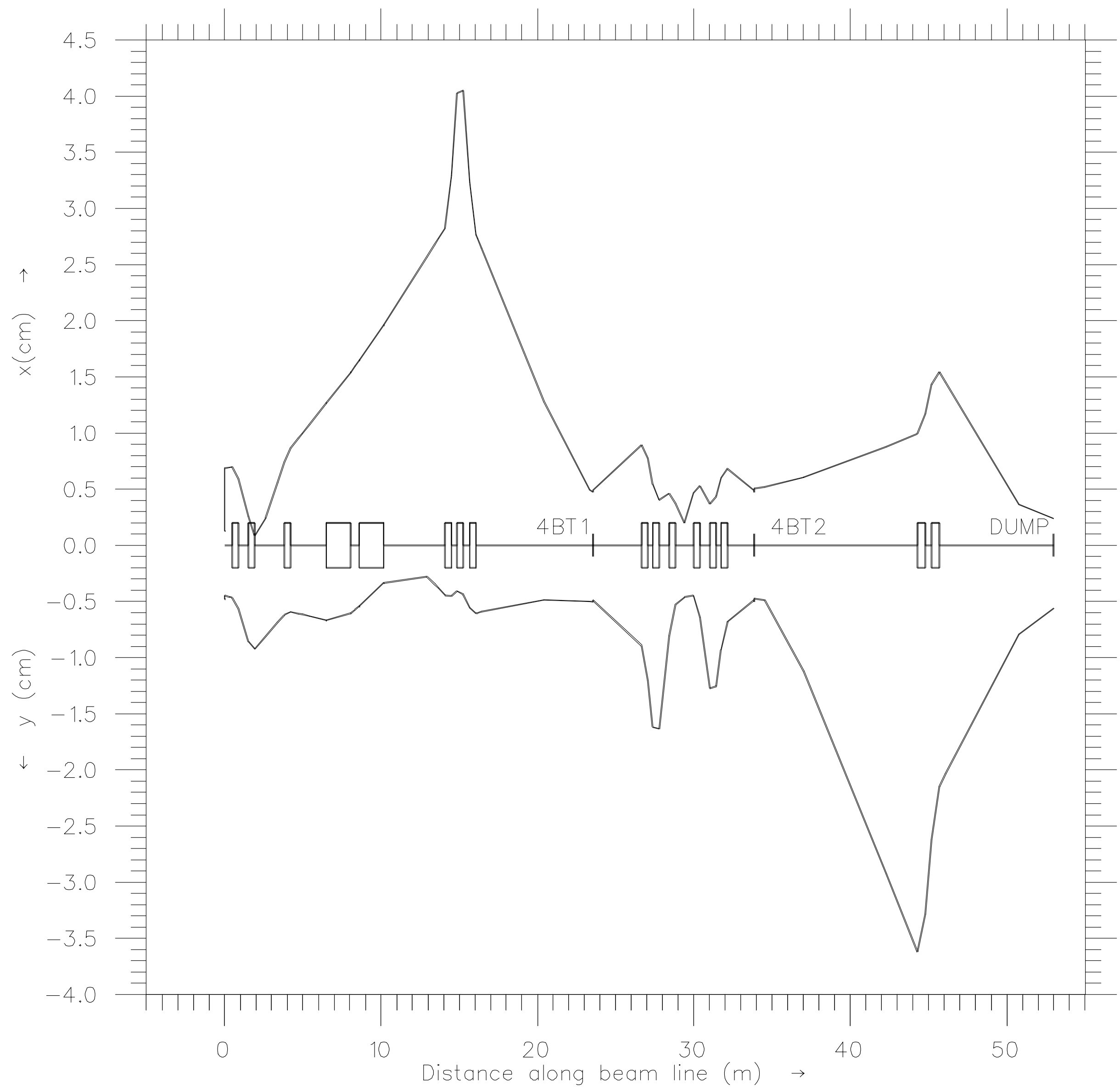



Fig. 8. 400 MeV beam profiles on beam line 4B for $R_{36} = -$ cm/% and $R_{31} \leq 0.5$

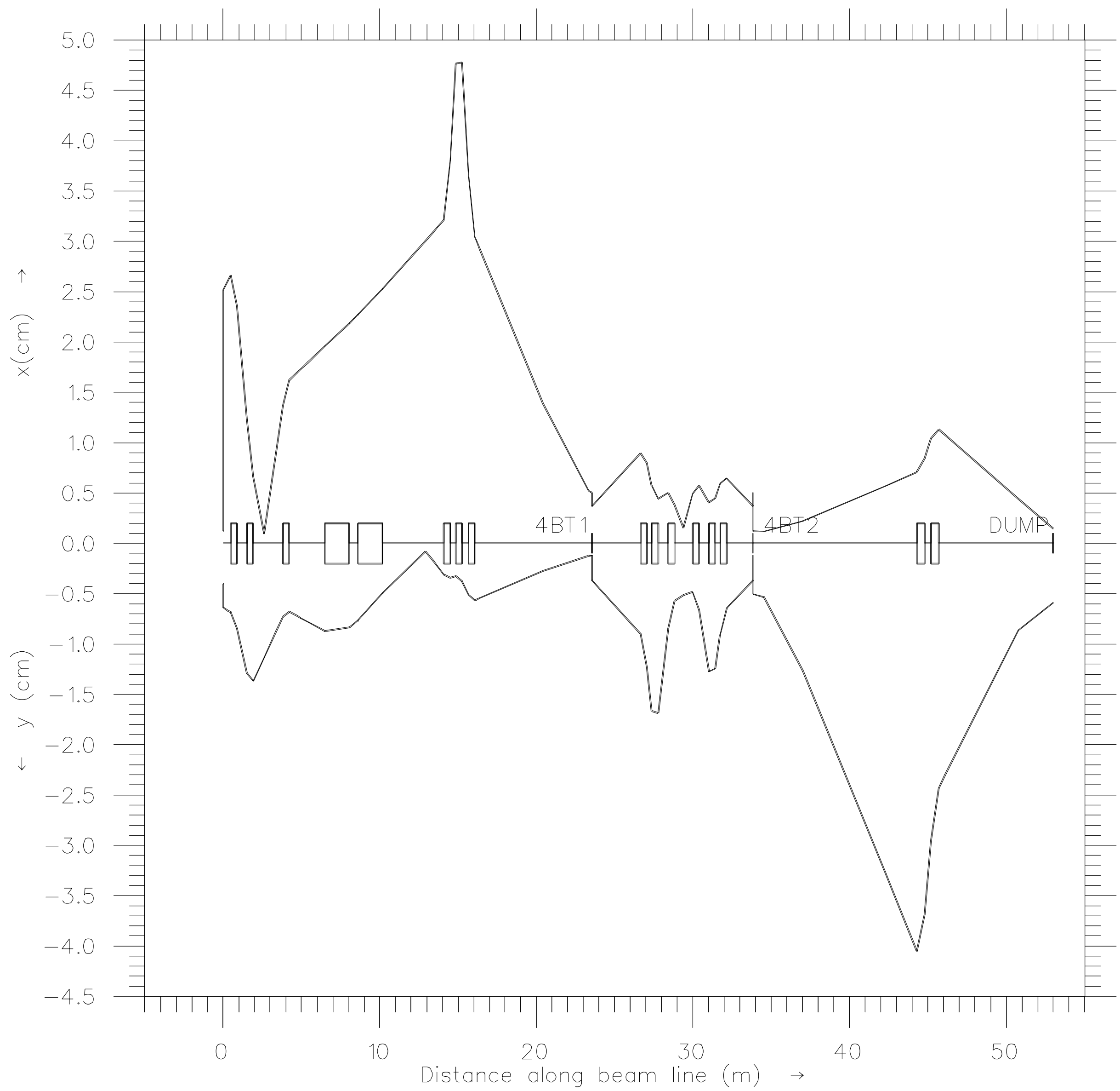



Fig. 9. 300 MeV beam profiles on beam line 4B for $R_{36} = -$ cm/% and $R_{31} \leq 0.5$

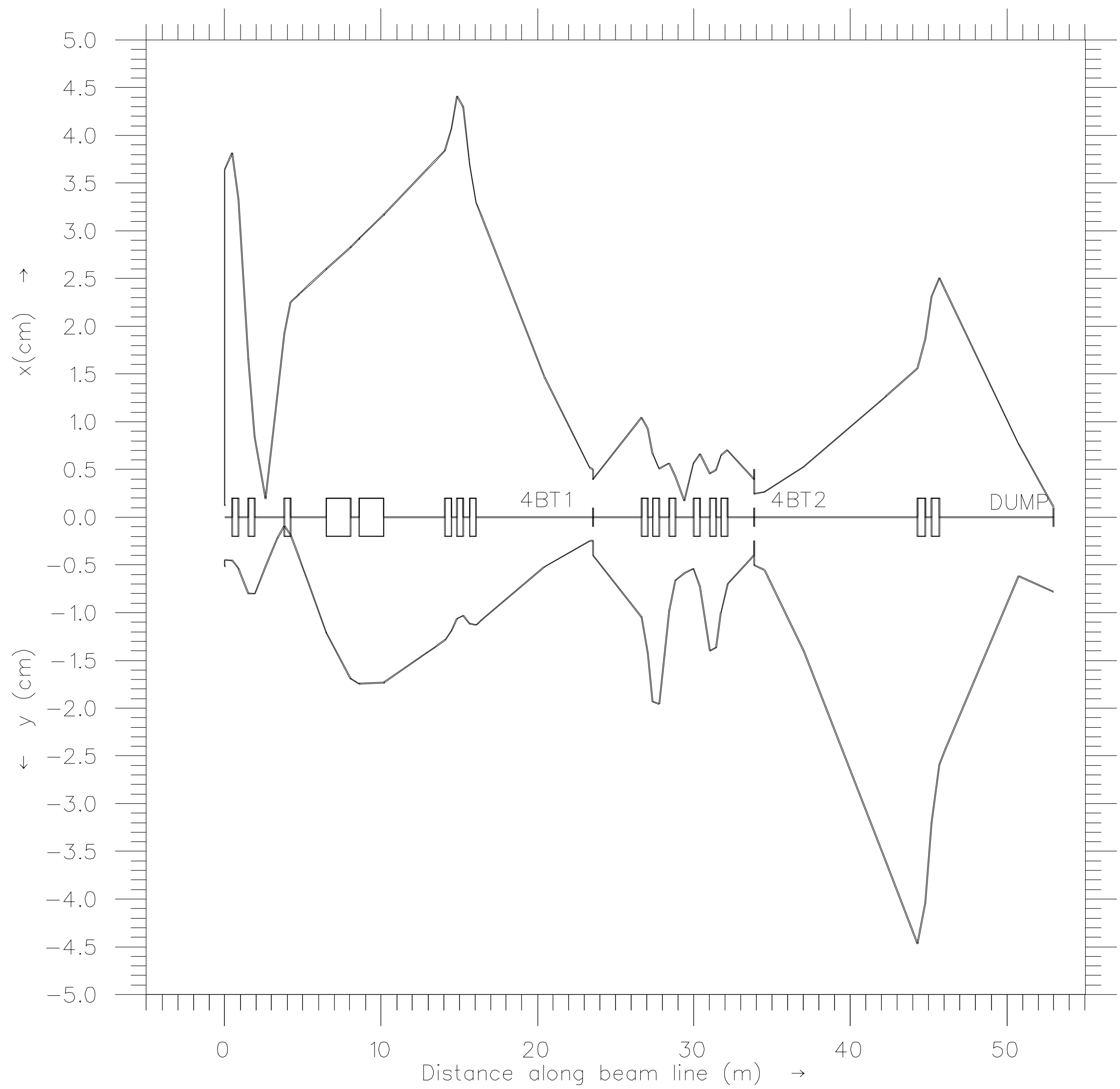



Fig. 10. 200 MeV beam profiles on beam line 4B for $R_{36} = -$ cm/% and $R_{31} \leq 0.5$

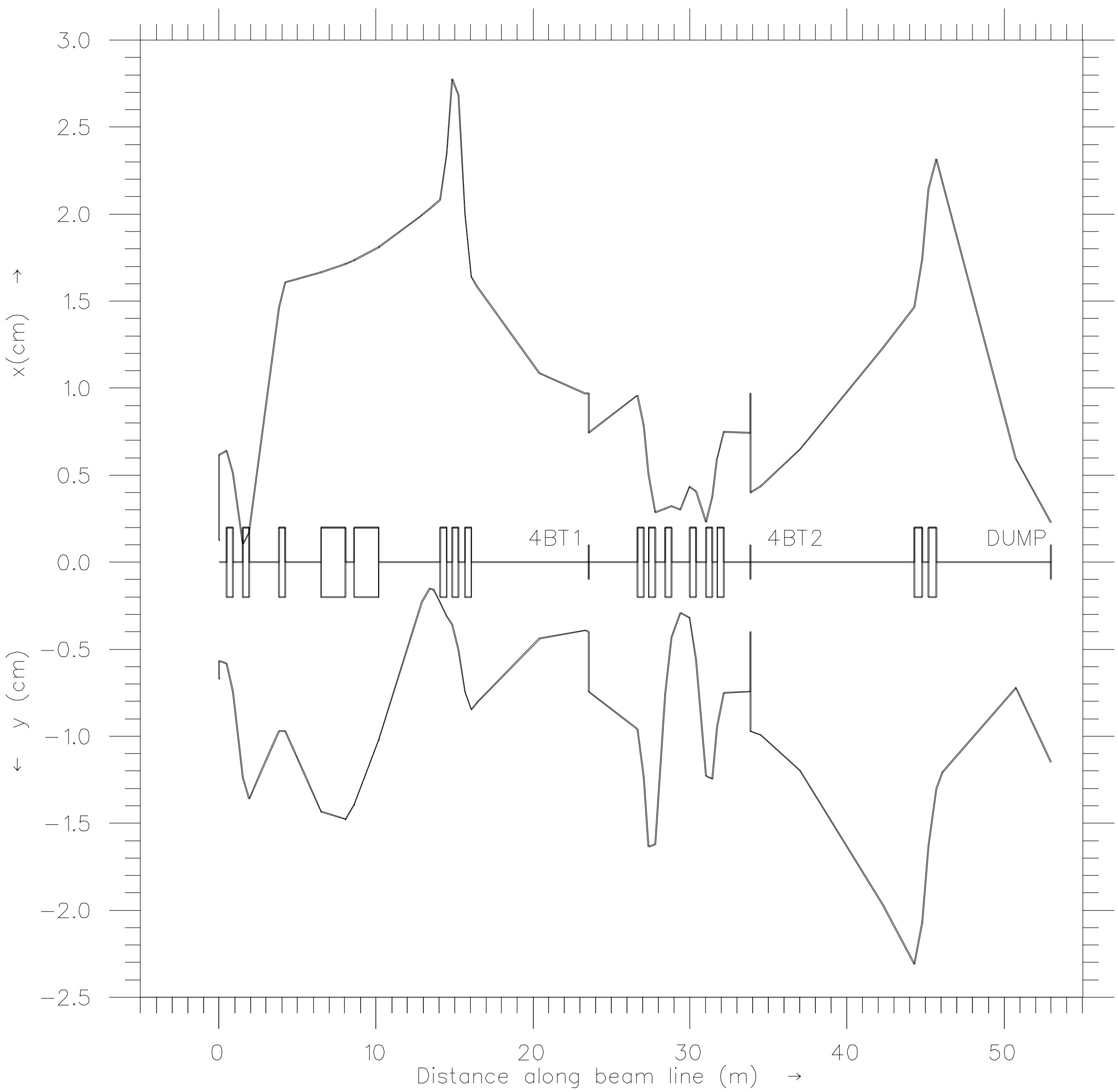



Fig. 11. 500 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

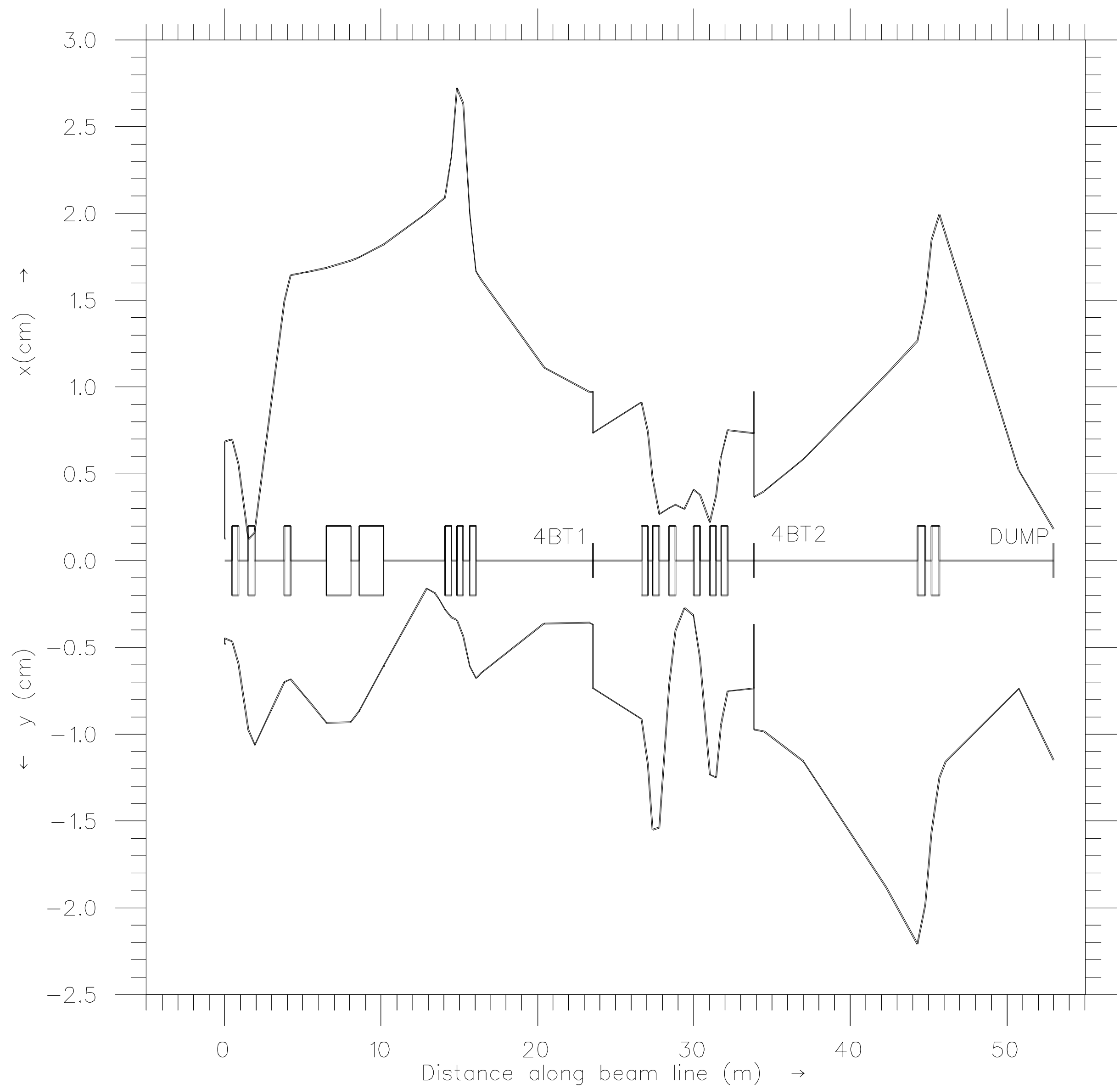



Fig. 12. 400 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

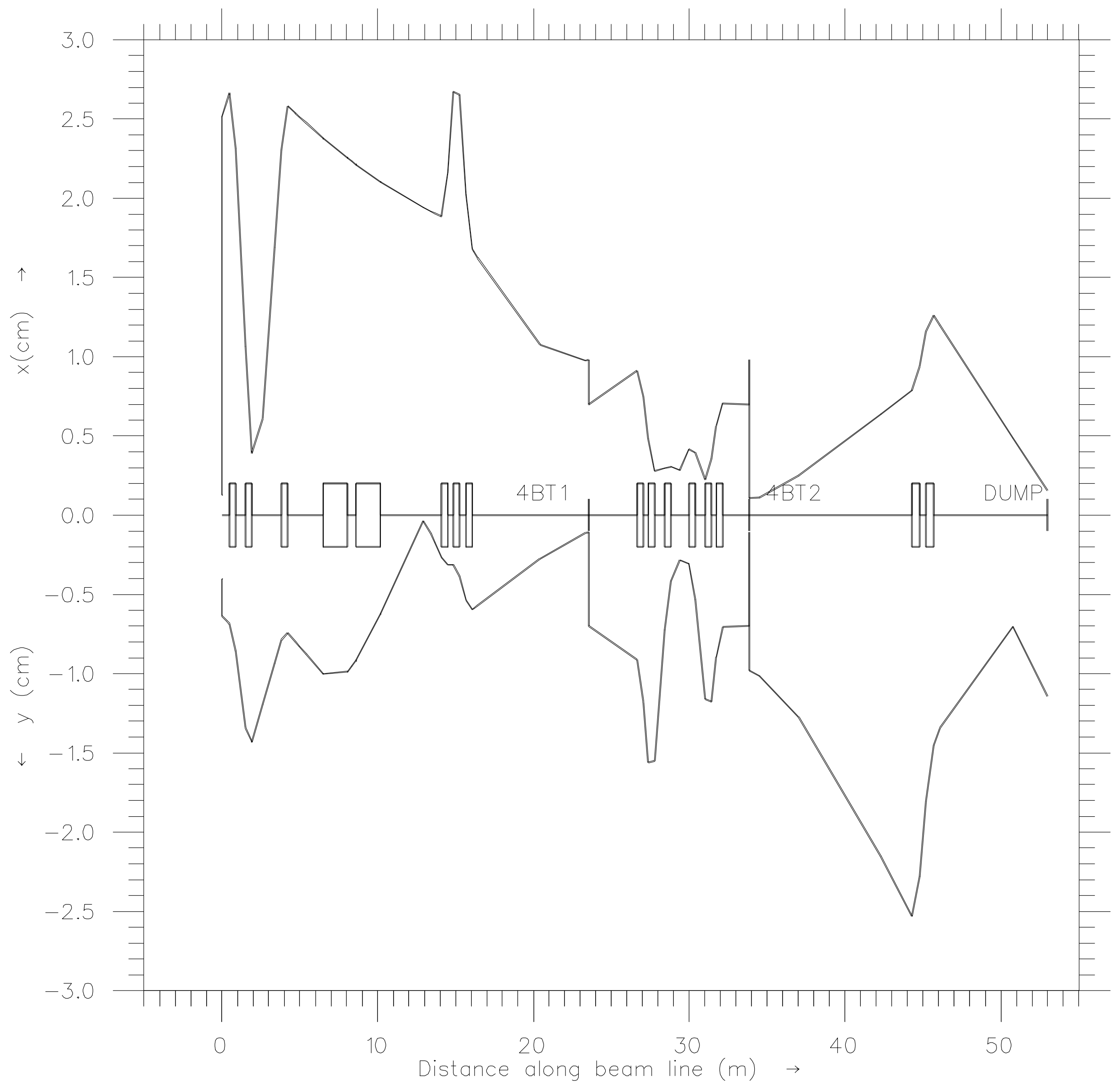



Fig. 13. 300 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.

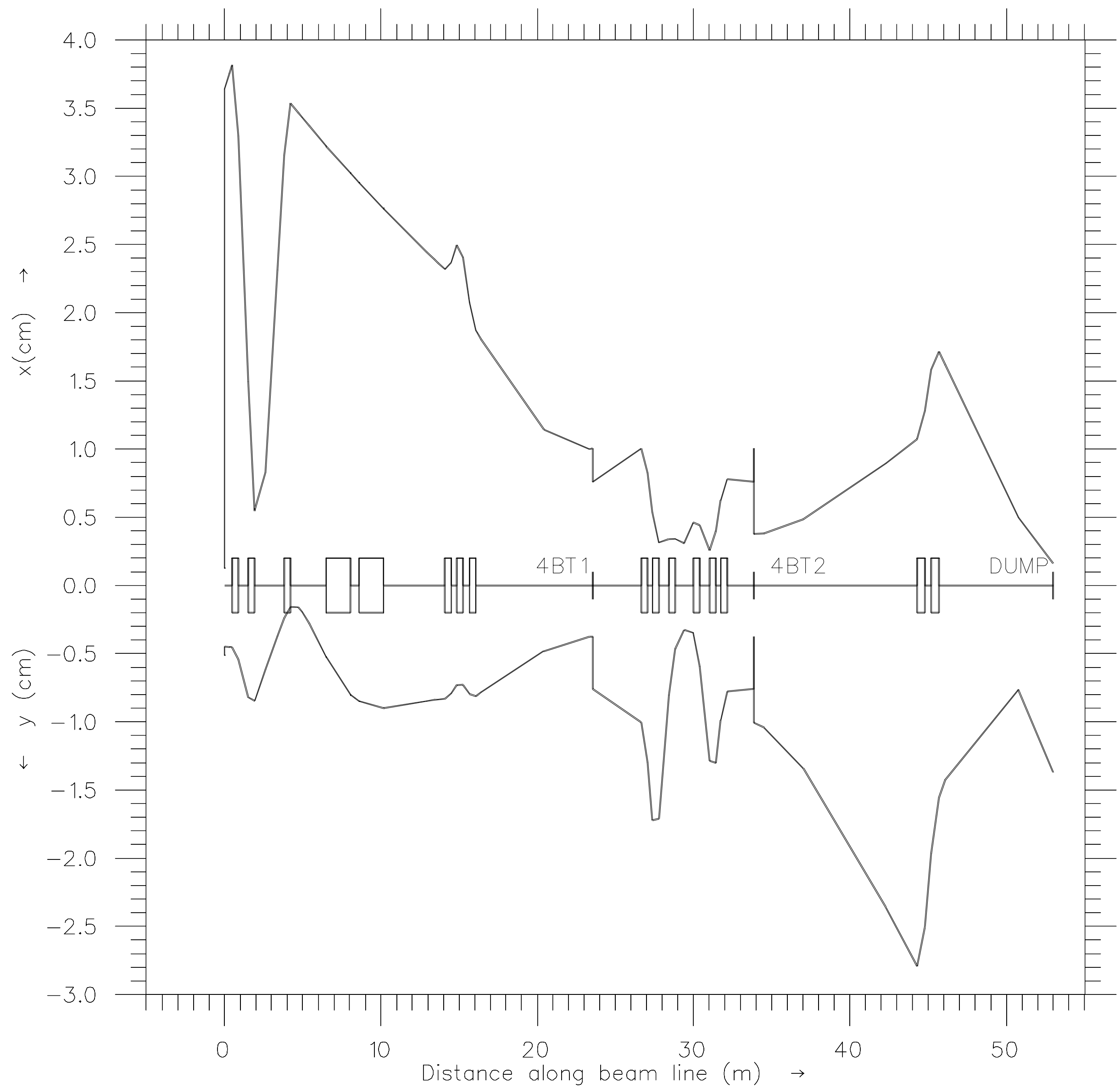



Fig. 14. 200 MeV beam profiles on beam line 4B for $R_{36} = -10$ cm/% and $R_{31} = 1$.