1. Introduction

An earlier report [1] presented the theoretical basis for the production of complete longitudinal polarization at TRIUMF by a solenoid-dipole solenoid-dipole (SDSD) system. Later, this treatment was extended to include (horizontal) transverse polarization [2]. A mechanism for the implementation of these schemes at TRIUMF was given in ref [3].

Each of these references assumed that the polarization $\mathbf{P_i}$ of the extracted beam was vertical. Thus, in a Cartesian coordinate system in which the z-axis is vertical and the y-axis points along the beam direction, the initial polarization was represented by the vector

$$\mathbf{P_i} = (P_{xi}, P_{yi}, P_{zi}) = (0, 0, 1)$$

Further, only those particles in the extracted beam whose momenta are equal to that to which the beamline is tuned will have their polarizations precessed completely longitudinally or horizontally.

Recently, the proponents of a (proposed) TRIUMF experiment approached the author as to the effect of the SDSD system on particles whose polarizations were not completely vertical and whose momemta differed from that to which the beamline was tuned. This report is the result of that request.

Implicit in this study is the assumption that the polarization of the extracted beam is mainly vertical. This being the case, the criteria of ref [1] and ref [2] for the definition of pure longitudinal and horizontal transverse polarizations are valid. Thus it is assumed that *any* initial vertical polarization is to be rotated into either the longitudinal or the transverse direction.

Given that the SDSD system is tuned to produce the appropriate polarization for particles of a specific momentum and an initial completely vertical polarization, this report presents the results of an investigation of the effects on the final polarization of initial small, non-vertical polarization components and of momemtum deviation from the beamline tune. It is noted that this study is concerned with a case in which the polarizations of all beam particles have the same, specific orientation with respect to the vertical. Should beam polarization be uniformly distributed on the surface of a cone whose axis is vertical, symmetry of the problem is such that complete longitudinal or horizontal transverse polarization will be retained.

2. Method of Calculation

Here the technique of ref [1] and ref [2] is abandoned and a general spin-rotation matrix is developed for tracing the precession of spin through the SDSD system. For consistency, however, the notation used in these references is retained. Thus the angle of spin precession in a solenoid is designated ϕ and that in a dipole Θ. These angles are related to the solenoid and dipole properties by

$$\phi = \mu \left[\frac{B_0 L}{(B\rho)_0}\right] \tag{1}$$

and

$$\Theta = \gamma G \theta \tag{2}$$

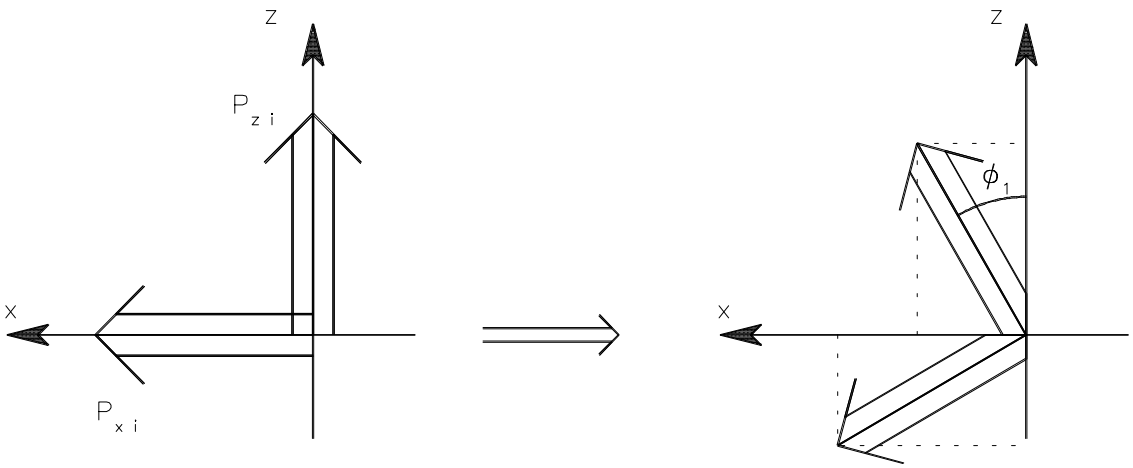
in which: μ = magnetic moment of the particle
B_0 = axial field of the solenoid

L = effective length of the solenoid
$(B\,\rho)_0$ = magnetic rigidity of the particle
G = ($g/2$ - 1) = 1.79285 for protons
θ = bend angle of the dipole

In the sections below, derivation of the spin transfer-matrix through a SDSD system is developed. The coordinate system used is one in which the positive z-axis points vertically up, the x-y plane is horizontal and the beam travels along the positive y-axis.

2.1 Precession of Spin in a Solenoid

If a particle with polarization $\mathbf{P_i} = (P_{xi}, P_{yi}, P_{zi})$ enters a solenoid, its spin component P_{yi} parallel to the axial field of the solenoid does not precess. However, its spin components along the x- and z-axes will precess through an angle ϕ_1 as shown in the diagram below.



From the diagram it is seen that if $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the x-, y- and z-axes, then

$$\begin{array}{lcccc} P_x\,\mathbf{i} & \longrightarrow & P_{xi}\cos\phi_1\,\mathbf{i} & & - \quad P_{xi}\sin\phi_1\,\mathbf{k} \\ P_y\,\mathbf{j} & \longrightarrow & & P_{yi}\,\mathbf{j} & \\ P_z\,\mathbf{k} & \longrightarrow & P_{zi}\sin\phi_1\,\mathbf{i} & & + \quad P_{zi}\cos\phi_1\,\mathbf{k} \end{array}$$

This may be written in matrix form as

$$\mathbf{P_S} = \mathbf{R_S}(\phi_1)\ \mathbf{P_i}$$

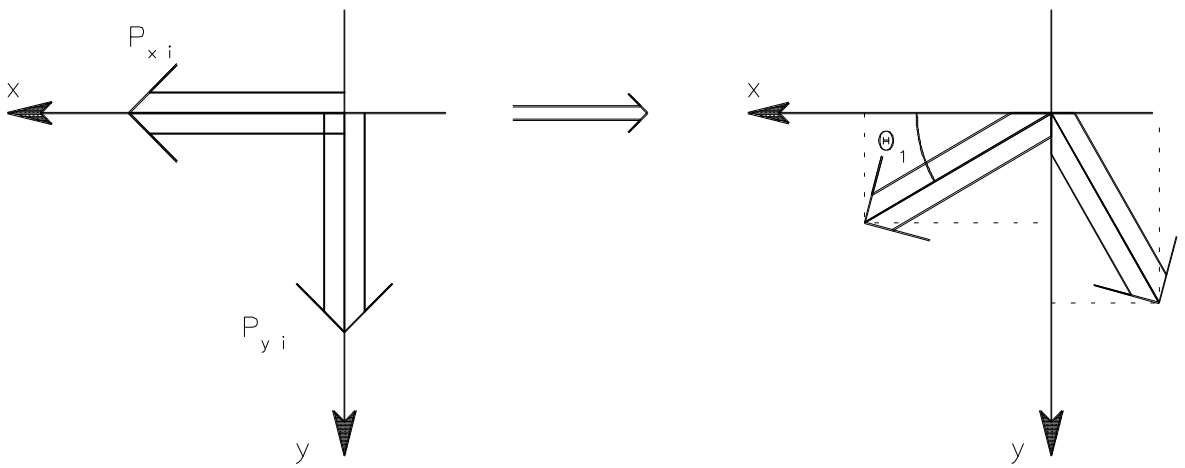
or

$$\begin{bmatrix} P_{xS} \\ P_{yS} \\ P_{zS} \end{bmatrix} = \begin{bmatrix} \cos\phi_1 & 0 & \sin\phi_1 \\ 0 & 1 & 0 \\ -\sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix} \begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix}$$

2.2 Precession of Spin in a Dipole

In a dipole that bends horizontally, the vertical (z-) component of initial polarization $\mathbf{P_i}$ that is parallel to the field of the dipole does not precess. Its spin components along the x- and y-axes precess through an angle Θ_1 as shown in the diagram on the next page.

From that diagram it follows that



$$
\begin{array}{lcccc}
P_x\,\mathbf{i} & \longrightarrow & P_{xi}\cos\Theta_1\,\mathbf{i} & + & P_{xi}\sin\Theta_1\,\mathbf{j} \\
P_y\,\mathbf{j} & \longrightarrow & -\,P_{yi}\sin\Theta_1\,\mathbf{i} & + & P_{yi}\cos\Theta_1\,\mathbf{j} \\
P_z\,\mathbf{k} & \longrightarrow & & & P_{zi}\,\mathbf{k}
\end{array}
$$

This may be written in matrix form as

$$\mathbf{P_D} = \mathbf{R_D}(\Theta_1)\ \mathbf{P_i}$$

or

$$\begin{bmatrix} P_{xD} \\ P_{yD} \\ P_{zD} \end{bmatrix} = \begin{bmatrix} \cos\Theta_1 & -\sin\Theta_1 & 0 \\ \sin\Theta_1 & \cos\Theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix}$$

2.2 Precession of Spin in a Solenoid-Dipole System

The matrix for spin rotation of a particle first passing through a solenoid and then a dipole is given by the matrix product

$$\mathbf{P_{DS}} = \mathbf{R_D}(\Theta_1)\ \mathbf{P_S} = \mathbf{R_D}(\Theta_1)\ \mathbf{R_S}(\phi_1)\ \mathbf{P_i} = \mathbf{R_{DS}}(\Theta_1,\phi_1)\ \mathbf{P_i}$$

or, in matrix form,

$$\begin{bmatrix} P_{xDS} \\ P_{yDS} \\ P_{zDS} \end{bmatrix} = \begin{bmatrix} \cos\Theta_1 & -\sin\Theta_1 & 0 \\ \sin\Theta_1 & \cos\Theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi_1 & 0 & \sin\phi_1 \\ 0 & 1 & 0 \\ -\sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix} \begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix}$$

When the matrix multiplication is done, the following result is obtained

$$\begin{bmatrix} P_{xDS} \\ P_{yDS} \\ P_{zDS} \end{bmatrix} = \begin{bmatrix} \cos\phi_1\cos\Theta_1 & -\sin\Theta_1 & \sin\phi_1\cos\Theta_1 \\ \cos\phi_1\sin\Theta_1 & \cos\Theta_1 & \sin\phi_1\sin\Theta_1 \\ -\sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix} \begin{bmatrix} P_{xi} \\ P_{yi} \\ P_{zi} \end{bmatrix}$$

A similar situation holds for the passage the combination of the second solenoid and then the second dipole:

$$\mathbf{P_{DS}} = \mathbf{R_{DS}}(\Theta_2,\phi_2)\ \mathbf{P_i}$$

Thus the final spin vector $\mathbf{P_f}$ is related to the initial spin vector $\mathbf{P_i}$ by the relation

$$\mathbf{P_f} = \mathbf{R_{DS}}(\Theta_2, \phi_2)\ \mathbf{R_{DS}}(\Theta_1, \phi_1)\ \mathbf{P_i} = \mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1)\ \mathbf{P_i}$$

in which the elements of $\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1)$ are

$$\begin{aligned}
R_{11} &= \cos\phi_1[\cos\phi_2\cos\Theta_1\cos\Theta_2 - \sin\Theta_1\sin\Theta_2] - \sin\phi_1\sin\phi_2\cos\Theta_2 \\
R_{21} &= \cos\phi_1[\cos\phi_2\cos\Theta_1\sin\Theta_2 + \sin\Theta_1\sin\Theta_2] - \sin\phi_1\sin\phi_2\sin\Theta_2 \\
R_{31} &= -[\sin\phi_1\cos\phi_2 + \cos\phi_1\sin\phi_2\cos\Theta_1] \\
\\
R_{12} &= -[\cos\Theta_1\sin\Theta_2 + \cos\phi_2\sin\Theta_1\cos\Theta_2] \\
R_{22} &= \cos\Theta_1\cos\Theta_2 - \cos\phi_2\sin\Theta_1\sin\Theta_2 \\
R_{32} &= \sin\phi_2\sin\Theta_1 \\
\\
R_{13} &= \sin\phi_1[\cos\phi_2\cos\Theta_1\cos\Theta_2 - \sin\Theta_1\sin\Theta_2] + \cos\phi_1\sin\phi_2\cos\Theta_2 \\
R_{23} &= \sin\phi_1[\cos\phi_2\cos\Theta_1\sin\Theta_2 + \sin\Theta_1\cos\Theta_2] + \cos\phi_1\sin\phi_2\sin\Theta_2 \\
R_{33} &= \cos\phi_1\cos\phi_2 - \sin\phi_1\sin\phi_2\cos\Theta_1
\end{aligned}$$

2.4 Conditions for Complete Longitudinal Polarization

Assuming that the polarization of the extracted beam is initially completely vertical, the conditions for its rotation into the beam direction—that is, along the y-axis—are

$$R_{21} = R_{22} = 0 \qquad \text{and} \qquad R_{23} = 1$$

Using the matrix elements listed above, this leads to the results given in ref[1)]:

$$\sin\phi_1 = \frac{\cos\Theta_2}{\sin\Theta_1} \qquad \text{and} \qquad \cos\phi_2 = \frac{1}{\tan\Theta_1 \cdot \tan\Theta_2} \tag{3}$$

Under these conditions, the matrix $\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1)$ takes the form

$$\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1) = \begin{bmatrix} -\sin\phi_2\sin\Theta_1 & -\dfrac{\cos\Theta_1}{\sin\Theta_2} & 0 \\ 0 & 0 & 1 \\ -\dfrac{\cos\Theta_1}{\sin\Theta_2} & \sin\phi_2\sin\Theta_1 & 0 \end{bmatrix}$$

with

$$\sin\phi_2 = \frac{\sqrt{\sin^2\Theta_1 - \cos^2\Theta_2}}{\sin\Theta_1\sin\Theta_2}$$

2.5 Conditions for (Horizontal) Transverse Polarization

Similarly, the conditions for the rotation of an initial vertical polarization into a horizontal transverse polarization—that is, along the positive x-axis—are

$$R_{11} = R_{12} = 0 \qquad \text{and} \qquad R_{13} = 1$$

With the values given above for these matrix elements, the relations given in ref[2)] are obtained:

$$\sin\phi_1 = -\frac{\sin\Theta_2}{\sin\Theta_1} \qquad \text{and} \qquad \cos\phi_2 = -\frac{\tan\Theta_2}{\tan\Theta_1} \tag{4}$$

In this case, the matrix $\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1)$ takes the form

$$\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1) = \begin{bmatrix} 0 & 0 & 1 \\ \sin\phi_2 \sin\Theta_1 & \frac{\cos\Theta_1}{\cos\Theta_2} & 0 \\ -\frac{\cos\Theta_1}{\cos\Theta_2} & \sin\phi_2 \sin\Theta_1 & 0 \end{bmatrix}$$

where, in this instance,

$$\sin\phi_2 = \frac{\sqrt{\sin^2\Theta_1 - \sin^2\Theta_2}}{\sin\Theta_1 \cos\Theta_2}$$

It is noted in passing that in the special case of $\Theta_1 = \Theta_2 = \Theta$, the matrix $\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1)$ reduces to

$$\mathbf{R_{DSDS}}(\Theta_2, \phi_2, \Theta_1, \phi_1) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

3. Results of Calculations

The spin-rotation matrix developed above was used to calculate the effect of the SDSD system on particles whose polarizations were (initially non-vertical and/or whose momenta were not that to which the external beamline was tuned. The following procedure was used.

For a given central momentum $\boldsymbol{p}_0$, the fields of the solenoids were calculated from the (appropriate) relations given in equations (3) and (4) above. Assuming that the effective lengths of the solenoids and of the dipoles were not a function of momentum—at least, not for small variations around the central momentum—the spin-precession angles for the different momenta were calculated from the relationships given by equations (1) and (2). Thus, because the magnetic rigidity of a particle is proportional to its momentum and the bend angle of a dipole is inversely proportional thereto, a particle of momemtum $\boldsymbol{p_1}$ will have its spin precessed through angles ϕ_1 and Θ_1 given by

$$\frac{\phi_1}{\phi_0} = \frac{p_1}{p_0} \qquad \text{and} \qquad \frac{\Theta_1}{\Theta_0} = \frac{\gamma_1}{\gamma_0} \cdot \frac{p_1}{p_0}$$

where the subscript '0' refers to those quantities of the "on-momentum" particle–that is, the "tune" of the SDSD system. These values of ϕ_1 and Θ_1 were substituted into the spin-precession matrix and used in further calculations. Notice that the inclusion of the momentum factor in the calculation of Θ_1 taken into account the (small) change in the bend angle of the dipole.

At each energy the spin-precession matrix was used to give the polarization components at the exit of the SDSD system. The orientation of the initial polarization vector was varied from completely vertical to lying at a 45° angle with respect to the z-axis. In the latter case, the component of polarization in the x-y plane was traced as it was rotated from along the x-axis to along the y-axis.

These procedures clearly generate a large amount of computer output and, therefore, results are difficult to present. In the following sections a few samples of output are given. Tables are also given from which the interested reader may calculate the non-longitudinal and/or non-transverse components of polarization

after passage through the SDSD system.

In the calculations a momentum spread of $\pm 0.1\%$ about the central momentum was used. The magnitude of the initial polarization is taken in the following to be 1,000,000. "Cone angle" is defined as the angle between the positive x-axis and the projection of $\boldsymbol{P}$ on the x-y plane.

3.1 Longitudinal Polarization

Consider a 200 MeV proton traversing a SDSD system that is adjusted to produce pure longitudinal polarization. Table 1 lists data calculated for a direction of initial polarization ranging from purely vertical to a cone angle of 15°. For each orientation (excluding purely vertical), data is listed for polar angles from 0° to 90°. It is noticed that, as required, the initial z-component of polarization is completely transformed into a y-component by the SDSD system. Its effect on the initially non-vertical spin components is to mix them on traversal of the SDSD configuration.

Similar data for and energy of 199.653MeV is listed in table 2. [Remember that the data of this table is calculated for dipole and solenoid fields fixed at their values for 200 MeV.] For an initially completely vertically polarized particle of this energy, the SDSD system induces non-longitudinal components in the process of producing longitudinal polarization. The magnitude of these components is of the same magnitude as is the momentum spread of the primary beam—in this case, $\approx \pm 0.1\%$. The effect on the longitudinal component of polarization is minimal—at least for the momentum spread considered here. It is seen that changes in the precession angles Θ_1, Θ_2, ϕ_1 and ϕ_2 are roughly of the same magnitude as are the momentum changes ($\pm 0.1\%$).

The calculated spin-precession angles for the various momenta around 200 MeV are listed in table 3-a. Table 3-b lists elements of the polarization-rotation matrix for these energies. From this table one may ascertain the extent to which, for off-momentum particles, the SDSD system admixes initial x- and y-components into the final y-component and the initial z-component into the final x- and y-components.

Similar data for energies of 300 MeV, 400 MeV and 500 MeV are presented in tables 4-6.

3.2 Transverse Polarization

Listed in table 7 are parameters for the transverse polarization and an energy of 500 MeV. Again, final polarization components are calculated for initial cone angles of 0°, 5°, 10° and 15°. Similar calculations for and energy of 500.826 MeV are given in table 8. Again it is noticed that even for initial complete vertical polarization off-momentum particles will have non-longitudinal components of polarization after passing through the SDSD system. The magnitude of those components is of the order of the momentum spread of the primary beam.

Table 9-a lists, as a function of energy, the angles of polarization rotation in the solenoids and dipoles for energies around 200 MeV. Elements of the polarization-rotation matrix are given for these energies in table 9-b.

Similar data for energies for 300 MeV, 400 MeV and 500 MeV are presented in tables 10-12.

4. Discussion

This report has presented a study of the effects of a SDSD system on a particle whose initial polarization is not vertical and whose momentum differs from the tune of the SDSD configuration. Implicit in this study was the assumption that one desired that only the initial vertical component of polarization be precessed into the longitudinal or the transverse direction. It has also been assumed that the initial polarization of all particles of the beam had a specific orientation with respect to the vertical.

Similar data for energies for 300 MeV, 400 MeV and 500 MeV are presented in tables 4-6.

For on-momentum particles the results show that, as one would expect, non-vertical initial components of polarization are mixed together in the final polarization; they do not, however, couple to the desired (longitudinal or transverse) component. On the other hand, the final polarization of the off-momentum particles will have components along each of the coordinate axes—even if the initial polarization was completely vertical. The magnitude of these "unwanted" components is roughly that of the momentum spread of the extracted beam—at least for the small ($\pm 0.1\%$) momentum spread considered here.

Tables of the polarization-rotation matrix have been presented in order that the interested party may determine for himself whether these effects could be detrimental to his experiment.

References

1. MK Craddock, TRI-DN-84-8.
2. GM Stinson, TRI-DNA-84-3.
3. GM Stinson, TRI-DNA-85-5.

Table 1

Calculated parameters for a 200 MeV particle on beamline 4-B
— Longitudinal Polarization† —

Cone Angle	Polar Angle	Initial Polarization P_x	P_y	P_z	Final Polarization P_x	P_y	P_z
0	0	0	0	1000000	0	1000000	0
5	0	87155	0	996194	−86974	996194	−5616
5	10	85831	15134	996194	−86628	996194	9571
5	20	81899	29809	996194	−83650	996194	24468
5	30	75479	43577	996194	−78130	996194	38622
5	40	66765	56022	996194	−70236	996194	51603
5	50	56022	66765	996194	−60208	996194	63015
5	60	43577	75479	996194	−48351	996194	72513
5	70	29809	81899	996194	−35025	996194	79808
5	80	15134	85931	996194	−20634	996194	84677
5	90	0	87155	996194	−5616	996194	86974
10	0	173648	0	984807	−173287	984807	−11191
10	10	171010	30153	984807	−172597	984807	19069
10	20	163175	59391	984807	−166664	984807	48751
10	30	150383	86824	984807	−155666	984807	76951
10	40	133022	111618	984807	−139939	984807	102813
10	50	111618	133022	984807	−119959	984807	125552
10	60	86824	150383	984807	−96335	984807	144475
10	70	59391	163175	984807	−69783	984807	159009
10	80	30153	171010	984807	−41112	984807	168711
10	90	0	173648	984807	−11191	984807	173287
15	0	258819	0	965925	−258280	965925	−16680
15	10	254886	44943	965925	−257253	965925	28423
15	20	243210	88521	965925	−248409	965925	72662
15	30	224143	129409	965925	−232018	965925	114694
15	40	198266	166365	965925	−208576	965925	153242
15	50	166365	198266	965925	−178797	965925	187132
15	60	129409	224143	965925	−143586	965925	215337
15	70	88521	243210	965925	−104011	965925	236999
15	80	44943	254886	965925	−61276	965925	251460
15	90	0	258819	965925	−16680	965925	258280

† At 200 MeV, $\Theta_1 = 86.99720°$, $\Theta_2 = 54.37323°$, $\phi_1 = 35.68323°$ and $\phi_2 = 87.84560°$. The first and second solenoid fields are 8.11718 kG and 19.98301 kG respectively.

Table 2

Calculated parameters for a 199.635 MeV particle on beamline 4-B
— Longitudinal Polarization† —

Cone Angle	Polar Angle	Initial Polarization P_x	P_y	P_z	Final Polarization P_x	P_y	P_z
0	0	0	0	1000000	−1306	999998	−685
5	0	87155	0	996194	−88285	996076	−6148
5	10	85831	15134	996194	−87913	996087	9038
5	20	81899	29809	996194	−86909	996101	23931
5	30	75479	43577	996194	−79364	996118	38075
5	40	66765	56022	996194	−71448	996137	51042
5	50	56022	66765	996194	−61401	996158	62347
5	60	43577	75479	996194	−49527	996180	71914
5	70	29809	81899	996194	−36188	996202	79186
5	80	15134	85931	996194	−21789	996224	84030
5	90	0	87155	996194	−6767	996246	86301
10	0	173648	0	984807	−174592	984572	−11565
10	10	171010	30153	984807	−173850	984594	18694
10	20	163175	59391	984807	−167865	984622	48365
10	30	150383	86824	984807	−156819	984656	76546
10	40	133022	111618	984807	−141047	984694	102381
10	50	111618	133022	984807	−121028	984736	125084
10	60	86824	150383	984807	−97371	984780	143967
10	70	59391	163175	984807	−70794	984824	158454
10	80	30153	171010	984807	−42106	984869	168107
10	90	0	173648	984807	−12177	984911	172631
15	0	258819	0	965925	−259571	965576	−16894
15	10	254886	44943	965925	−258465	965608	28207
15	20	243210	88521	965925	−249545	965650	72431
15	30	224143	129409	965925	−233080	965700	114435
15	40	198266	166365	965925	−209572	965757	152941
15	50	166365	198266	965925	−179734	965820	186780
15	60	129409	224143	965925	−144474	965885	214924
15	70	88521	243210	965925	−104862	965951	236517
15	80	44943	254886	965925	−62102	966017	250904
15	90	0	258819	965925	−17494	966080	257647

† At 199.635 MeV, $\Theta_1 = 87.05638°$, $\Theta_2 = 54.41024°$, $\phi_1 = 35.71894°$ and $\phi_2 = 87.93354°$. The first and second solenoid fields are 8.11718 kG and 19.98301 kG respectively.

Table 3

Longitudinal Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 200 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
199.635	1.21277	2.17423	87.0564	54.41024	35.71894	87.93354
199.708	1.21285	2.17437	87.0445	54.40283	35.71180	87.91594
199.781	1.21293	2.17451	87.0327	54.39543	35.70465	87.89834
199.854	1.21301	2.17465	87.0209	54.38803	35.69750	87.88076
199.927	1.21308	2.17479	87.0090	54.38064	35.69036	87.86318
200.000	1.21316	2.17493	86.9972	54.37325	35.68323	87.84560
200.073	1.21324	2.17507	86/9854	54.36586	35.67609	87.82804
200.146	1.21332	2.17521	86.9736	54.35848	35.66896	87.81048
200.219	1.21339	2.17535	86.9618	54.35110	35.66183	87.79293
200.292	1.21347	2.17549	86.9500	54.34372	35.65470	87.77538
200.364	1.21355	2.17563	86.9382	54.33635	35.64758	87.75785

— b) Polarization matrix elements for energies around 200 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
199.635	−998.030	−62.718	−1.306	−1.347	0.602	999.999	−62.718	998.031	−0.685
199.708	−998.009	−63.065	−1.045	−1.077	0.481	999.999	−63.064	998.009	−0.548
199.781	−997.987	−63.410	−0.783	−0.808	0.361	1000.000	−63.410	997.987	−0.411
199.854	−997.965	−63.756	−0.522	−0.539	0.240	1000.000	−63.756	997.965	−0.274
199.927	−997.943	−64.102	−0.261	−0.269	0.120	1000.000	−64.102	997.943	−0.137
200.000	−997.921	−64.448	0.0	0.0	0.0	1000.000	−64.448	997.921	0.0
200.073	−997.899	−64.793	0.261	0.269	−0.120	1000.000	−64.793	997.899	0.137
200.146	−997.876	−65.138	0.522	0.539	−0.240	1000.000	−65.138	997.876	0.274
200.219	−997.853	−65.483	0.783	0.808	−0.360	1000.000	−65.483	997.854	0.412
200.292	−997.830	−65.828	1.043	1.077	−0.479	999.999	−65.828	997.831	0.549
200.365	−997.807	−66.173	1.304	1.346	−0.599	999.999	−66.173	997.808	0.686

† At 200 MeV, the first and second solenoid fields are 8.11718 kG and 19.98301 kG respectively.

Table 4

Longitudinal Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 300 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
299.473	1.31918	2.36500	94.6946	59.18413	30.99347	92.87163
299.578	1.31929	2.36520	94.6837	59.17732	30.98727	92.85304
299.684	1.31940	2.36540	94.6728	59.17052	30.98017	92.83446
299.789	1.31952	2.36560	94.6619	59.16371	30.97487	92.81588
299.895	1.31963	2.36580	94.6511	59.15692	30.96867	92.79731
300.000	1.31974	2.36600	94.6402	59.15012	30.96248	92.77876
300.105	1.31985	2.36621	94.6293	59.14333	30.95629	92.76020
300.211	1.31997	2.36641	94.6185	59.13655	30.95010	92.74166
300.316	1.32008	2.36661	94.6076	59.12976	30.94391	92.72312
300.422	1.32019	2.36681	94.5968	59.12298	30.93773	92.70459
300.527	1.32030	2.36701	94.5859	59.11620	30.93155	92.68607

— b) Polarization matrix elements for energies around 300 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
299.473	−995.393	95.868	−1.086	−0.999	0.954	999.999	95.869	995.394	−0.853
299.578	−995.425	95.540	−0.868	−0.799	0.763	999.999	95.541	995.425	−0.683
299.684	−995.457	95.213	−0.651	−0.600	0.572	1000.000	95.213	995.457	−0.512
299.789	−995.488	94.885	−0.434	−0.400	0.381	1000.000	94.885	995.488	−0.341
299.895	−995.519	94.558	−0.217	−0.200	0.190	1000.000	94.558	995.519	−0.171
300.000	−995.550	94.231	0.0	0.0	0.0	1000.000	94.231	995.550	0.0
300.105	−995.581	93.903	0.217	0.200	−0.190	1000.000	93.903	995.581	0.171
300.211	−995.612	93.576	0.434	0.400	−0.381	1000.000	93.576	995.612	0.341
300.316	−995.643	93.249	0.650	0.600	−0.571	1000.000	93.250	995.643	0.512
300.422	−995.673	92.922	0.867	0.800	−0.760	999.999	92.923	995.673	0.683
300.527	−995.703	92.595	1.084	1.000	−0.950	999.999	92.596	995.703	0.854

† At 300 MeV, the first and second solenoid fields are 8.83152 kG and 26.46356 kG respectively.

Table 5

Longitudinal Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 400 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
399.320	1.42560	2.55578	102.3335	63.95846	26.75845	96.21192
399.456	1.42574	2.55604	102.3235	63.95216	26.75309	96.19266
399.592	1.42589	2.55630	102.3134	63.94587	26.74774	96.17341
399.728	1.42603	2.55656	102.3033	63.93957	26.74239	96.15417
399.864	1.42618	2.55682	102.2933	63.93329	26.73704	96.13494
400.000	1.42632	2.55708	102.2832	63.92700	26.73169	96.11571
400.136	1.42647	2.55734	102.2731	63.92072	26.72635	96.09649
400.272	1.42661	2.55760	102.2631	63.91444	26.72100	96.07728
400.408	1.42676	2.55786	102.2531	63.90816	26.71566	96.05807
400.544	1.42690	2.55812	102.2430	63.90189	26.71032	96.03888
400.681	1.42705	2.55838	102.2330	63.89562	26.70499	96.01969

— b) Polarization matrix elements for energies around 400 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
399.320	−971.185	238.325	−0.909	−0.642	1.201	999.999	238.326	971.185	−1.013
399.456	−971.258	238.029	−0.727	−0.513	0.960	999.999	238.030	971.258	−0.810
399.592	−971.330	237.734	−0.545	−0.385	0.720	1000.000	237.734	971.330	−0.608
399.728	−971.403	237.438	−0.363	−0.257	0.480	1000.000	237.438	971.403	−0.405
399.864	−971.475	237.102	−0.182	−0.128	0.240	1000.000	237.142	971.475	−0.203
400.000	−971.547	236.847	0.0	0.0	0.0	1000.000	236.847	971.547	0.0
400.136	−971.619	236.551	0.182	0.129	−0.240	1000.000	236.551	971.619	0.203
400.272	−971.691	236.256	0.363	0.257	−0.479	1000.000	236.256	971.691	0.405
400.408	−971.763	235.960	0.545	0.386	−0.719	1000.000	235.961	971.762	0.608
400.544	−971.834	235.665	1.726	0.515	−0.958	999.999	235.665	971.834	0.810
400.681	−971.906	235.370	1.907	0.643	−1.198	999.999	235.370	971.905	1.013

† At 400 MeV, the first and second solenoid fields are 9.00430 kG and 32.37560 kG respectively.

Table 6

Longitudinal Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 500 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
499.174	1.53202	2.74658	109.9730	68.73315	22.74821	98.22184
499.339	1.53220	2.74689	109.9637	68.72729	22.74366	98.20218
499.504	1.53237	2.74721	109.9543	68.72143	22.73911	98.18253
499.670	1.53255	2.74752	109.9449	68.71558	22.73456	98.16289
499.835	1.53273	2.74784	109.9356	68.70972	22.73001	98.14325
500.000	1.53290	2.74815	109.9262	68.70387	22.72547	98.12362
500.165	1.53308	2.74847	109.9168	68.69803	22.72092	98.10400
500.330	1.53325	2.74879	109.9075	68.69218	22.71638	98.08439
500.496	1.53343	2.74910	109.8982	68.68634	22.71184	98.06478
500.661	1.53361	2.74942	109.8888	68.68051	22.70730	98.04518
500.826	1.53378	2.74973	109.8795	68.67467	22.70276	98.02560

— b) Polarization matrix elements for energies around 500 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
499.174	−930.194	367.067	−0.765	−0.287	1.358	999.999	367.068	930.193	−1.158
499.339	−930.295	366.811	−0.612	−0.229	1.086	999.999	366.812	930.295	−0.926
499.504	−930.396	366.555	−0.459	−0.172	0.814	1000.000	366.555	930.396	−0.694
499.670	−930.497	366.299	−0.306	−0.115	0.543	1000.000	366.299	930.497	−0.463
499.835	−930.598	366.043	−0.153	−0.058	0.271	1000.000	366.043	930.598	−0.231
500.000	−930.699	365.787	0.0	0.0	0.0	1000.000	365.787	930.699	0.0
500.165	−930.799	365.531	0.153	0.058	−0.271	1000.000	365.531	930.799	0.231
500.330	−930.900	365.275	0.305	0.115	−0.542	1000.000	365.275	930.899	0.463
500.496	−931.000	365.019	0.458	0.173	−0.813	1000.000	365.019	931.000	0.694
500.661	−931.100	364.763	0.611	0.231	−1.084	999.999	364.764	931.100	0.925
500.826	−931.200	364.508	0.763	0.289	−1.355	999.999	364.508	931.199	1.156

† At 500 MeV, the first and second solenoid fields are 8.74433 kG and 37.76510 kG respectively.

Table 7

Calculated parameters for a 500 MeV particle on beamline 4-B
— Transverse Polarization† —

Cone Angle	Polar Angle	Initial Polarization P_x	P_y	P_z	Final Polarization P_x	P_y	P_z
0	0	0	0	1000000	1000000	0	0
5	0	87155	0	996194	996194	30120	81785
5	10	85831	15134	996194	996194	15461	85773
5	20	81899	29809	996194	996194	332	87155
5	30	75479	43577	996194	996194	−14807	85888
5	40	66765	56022	996194	996194	−29496	82012
5	50	56022	66765	996194	996194	−43289	75644
5	60	43577	75479	996194	996194	−55767	66978
5	70	29809	81899	996194	996194	−66551	56276
5	80	15134	85931	996194	996194	−75312	43865
5	90	0	87155	996194	996194	−81785	30120
10	0	173648	0	984807	984807	60012	162948
10	10	171010	30153	984807	984807	30805	170893
10	20	163175	59391	984807	984807	661	173646
10	30	150383	86824	984807	984807	−29501	171123
10	40	133022	111618	984807	984807	−58768	163401
10	50	111618	133022	984807	984807	−86250	150713
10	60	86824	150383	984807	984807	−111111	133446
10	70	59391	163175	984807	984807	−132595	112124
10	80	30153	171010	984807	984807	−150051	87396
10	90	0	173648	984807	984807	−162948	60012
15	0	258819	0	965925	965925	89447	242871
15	10	254886	44943	965925	965925	45913	254713
15	20	243210	88521	965925	965925	986	258817
15	30	224143	129409	965925	965925	−43971	255056
15	40	198266	166365	965925	965925	−87593	243545
15	50	166365	198266	965925	965925	−128554	224635
15	60	129409	224143	965925	965925	−165608	198899
15	70	88521	243210	965925	965925	−197631	167120
15	80	44943	254886	965925	965925	−223649	130262
15	90	0	258819	965925	965925	−242871	89447

† At 500 MeV, $\Theta_1 = 109.92920°$, $\Theta_2 = 68.70387°$, $\phi_1 = -82.32751°$ and $\phi_2 = 21.56807°$. The first and second solenoid fields are −31.67806 kG and 8.29898 kG respectively.

Table 8

Calculated parameters for a 500.826 MeV particle on beamline 4-B
— Transverse Polarization† —

Cone Angle	Polar Angle	Initial Polarization P_x	P_y	P_z	Final Polarization P_x	P_y	P_z
0	0	0	0	1000000	999998	−772	1762
5	0	87155	0	996194	996071	29331	83548
5	10	85831	15134	996194	996053	14671	87532
5	20	81899	29809	996194	996039	−458	88911
5	30	75479	43577	996194	996030	−15597	87641
5	40	66765	56022	996194	996025	−30285	83761
5	50	56022	66765	996194	996026	−44077	77390
5	60	43577	75479	996194	996031	−56553	68720
5	70	29809	81899	996194	996042	−67334	58016
5	80	15134	85931	996194	996057	−76092	45603
5	90	0	87155	996194	996076	−82561	31857
10	0	173648	0	984807	984564	59212	164698
10	10	171010	30153	984807	984528	30003	172637
10	20	163175	59391	984807	984500	−140	175382
10	30	150383	86824	984807	984481	−30303	172852
10	40	133022	111618	984807	984472	−59568	165122
10	50	111618	133022	984807	984473	−87047	152428
10	60	86824	150383	984807	984484	−111903	135155
10	70	59391	163175	984807	984505	−133383	113828
10	80	30153	171010	984807	984536	−150833	89096
10	90	0	173648	984807	984574	−163723	61709
15	0	258819	0	965925	965564	88643	244595
15	10	254886	44943	965925	965510	45107	256427
15	20	243210	88521	965925	965468	178	260519
15	30	224143	129409	965925	965440	−44778	256748
15	40	198266	166365	965925	965427	−88398	245227
15	50	166365	198266	965925	965428	−129354	226306
15	60	129409	224143	965925	965445	−166402	200561
15	70	88521	243210	965925	965476	−198417	168774
15	80	44943	254886	965925	965521	−224426	131911
15	90	0	258819	965925	965578	−243638	91091

† At 500.826 MeV, $\Theta_1 = 109.87948°$, $\Theta_2 = 68.67467°$, $\phi_1 = -82.24527°$ and $\phi_2 = 21.54652°$. The first and second solenoid fields (at 500 MeV) are −31.67806 kG and 8.29898 kG respectively.

Table 9

Transverse Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 200 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
199.635	1.21277	2.17423	87.0564	54.41024	−54.53786	94.29202
199.708	1.21285	2.17437	87.0445	54.40283	−54.52694	94.27315
199.781	1.21293	2.17451	87.0327	54.39543	−54.51603	94.25428
199.854	1.21301	2.17465	87.0209	54.38803	−54.50512	94.23542
199.927	1.21308	2.17479	87.0090	54.38064	−54.49422	94.21657
200.000	1.21316	2.17493	86.9972	54.37325	−54.48332	94.19773
200.073	1.21324	2.17507	86/9854	54.36586	−54.47243	94.17889
200.146	1.21332	2.17521	86.9736	54.35848	−54.46154	94.16007
200.219	1.21339	2.17535	86.9618	54.35110	−54.45065	94.14125
200.292	1.21347	2.17549	86.9500	54.34372	−54.43977	94.12243
200.364	1.21355	2.17563	86.9382	54.33635	−54.42889	94.10365

— b) Polarization matrix elements for energies around 200 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
199.635	0.223	1.737	999.998	995.881	90.666	−0.380	−90.667	995.880	−1.710
199.708	0.178	1.389	999.999	995.895	90.519	−0.303	−90.519	995.894	−1.367
199.781	0.134	1.042	999.999	995.908	90.372	−0.227	−90.372	995.908	−1.025
199.854	0.089	0.694	1000.000	995.921	90.224	−0.151	−90.225	995.921	−0.684
199.927	0.045	0.347	1000.000	995.935	90.077	−0.076	−90.077	995.935	−0.342
200.000	0.0	0.0	1000.000	995.948	89.931	0.0	−89.931	995.948	0.0
200.073	−0.045	−0.347	1000.000	995.961	89.784	0.075	−89.784	995.961	0.342
200.146	−0.089	−0.694	1000.000	995.974	89.637	0.151	−89.637	995.974	0.683
200.219	−0.133	−1.041	999.999	995.988	89.491	0.226	−89.491	995.987	1.025
200.292	−0.178	−1.388	999.999	996.001	89.344	0.301	−89.345	996.000	1.366
200.365	−0.222	−1.734	999.998	996.014	89.198	0.376	−89.199	996.012	1.707

† At 200 MeV, the first and second solenoid fields are −12.39380 kG and 21.42798 kG respectively.

Table 10

Transverse Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 300 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
299.473	1.31918	2.36500	94.6946	59.18413	−59.52656	82.27246
299.578	1.31929	2.36520	94.6837	59.17732	−59.51464	82.25599
299.684	1.31940	2.36540	94.6728	59.17052	−59.50273	82.23953
299.789	1.31952	2.36560	94.6619	59.16371	−59.49083	82.22308
299.895	1.31963	2.36580	94.6511	59.15692	−59.47893	82.20663
300.000	1.31974	2.36600	94.6402	59.15012	−59.46703	82.19019
300.105	1.31985	2.36621	94.6293	59.14333	−59.45514	82.17375
300.211	1.31997	2.36641	94.6185	59.13655	−59.44325	82.15733
300.316	1.32008	2.36661	94.6076	59.12976	−59.43137	82.14090
300.422	1.32019	2.36681	94.5968	59.12298	−59.41950	82.12449
300.527	1.32030	2.36701	94.5859	59.11620	−59.40762	82.10808

— b) Polarization matrix elements for energies around 300 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
299.473	0.569	1.638	999.998	987.596	−157.019	−0.305	157.018	987.594	−1.707
299.578	0.455	1.310	999.999	987.572	−157.167	−0.244	157.167	987.571	−1.366
299.684	0.341	0.983	999.999	987.548	−157.316	−0.183	157.316	987.548	−1.024
299.789	0.228	0.655	1000.000	987.525	−157.464	−0.122	157.464	987.524	−0.683
299.895	0.114	0.327	1000.000	987.501	−157.613	−0.061	157.613	987.501	−0.341
300.000	0.0	0.0	1000.000	987.477	−157.761	0.0	157.761	987.477	0.0
300.105	−0.114	−0.327	1000.000	987.454	−157.909	0.061	157.909	987.454	0.341
300.211	−0.227	−0.654	1000.000	987.430	−158.057	0.121	158.057	987.430	0.682
300.316	−0.341	−0.982	999.999	987.406	−158.204	0.191	158.204	987.406	1.023
300.422	−0.455	−1.309	999.999	987.383	−158.352	0.242	158.352	987.382	1.364
300.527	−0.568	−1.635	999.999	987.359	−158.499	0.302	158.499	987.358	1.705

† At 300 MeV, the first and second solenoid fields are −16.96196 kG and 23.44335 kG respectively.

Table 11

Transverse Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 400 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
399.320	1.42560	2.55578	102.3335	63.95846	−66.88778	63.64229
399.456	1.42574	2.55604	102.3235	63.95216	−66.87439	63.62955
399.592	1.42589	2.55630	102.3134	63.94587	−66.86101	63.61682
399.728	1.42603	2.55656	102.3033	63.93957	−66.84763	63.60409
399.864	1.42618	2.55682	102.2933	63.93329	−66.83426	63.59137
400.000	1.42632	2.55708	102.2832	63.92700	−66.82089	63.57865
400.136	1.42647	2.55734	102.2731	63.92072	−66.80753	63.56594
400.272	1.42661	2.55760	102.2631	63.91444	−66.79417	63.55323
400.408	1.42676	2.55786	102.2531	63.90816	−66.78082	63.54052
400.544	1.42690	2.55812	102.2430	63.90189	−66.76748	63.52783
400.681	1.42705	2.55838	102.2330	63.89562	−66.75414	63.51513

— b) Polarization matrix elements for energies around 400 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
399.320	0.923	1.500	999.998	875.361	−483.470	−0.083	483.469	875.360	−1.760
399.456	0.738	1.200	999.999	875.298	−483.584	−0.066	483.584	875.297	−1.407
399.592	0.554	0.900	999.999	875.235	−483.699	−0.049	483.698	875.234	−1.055
399.728	0.369	0.600	1000.000	875.171	−483.813	−0.033	483.813	875.171	−0.703
399.864	0.185	0.300	1000.000	875.108	−483.927	−0.016	483.927	875.108	−0.352
400.000	0.0	0.0	1000.000	875.045	−484.041	0.0	484.041	875.045	0.0
400.136	−0.184	−0.300	1000.000	874.982	−484.155	0.016	484.155	874.982	0.352
400.272	−0.369	−0.599	1000.000	874.919	−484.269	0.032	484.269	874.919	0.703
400.408	−0.553	−0.899	999.999	874.856	−484.383	0.049	484.382	874.856	1.543
400.544	−0.737	−1.198	999.999	874.793	−484.496	0.065	484.496	874.793	1.405
400.681	−0.922	−1.497	999.998	874.730	−484.610	0.081	484.609	874.729	1.756

† At 400 MeV, the first and second solenoid fields are −22.50794 kG and 21.41582 kG respectively.

Table 12

Transverse Polarization on Beamline 4-B

— a) Parameters of the SDSD system for energies around 500 MeV † —

Energy (MeV)	γ	γG (°)	Θ_1 (°)	Θ_2 (°)	ϕ_1 (°)	ϕ_2 (°)
499.174	1.53202	2.74658	109.9730	68.73315	−82.40992	21.58966
499.339	1.53220	2.74689	109.9637	68.72729	−82.39343	21.58534
499.504	1.53237	2.74721	109.9543	68.72143	−82.37694	21.58102
499.670	1.53255	2.74752	109.9449	68.71558	−82.36046	21.57670
499.835	1.53273	2.74784	109.9356	68.70972	−82.35398	21.57238
500.000	1.53290	2.74815	109.9262	68.70387	−82.32751	21.56807
500.165	1.53308	2.74847	109.9168	68.69803	−82.31105	21.56375
500.330	1.53325	2.74879	109.9075	68.69218	−82.29460	21.55944
500.496	1.53343	2.74910	109.8982	68.68634	−82.27815	21.55513
500.661	1.53361	2.74942	109.8888	68.68051	−82.26170	21.55083
500.826	1.53378	2.74973	109.8795	68.67467	−82.24527	21.54652

— b) Polarization matrix elements for energies around 500 MeV † —

Energy MeV	R_{11}	R_{12}	R_{13}	R_{21}	R_{22}	R_{23}	R_{31}	R_{32}	R_{33}
	All matrix elements have been multiplied by 10^3								
499.174	1.390	1.337	999.998	345.827	−938.298	0.774	938.297	345.825	−1.767
499.339	1.112	1.069	999.999	345.781	−938.315	0.619	938.315	345.780	−1.413
499.504	0.834	0.802	999.999	345.735	−938.332	0.464	938.332	345.735	−1.060
499.670	0.556	0.534	1000.000	345.690	−938.349	0.309	938.349	345.689	−0.706
499.835	0.278	0.267	1000.000	345.644	−938.366	0.155	938.366	345.644	−0.353
500.000	0.0	0.0	1000.000	345.599	−938.382	0.0	938.382	345.599	0.0
500.165	−0.278	−0.267	1000.000	345.553	−938.399	−0.155	938.399	345.553	0.353
500.330	−0.555	−0.534	1000.000	345.508	−938.416	−0.309	938.416	345.508	0.706
500.496	−0.833	−0.801	999.999	345.463	−938.432	−0.464	938.432	345.462	1.058
500.661	−1.110	−1.067	999.999	345.418	−938.449	−0.618	938.448	345.417	1.411
500.826	−1.388	−1.334	999.998	345.373	−938.465	−0.772	938.464	345.372	1.763

† At 500 MeV, the first and second solenoid fields are −31.67806 kG and 8.29898 kG respectively.