1. Introduction

Protons are to be extracted from extraction port 2A to deliver beam to an external ISAC facility. The current beamline design calls for two 27.5° dipoles to be located in the cyclotron vault and a 30° switching magnet upstream of the targets. This report presents a design for an H-magnet that would be suitable for the vault; a design for the switching magnet will be given separately.

2. Design parameters for the vault dipoles

The TRANSPORT calculations for the beamline require a magnet with an effective length 1.246 m that is capable of producing a field of 14 kG at 500 MeV. We design the magnet for a maximum energy of 520 MeV and field of 14.344 kG in order that to have sufficient range. We also add the following additional parameters.

B_0	=	Maximum magnetic field	=	14.344 kG
g	=	Maximum air gap	=	10.160 cm
θ	=	Maximum bend angle	=	27.500°
s	=	Length of the central trajectory	=	1.246 m

We first calculate the basic properties of these magnets.

$$\rho = \text{radius of curvature of the central trajectory} = \frac{s}{\theta} = \frac{(180.0)(1.24595)}{(27.5)(\pi)} = 2.59592 \text{ m} = 102.202 \text{ in.}$$

$$\boxed{\text{Radius of curvature of the central trajectory} = \rho = 2.596 \text{ m} = 102.2 \text{ in.}}$$

We take the effective straight-line length of the magnet to be

$$l_e = 2\rho \sin\frac{\theta}{2} = 2(2.59592)(0.23769) = 1.23403 \text{ m} = 48.58375 \text{ in.}$$

$$\boxed{\text{Straight-line effective length of the magnet} = l_e = 1.234 \text{ m} = 48.584 \text{ in.}}$$

and assume that the the iron length, l_i, is obtained from

$$l_e = l_i + g$$

so that

$$l_i = l_e - g = 1.23403 - 0.1016 = 1.13242 \text{ m} = 44.58346 \text{ in.}$$

$$\boxed{\text{Iron length of the magnet} = l_i = 1.135 \text{ m} = 44.600 \text{ in.}}$$

The deviation, Δ, of the central trajectory from a line drawn through the points of entry and exit is found from the relation

$$\Delta = \rho\left[1 - \cos\frac{\theta}{2}\right] = 2.59592(1 - 0.97134) = 0.07439 \text{ m} = 2.92889 \text{ in.}$$

Maximum deviation of central trajectory from straight line = Δ = 0.075 m = 2.953 in.

2.1 Ampere-turns per coil

The required Ampere-turns per coil are calculated from the relation

$$NI \text{ per pole} = \frac{1}{2}\left[1.1\,\frac{B_0\,g}{\mu_0}\right] = \frac{1}{2}\,\frac{(1.1)(1.43442)(0.1016)}{4\pi\times10^{-7}} = 63786 \text{ A-t}$$

where we have allowed for a 10% flux leakage. We take

$$NI \text{ per pole} = 64000 \text{ Ampere-turns}$$

and generate the following table

I (Amperes)	100	200	300	400	500	600	700	800	900	1000
N (turns)	640	320	213	160	128	107	91	80	71	64

Because a supplier of an inexpensive 600 A, 100 V power supply has been found, we choose

I	=	595 Amperes
Coil configuration		9 turns wide by 12 turns high

2.2 Coil design

We assume a current density of 3000 A/in^2 = 4.65 A/mm^2 and calculate the required conductor area from

$$\text{Conductor area} = \frac{600\ A}{3000\ A/\text{in}^2} = 0.2000 \text{ in.}^2 = 129.03 \text{ mm}^2$$

This is satisfied within 10% by Ananconda 0.5160 in. conductor; its parameters are listed as

OD	0.5160	in.	13.106	mm
ID	0.2870	in.	7.290	mm
Copper area	0.1940	in.^{2}	125.161	mm^2
Cooling area	0.06469	in.^{2}	41.735	mm^2
Mass	0.7495	lb/ft	1.115	kg/m
Resistance at 20° C	41.99	$\mu\Omega$/ft	137.762	$\mu\Omega$/m
k (British units)	0.01520			

We assume that each conductor is double-wrapped with insulation that is 0.007 in. (0.178 mm) thick with a tolerance of 0.0015 in. (0.038 mm). Then the *total* insulation per conductor has:

Minimum thickness	4(0.007 − 0.0015) in.	=	0.022 in. = 0.559 mm
Nominal thickness	4(0.007) in.	=	0.028 in. = 0.711 mm
Maximum thickness	4(0.007 + 0.0015) in.	=	0.034 in. = 0.864 mm

The tolerance of the outer dimension of the conductor is listed as 0.004 in. = 0.100 mm so that the dimensions of a *wrapped* conductor are:

Minimum	0.516 in. + 0.022 in. − 0.004 in.	=	0.534 in. = 13.56 mm
Nominal	0.516 in. + 0.028 in.	=	0.544 in. = 13.82 mm
Maximum	0.516 in. + 0.034 in. + 0.004 in.	=	0.554 in. = 14.07 mm

We further allow

a) a gap between layers of 0.010 in. (0.254 mm) maximum
b) for keystoning, assume 0.010 in. (0.254 mm)
c) a 4-turn ground wrap of 0.007 in. (0.178 mm) tape

Then the *width* of the coil is obtained from

	Maximum		Minimum	
	in.	mm	in.	mm
Wrapped conductor	4.986	126.644	4.806	122.072
Gapping (8x0.10)	0.080	2.032		
Ground wrap (4x0.178x2)	0.056	1.422	0.056	1.422
Total (mm)	5.122	130.099	4.862	123.495

The average coil width is 4.992 in. = 126.797 mm. We take

Maximum coil width	=	5.150 in.	=	130.8 mm.
Nominal coil width	=	5.000 in.	=	127.0 mm.

The *height* of the coil is

	Maximum		Minimum	
	in.	mm	in.	mm
Wrapped conductor	6.648	168.859	6.408	162.763
Gapping (11x0.010)	0.110	2.794		
Keystoning (12x0.010)	0.120	3.048	0.060	1.524
Ground wrap (4x0.178x2)	0.056	1.422	0.056	1.422
Total (mm)	6.934	176.124	6.524	165.710

The average coil height is 6.729 in. = 170.917 mm. We take

Maximum coil height	=	7.000 in.	=	177.8 mm.
Nominal coil height	=	6.800 in.	=	172.7 mm.

We take the conductor dimension D to be

$$\begin{aligned} D &= \text{Nominal dimension} + 4(\text{Insulation thickness}) + \text{Turn separation} \\ &= 0.516 \text{ in.} + 0.028 \text{ in.} + 0.010 \text{ in.} \\ &= 0.554 \text{ in.} = 14.07 \text{ mm} \end{aligned}$$

and further assume a pole-coil gap of $G = 0.75$ in. = 19.05 mm and that the pole corners are rounded with a radius

$$\text{Pole radius} = R_{pole} = 4\,D - G = 4(0.554) \text{ in.} - 0.750 \text{ in.} = 1.466 \text{ in.} = 37.24 \text{ mm.}$$

Then the n^{th} conductor is a distance

$$D_n = n\,D + G + \text{Polewidth}/2 + 4(\text{insulation thickness})$$

from the longitudinal center-line of the pole and its (outer) radius of curvature is

$$R_n = R_{pole} + n\,D + G + 4(\text{insulation thickness})$$

The length of the straight longitudinal section of the winding is

$$L_{length} = L_{iron} - 2\,R_{pole}$$

and that of the straight section along the pole width is

$$L_{width} = W_{iron} - 2\,R_{pole}.$$

Thus the length of the n^{th} turn is

$$\begin{aligned} l_n &= 2[L_{length} + L_{width}] + 2\,\pi\,R_n \\ &= 2[L_{iron} + W_{iron} + (\pi - 4)R_{pole} + \pi(4(\text{insulation}) + G)] + 2\,\pi\,n\,D \end{aligned}$$

and the length of an N-turn layer is

$$L_N = \sum_{n=1}^{N} l_n = 2\,N[L_{iron} + W_{iron} + (\pi - 4)R_{pole} + \pi(4(\text{insulation} + G)] + \pi\,N(N+1)D \qquad (1)$$

We now consider properties specific to the magnets.

3. H-Frame dipole design

3.1 Pole width

We allow for a maximum beam width, x, of 10.16 cm and assume a 0.625 in. (15.875 mm) chamfer, c, at 45° to the each pole edge. Because the field is relatively uniform in an H-frame magnet, we take the pole width, W_{iron}, to be

$$W_{iron} = 2\,g + \Delta + x + 2\,c = 2(4.000)\text{ in.} + 2.929\text{ in.} + 4.000\text{ in.} + 2(0.625)\text{ in.} = 16.179\text{ in.}$$

$$\boxed{\text{Pole width} = W_{iron} = 16.250\text{ in.} = 412.75\text{ mm.}}$$

Substituting the following values into equation (1)

$$\begin{aligned} L_{iron} &= 44.600\text{ in.} &\approx 1132.8\text{ mm} \\ W_{iron} &= 16.250\text{ in.} &\approx 412.8\text{ mm} \\ R_{pole} &= 1.466\text{ in.} &\approx 37.2\text{ mm} \\ G &= 0.750\text{ in.} &\approx 19.1\text{ mm} \\ D &= 0.554\text{ in.} &\approx 14.1\text{ mm} \\ \text{Insulation} &= 0.007\text{ in.} &\approx 0.2\text{ mm} \end{aligned}$$

we find that the length of a 9-turn layer is 1271.7 in. [32,301 mm] = 106.0 ft [32.3 m]. We take

$$\boxed{\text{Length of 9-turn layer} = 110\text{ ft} \approx 33.5\text{ m}}$$

and the length per coil becomes

$$\boxed{\text{Length per coil} = 1320\text{ ft} \approx 405\text{ m.}}$$

Because two coils are required per dipole, then

Total length per dipole	2640 ft	≈	810 m
Allow 10% for winding losses	264 ft	≈	81 m
Total	2904 ft	≈	891 m

Then order

$$\boxed{\text{Total length of copper} = 2910\text{ ft} \approx 900\text{ m}}$$

of conductor of mass 0.7495 lb/ft for a total mass of

$$\boxed{\text{Total mass} = 2190\text{ lb} \approx 995\text{ kg.}}$$

3.2 Power requirements

At 20°C, the resistance of the coil is:

$$R_{20^\circ} = 41.99\times10^{-6}\ \Omega/\text{ft}\times1320\ \text{ft} = 0.05543\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40° C coolant temperature rise). Then the mean coil temperature will be 50°C.

With a 30°C rise above ambient of the coil we then have:

$$\begin{aligned} R_{hot} &= R_{20^\circ}[1 + (\text{Temp. coeff}/^\circ\text{C})dT(^\circ\text{C})] \\ &= 0.05543[1 + (0.00393)(30)] \\ &= 0.0620\ \Omega \text{ per coil} \end{aligned}$$

Thus, at a current of 595 A, we obtain

$$\text{Voltage per coil} = 38.87 \text{ Volts}$$

Therefore, allowing for a 10% lead loss, we choose a power supply that has:

I	=	600	A minimum
V	=	82.5	V minimum
P	=	49.5	kW minimum

3.3 Cooling requirements

In these calculations we use the British system of units.

The power required per coil is:

$$\text{Power per coil} = I^2 R_{hot} = (595)(595)(0.06196) = 21.94 \text{ kW}.$$

The required flow rate is given by:

$$\begin{aligned} v\ (\text{ft/sec}) &= \frac{2.19}{\Delta T(^\circ\ \text{F})}\times\frac{P(\text{kW})}{\text{Cooling area (in}^2)} \\ &= 0.47019\times P(\text{kW}) \end{aligned}$$

for $\Delta T = 72°\text{F} = 40°\text{C}$ and $A = 0.06469\ \text{in}^2 = 41.735\ \text{mm}^2$. Choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit we have:

$$P_{max} = \frac{(2.50)(72)(0.06469)}{2.19} = 5.317 \text{ kW/water circuit}$$

from which we calculate the number of cooling circuits per coil (excluding lead loss) as

$$\begin{aligned} &P &&= \text{Total power per coil} &&= 21.94 \text{ kW} \\ &\text{Number of circuits} &&= P\ /\ P_{max} &&= 4.13 \end{aligned}$$

Thus we take

$$\boxed{\text{Number of cooling circuits per coil} = 6}$$

This requires a flow rate of $v = 1.719$ ft/sec per water circuit.

The volume of flow required per circuit is

$$\begin{aligned} \text{Volume/circuit} &= 2.6\,v\,(\text{ft/sec}) \times \text{Cooling area (in}^2) \\ &= 2.6(1.719)(0.06469) = 0.2892 \text{ IGPM} \end{aligned}$$

Thus we have the following volumes of flow.

Volume per cooling circuit	=	0.289 IGPM	=	1.315 ℓ/min	=	0.347 USGPM
Volume per coil	=	1.735 IGPM	=	7.887 ℓ/min	=	2.084 USGPM
Volume per magnet	=	3.470 IGPM	=	15.774 ℓ/min	=	4.167 USGPM

3.4 Pressure drop

The pressure drop is given by

$$dP = k\,v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case with $k = 0.0152$ we obtain:

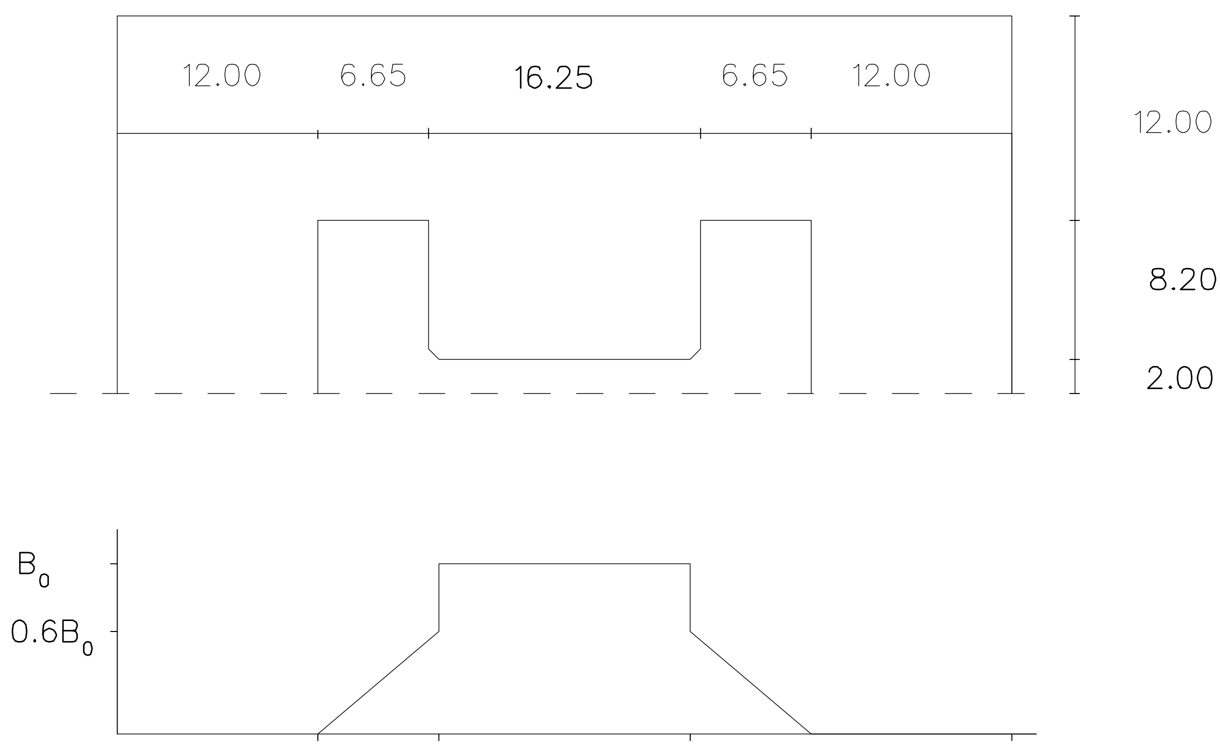
$$\Delta P = (0.0152)(1.719)^{1.79} = 0.0401 \text{ psi/ft} = 0.1315 \text{ psi/m}$$

and the total pressure drop across one cooling circuit is:

$$\boxed{\text{Pressure drop per cooling circuit} = 0.0401 \text{ psi/ft} \times 2(110) \text{ ft} = 8.82 \text{ psi}}$$

3.5 Iron dimensions

A cross section of the dipole and an assumed field profile is shown in the figure below.



Cross section of the H magnet and assumed field profile.

As is illustrated in the figure, the magnetic field profile has been assumed to rise linearly from zero at the inside edge of the yoke to a value of $0.6B_g$ at the flat part of the pole. At that point the field rises to the full value in the gap B_g and remains at that value to the outer edge of the pole. At that point the field is assumed to abruptly drop to a value of $0.6B_g$ and to fall linearly to zero at the outer edge of the outer coil. This assumption is made for ease of further calculation.

We have assumed a maximum coil width of 5.150 in. and allowed 0.750 in. clearance between the coil and the yoke and the coil and the pole. Consequently, the widths of the coil slots are 6.650 in. To this we add $c = 0.625$ in. for the chamfer to find that the edges of the flat portion of the pole are a distance $l_1 = 7.275$ in. from the inside edges of the yoke.

Calling the field in the yoke B_y and the yoke thickness t and assuming that the flux divides equally between the vertical yokes, we equate the flux densities in the yoke and in the gap to obtain a relation between the yoke field and the yoke thickness. We have

$$\begin{aligned} 2\,B_y\,t &= \frac{l_1}{2}\,(0.6B_g) + (W_{iron} - 2c)B_g + \frac{l_1}{2}\,(0.6B_g) \\ &= \left[\frac{7.275(0.6)}{2} + (16.250 - 2(0.625)) + \frac{7.275(0.6)}{2}\right] B_g \\ &= 19.365\,B_g = (19.365)(14.34418) \text{ kG-in.} \\ &= 277.775 \text{ kG-in.} = 0.70555 \text{ T-m} \end{aligned}$$

We thus make the following table.

B_y (kG)	10.	11.	12.	13.
t (in.)	13.889	12.626	11.574	10.684

We choose

Yoke field	=	B_y	=	11.574 kG	=	1.157 T
Yoke thickness	=	t	=	12.000 in.	=	0.305 m

In the above, the coil-slot width was calculated from

$$\begin{aligned} \text{Coil-slot width} &= \text{Maximum coil width} + 2(\text{Pole-coil separation}) \\ &= 5.150 \text{ in.} + 2(0.750) \text{ in.} \\ &= 6.650 \text{ in.} = 168.9 \text{ mm.} \end{aligned}$$

Also, the total dipole width is

$$\begin{aligned} \text{Dipole width} &= 2(\text{Coil-slot width} + \text{Yoke thickness}) + \text{Pole width} \\ &= 2(6.650 \text{ in.} + 12.000 \text{ in.}) + 16.250 \text{ in.} \\ &= 53.550 \text{ in.} = 1360.2 \text{ mm,} \end{aligned}$$

the overall length of the dipole is

$$\begin{aligned} \text{Dipole length} &= 2(\text{Pole-coil separation} + \text{Maximum coil width}) + \text{Pole length} \\ &= 2(0.750 \text{ in.} + 5.150 \text{ in.}) + 44.600 \text{ in.} \\ &= 56.400 \text{ in.} = 1432.6 \text{ mm,} \end{aligned}$$

and the pole-height is obtained from

$$\begin{aligned} \text{Pole height} &= \text{Maximum coil height} + \text{Chamfer} + 15 \text{ mm} \\ &= 7.000 \text{ in.} + 0.625 \text{ in.} + 0.591 \text{ in.} \\ &= 8.216 \text{ in.} = 208.7 \text{ mm,} \end{aligned}$$

and the lengths of the side yokes are

$$\begin{aligned} \text{Side-yoke height} &= 2(\text{Pole height}) + \text{Gap} \\ &= 2(8.200 \text{ in.}) + 4.000 \text{ in.} \\ &= 20.400 \text{ in.} = 518.2 \text{ mm.} \end{aligned}$$

where the pole height is taken as 8.200 in. (208.3 mm). We take

Coil-slot width	=	6.650 in.	=	168.9 mm
Pole height	=	8.200 in.	=	208.3 mm
Side-yoke height	=	20.400 in.	=	518.2 mm
Dipole width	=	53.550 in.	=	1360.2 mm
Dipole length	=	56.400 in.	=	1432.6 mm

3.6 Iron weight

The cross-sectional areas of the magnet components are tabulated below.

Section	Height		Width		Area	
	(in.)	(m)	(in.)	(m)	(in.^{2})	(m^2)
Top yoke	12.000	0.305	53.550	1.360	642.600	0.415
Bottom yoke	12.000	0.305	53.550	1.360	642.600	0.415
Vertical Yoke	20.400	0.518	12.000	0.305	244.800	0.158
Vertical Yoke	20.400	0.518	12.000	0.305	244.800	0.158
Top pole	8.200	0.208	16.250	0.413	133.250	0.086
Bottom pole	8.200	0.208	16.250	0.413	133.250	0.086
Total area					2041.300	1.318

The total volume of iron is then

$$\begin{aligned} \text{Volume of iron} &= \text{(Total area)(Iron length)} \\ &= (2041.300)(44.600)\ \text{in.}^3 \\ &= 91{,}055\ \text{in.}^3 = 52.694\ \text{ft}^3 = 1.492\ \text{m}^3 \end{aligned}$$

$$\boxed{\text{Volume of iron} = 53.0\ \text{ft}^3 = 1.501\ \text{m}^3}$$

and the iron mass at 7900 kg/m^3 is

$$\begin{aligned} \text{Iron mass} &= \text{(Iron volume)(Density)} \\ &= (1.501\ \text{m}^3)(7900\ \text{kg/m}^3) \\ &= 11.860 \times 10^3\ \text{kg} \\ &= 26.151 \times 10^3\ \text{lb} \end{aligned}$$

We take

$$\boxed{\text{Iron mass} = 26.250 \times 10^3\ \text{lb} = 11.910 \times 10^3\ \text{kg}}$$

A summary of the design parameters of this H-frame dipole is given in table 1.

4. Arc-shaped magnet design

4.1 Pole width

Here, the pole is curved to follow the beam curvature; consequently, we do not have to add the deviation Δ to the pole width as was required in the H-frame design. For consistency, we allow for a maximum beam width, x, of 10 cm and assume a 0.625 in. (15.875 mm) chamfer, c, at 45° to the each pole edge. Again, the field is relatively uniform and we take the pole width, W_{pole}, to be

$$W_{pole} = 2\,g + x + 2\,c = 2(4.000) \text{ in.} + 3.937 \text{ in.} + 2(0.625 \text{ in.}) = 13.187 \text{ in.}$$

$$\boxed{\text{Pole width } = W_{pole} = 13.5 \text{ in.} = 342.9 \text{ mm.}}$$

4.2 Coil design

Because the coil will be shaped to follow the curvature of the pole, we cannot use the formula given in section 2.2. We proceed as follows.

We assume that the magnet is centered about the y-axis of an x-y coordinate system such that the center of curvature of the central trajectory lies at (0, 0). Then the outer radius of the pole is

$$R_p^+ \quad = \quad \rho_0 + W_{pole}/2.$$

Making allowance for the pole-coil gap we then find that the *outer* radius of the outer n^{th} turn is

$$R_n^+ = R_p^+ + G + n\,D = \rho_0 + \frac{W}{2} + G + n\,D.$$

Similarly, the inner radius of the pole is

$$R_p^+ \quad = \quad \rho_0 - W_{pole}/2$$

and we find the *outer* radius of the inner n^{th} turn is

$$R_n^+ = \rho_0 - \frac{W_{pole}}{2} - G - (n-1)D.$$

These sections of coil are terminated at their intersection with the (vertical) lines $x = \pm x_0$ that represent the pole ends. Making an allowance of one pole-coil separation at the pole end—probably an underestimate—we have

$$x = \pm(x_0 + G) = \pm\left[\frac{l_i}{2} + G\right]$$

where l_i is defined in section 2. Then the absolute values of the half-angles subtended at the points of intersection of the arcs and these lines are given by

$$\theta_n^\pm = \sin^{-1}\left[\frac{x_0 + G}{R_n^\pm}\right]$$

where we measure the angles from the y-axis. The lengths of the curved sections are then

$$l_n^\pm = 2\,R_n^\pm\,\theta_n^\pm.$$

We can approximate the (two) straight sides of the coil by

$$l_n^s = W + 2(G + n\,D)$$

so that the overall length of the n^{th} turn is

$$L_n \quad = \quad l_n^+ + l_n^- + 2\,l_n^s$$

Substituting the following values into the above

$$
\begin{aligned}
\rho &= 102.202 \text{ in.} \approx 2595.9 \text{ mm} \\
l_i &= 44.600 \text{ in.} \approx 1132.8 \text{ mm} \\
W_{pole} &= 13.500 \text{ in.} \approx 342.9 \text{ mm} \\
G &= 0.750 \text{ in.} \approx 19.1 \text{ mm} \\
D &= 0.555 \text{ in.} \approx 14.1 \text{ mm}
\end{aligned}
$$

we obtain the following table for a 9-turn layer.

n	R_n^+	θ_n^+	l_n^+	R_n^-	θ_n^-	l_n^-	l_n^s	$\sum_{i=1}^{n} L_i$
1	110.256	0.2106	46.443	94.701	0.2459	46.568	16.110	125.230
2	110.811	0.2095	46.439	94.146	0.2473	46.573	17.220	252.683
3	111.366	0.2085	46.436	93.591	0.2488	46.579	18.330	382.358
4	111.921	0.2074	46.432	93.036	0.2504	46.585	19.440	514.255
5	112.476	0.2064	46.429	92.481	0.2519	46.591	20.550	648.375
6	113.031	0.2054	46.426	91.926	0.2535	46.597	21.660	784.718
7	113.586	0.2044	46.422	91.371	0.2550	46.604	22.770	923.284
8	114.141	0.2033	46.419	90.816	0.2566	46.610	23.880	1064.073
9	114.696	0.2023	46.416	90.261	0.2582	46.616	24.990	1207.086

Thus the length of a 9-turn layer is 1210 in. [30,734 mm] = 101 ft [30.7 m]. We take

$$\boxed{\text{Length of 9-turn layer} = 105 \text{ ft} = 32.0 \text{ m}}$$

and the length per coil becomes

$$\boxed{\text{Length per coil} = 1260 \text{ ft} = 385 \text{ m.}}$$

Because two coils are required per dipole, then

Total length per dipole	2520 ft	≈	770 m
Allow ≈10% for winding losses	250 ft	≈	75 m
Total	2770 ft	≈	845 m

Then order

$$\boxed{\text{Total length of copper} = 2775 \text{ ft} \approx 850 \text{ m}}$$

of conductor of mass 0.7495 lb/ft for a total mass of

$$\boxed{\text{Total mass} = 2080 \text{ lb} \approx 945 \text{ kg.}}$$

4.3 Power requirements

At 20°C, the resistance of the coil is:

$$R_{20^\circ} = 41.99 \times 10^{-6}\ \Omega/\text{ft} \times 1260 \text{ ft} = 0.05291\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40°C coolant temperature rise). Then the mean coil temperature will be 50°C. With a 30°C rise above ambient of the coil we then have:

$$
\begin{aligned}
R_{hot} &= R_{20^\circ}[1 + (\text{Temp. coeff}/^\circ\text{C})dT(^\circ\text{C})] \\
&= 0.05291[1 + (0.00393)(30)] \\
&= 0.0592\ \Omega \text{ per coil}
\end{aligned}
$$

Thus, at a current of 595 A, we obtain

$$\text{Voltage per coil} = 35.23 \text{ Volts}$$

Therefore, allowing for a 10% lead loss, we choose a power supply that has:

I	=	600	A minimum
V	=	77.5	V minimum
P	=	46.5	kW minimum

4.4 Cooling requirements

In these calculations we use the British system of units.

Using a current of 600 A, the power required per coil is:

$$\text{Power per coil} = I^2 R_{hot} = (600)(600)(0.0592) = 21.31 \text{ kW}.$$

The required flow rate is given by:

$$\begin{aligned} v \text{ (ft/sec)} &= \frac{2.19}{\Delta T(^\circ \text{ F})} \times \frac{P(\text{kW})}{\text{Cooling area (in}^2)} \\ &= 0.47019 \times P(\text{kW}) \end{aligned}$$

for $\Delta T = 72°\text{F} = 40°\text{C}$ and $A = 0.06469 \text{ in}^2 = 41.735 \text{ mm}^2$. Choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit we have:

$$P_{max} = \frac{(2.50)(72)(0.06469)}{2.19} = 5.317 \text{ kW/water circuit}$$

from which we calculate the number of cooling circuits per coil (excluding lead loss) as

$$\begin{aligned} P &= \text{Total power per coil} &= 21.312 \text{ kW} \\ \text{Number of circuits} &= P \;/\; P_{max} &= 4.01 \end{aligned}$$

Thus we take

$$\boxed{\text{Number of cooling circuits per coil} = 6}$$

This requires a flow rate of $v = 1.885$ ft/sec per water circuit. The volume of flow required per circuit is

$$\begin{aligned} \text{Volume/circuit} &= 2.6\, v(\text{ft/sec}) \times \text{Cooling area(in}^2) \\ &= 2.6(1.885)(0.06469) = 0.317 \text{ IGPM} \end{aligned}$$

Thus we have the following volumes of flow.

Volume per cooling circuit	=	0.317 IGPM	=	1.441 ℓ/min	=	0.381 USGPM
Volume per coil	=	1.902 IGPM	=	8.647 ℓ/min	=	2.285 USGPM
Volume per magnet	=	3.805 IGPM	=	17.294 ℓ/min	=	4.569 USGPM

4.5 Pressure drop

The pressure drop is given by

$$dP = k\, v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case with $k = 0.0152$ we obtain:

$$\Delta P = (0.0152)(1.885)^{1.79} = 0.0473 \text{ psi/ft} = 0.1551 \text{ psi/m}$$

and the total pressure drop across one cooling circuit is:

$$\boxed{\text{Pressure drop per cooling circuit} = 0.0473 \text{ psi/ft} \times 210 \text{ ft} = 9.93 \text{ psi}}$$

4.6 Iron dimensions

In principle, the all edges of the yoke components of the dipole should be curved. However, to save machining costs we consider a magnet with rectangular top and bottom yokes and outer (inner) spacers that have straight outer (inner) edges and curved inner (outer) edges. Thus the outer spacer has a plano-concave shape and the inner spacer a plano-convex one.

Given that the radius of curvature of the central trajectory is $\rho_0 = 202.2$ in. $= 2.596$ m, the pole width is $W_{pole} = 13.500$ in. $= 0.343$ m, the maximum coil width is $W_{coil} = 5.122$ in. $= 0.130$ m and the pole-coil gap is $G = 0.750$ in. $= 0.0191$ m, we then set

$$\begin{array}{lclcl}
\text{Radius of inner edge of outer yoke} & = & R_5 & = & \rho_0 + W_{pole}/2 + W_{coil} + 2G \\
\text{Radius of outer edge of pole} & = & R_4 & = & \rho_0 + W_{pole}/2 \\
\text{Radius of inner edge of pole} & = & R_3 & = & \rho_0 - W_{pole}/2 \\
\text{Radius of outer edge of inner yoke} & = & R_2 & = & \rho_0 - W_{pole}/2 - W_{coil} - 2G
\end{array}$$

We also define R_6 and R_1, respectively, as the perpendicular distances from the center of curvature to the outermost and innermost edges of the yoke. The figure on the next page shows a plan view of a section through the magnet gap and indicates these radii.

Then the area of the outer spacer A_{outer} is

$$A_{outer} = l_i[R_6 - R_5] - \frac{R_5^2[\theta_{outer} - \sin\theta_{outer}]}{2}$$

with l_i the iron length of the yoke as before and

$$\theta_{outer} = 2\sin^{-1}\frac{l_i}{2\,R_5}\,.$$

Similarly, the area of the inner spacer A_{inner} is

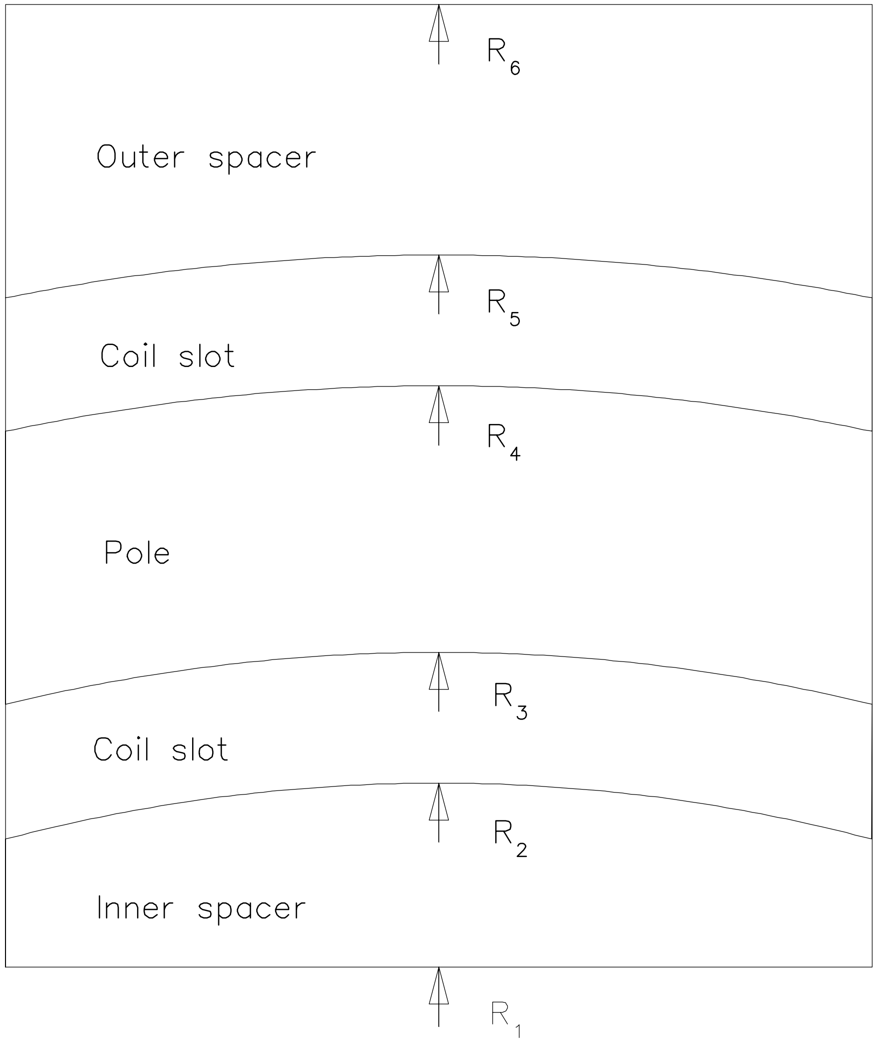
$$A_{inner} = l_i[R_2 - R_1] + \frac{R_2^2[\theta_{inner} - \sin\theta_{inner}]}{2}$$

with

$$\theta_{inner} = 2\sin^{-1}\frac{l_i}{2\,R_2}\,.$$

We wish to calculate the values of R_1 and R_6 such that the flux through each of the spacers is one-half of that through the gap. To calculate the magnetic flux passing through the gap region we assume, as before, that the field in the gap rises linearly from zero at the outer edge of the inner yoke to a value of $0.6B_0$ at the inner edge of the pole; B_0 is the field in the gap. The field rises abruptly to B_0 at the inner edge of the pole and remains constant across the pole itself. At the outer edge of the pole the field abruptly drops to a value of $0.6B_0$ and decreases linearly to zero at the inner edge of the outer spacer. The flux crossing the gap area is then

$$\Phi = \int B\,dA = \int B(r,\theta)\,r\,dr\,d\theta$$



Cross section of the H magnet and assumed field profile.

with

$$B(r) = \begin{cases} \dfrac{0.6B_0}{R_3 - R_2}(r - R_2) & R_2 \le r \le R_3 \\ B_0 & R_3 \le r \le R_4 \\ \dfrac{0.6B_0}{R_4 - R_5}(r - R_5) & R_4 \le r \le R_5 \end{cases}$$

Rather than doing a complete integration we proceed to estimate the gap flux as follows. We calculate the flux through the inner coil slot by approximating the coil-slot area by a rectangle of width $(R_3 - R_2)$ and of length $< R >_{inner} = [(R_3 + R_2)/2] < \theta >_{inner}$ with

$$\begin{aligned} < \theta >_{inner} &= 2\sin^{-1}\frac{l_i/2}{(R_3 + R_2)/2} \\ &= 2\sin^{-1}\frac{l_i}{R_3 + R_2} \end{aligned}$$

so that the area of the inner coil-slot is approximated as

$$\begin{aligned} < A >_{inner} &= [R_3 - R_2] < R >_{inner} \\ &= \frac{R_3^2 - R_2^2}{2} < \theta >_{inner} \end{aligned}$$

and with an average field of

$$< B >_{inner} = \frac{1}{2}[0.6\, B_0 - 0] = 0.3\, B_0.$$

Thus the flux through the inner coil slot is approximated by

$$\begin{aligned} \Phi_{inner} &= < B >_{inner} \;\; < A >_{inner} \\ &= 0.3\, B_0 \frac{R_3^2 - R_2^2}{2} < \theta >_{inner} \end{aligned}$$

A similar expression for the flux in the outer coil slot gives

$$\begin{aligned} \Phi_{outer} &= (B_{av})_{outer} < A >_{outer} \\ &= 0.3\, B_0 \frac{R_5^2 - R_4^2}{2} < \theta >_{outer} \end{aligned}$$

with

$$< \theta >_{outer} = 2 \sin^{-1} \frac{l_i}{R_5 + R_4}.$$

The flux across the pole face is

$$\begin{aligned} \Phi_{pole} &= B_0\, W_{pole}\, \rho_0\, \theta_0 \\ &= B_0 \frac{R_4^2 - R_3^2}{2} \theta_0 \end{aligned}$$

with θ_0 the bend angle of the dipole. Then the total flux crossing the gap is

$$\begin{aligned} \Phi &= \Phi_{inner} + \Phi_{pole} + \Phi_{outer} \\ &= 0.3\, B_0 \left[\frac{R_3^2 - R_2^2}{2} < \theta >_{inner} \; + \; \frac{R_5^2 - R_4^2}{2} < \theta >_{outer} \right] \; + \; B_0 \frac{R_4^2 - R_3^2}{2} \theta_0 \end{aligned}$$

We then obtain values for R_6 and R_1 from the following relations in which the fields in the outer and inner spacers are written as B_{outer} and B_{inner} respectively.

$$\frac{\Phi}{2\, B_{outer}} = A_{outer} = l_i[R_6 - R_5] - \frac{R_5^2[\theta_{outer} - \sin\theta_{outer}]}{2}$$

and

$$\frac{\Phi}{2\, B_{inner}} = A_{inner} = l_i[R_2 - R_1] + \frac{R_2^2[\theta_{inner} - \sin\theta_{inner}]}{2}$$

Using the values given at the beginning of this section we obtain

Radius of inner edge of outer yoke	=	R_5	=	2.93557 m	=	115.57358 in.
Radius of outer edge of pole	=	R_4	=	2.76737 m	=	108.95158 in.
Radius of inner edge of pole	=	R_3	=	2.42447 m	=	95.45158 in.
Radius of outer edge of inner yoke	=	R_2	=	2.25627 m	=	88.82958 in.

from which we calculate (using metric units)

$$\theta_{outer} = 2\sin^{-1}\frac{l_i}{2\,R_5} = 2\sin^{-1}\frac{1.132840}{2(2.935569)} = 0.388337 \text{ radian}\,,$$

$$\theta_{inner} = 2\sin^{-1}\frac{l_i}{2\,R_2} = 2\sin^{-1}\frac{1.132840}{2(2.256271)} = 0.507514 \text{ radian}\,,$$

$$<\theta>_{inner} \;=\; 2\sin^{-1}\frac{l_i}{R_3+R_2} = 2\sin^{-1}\frac{1.132840}{2(2.424470+2.25627`)} = 0.488897 \text{ radian}$$

and

$$<\theta>_{outer} \;=\; 2\sin^{-1}\frac{l_i}{R_5+R_4} = 2\sin^{-1}\frac{1.132840}{2(2.935569+2.767370)} = 0.399943 \text{ radian}.$$

Then

$$<A>_{inner} \;=\; \frac{R_3^2-R_2^2}{2}\;<\theta>_{inner} = \frac{(2.424470)^2-(2.256271)^2}{2}(0.488897) = 0.192453 \text{ m}^2$$

so that

$$\Phi_{inner} \;=\; <B>_{inner}<A>_{inner} = 0.3(1.434418)(0.192453) = 0.082818 \text{ T-m}^2 = 1283.67 \text{ kG-in.}^2$$

Also

$$<A>_{outer} \;=\; \frac{R_5^2-R_4^2}{2}\;<\theta>_{outer} = \frac{(2.935569)^2-(2.767370)^2}{2}(0.399943) = 0.191818 \text{ m}^2$$

so that

$$\Phi_{outer} \;=\; <B>_{outer}<A>_{outer} = 0.3(1.434418)(0.191818) = 0.082544 \text{ T-m}^2 = 1050.11 \text{ kG-in.}^2$$

The flux through the pole is

$$\begin{aligned}\Phi_{pole} &= B_0\,(R_4-R_3)\,\rho_0\,\theta_0\\ &= (1.434418)(2.767370-2.424470)(2.595920)\,\frac{27.5\,\pi}{180.}\\ &= 0.612836 \text{ T-m}^2 = 9498.98 \text{ kG-in.}^2\end{aligned}$$

Thus the total flux through the gap is

$$\Phi = \Phi_{inner} + \Phi_{outer} + \Phi_{pole} = 0.778198 \text{ T-m}^2 = 11,832.77 \text{ kG-in.}^2$$

We obtain a value for R_6 from the solution of

$$\frac{\Phi}{2\,B_{spacer}} = l_i[R_6-R_5] - \frac{R_5^2[\theta_{outer}-\sin\theta_{outer}]}{2}$$

and for R_1 from the solution of

$$\frac{\Phi}{2\,B_{spacer}} = l_i[R_2-R_1] + \frac{R_2^2[\theta_{inner}-\sin\theta_{inner}]}{2}$$

where B_{spacer} is the field in the appropriate spacer. We tabulate the results as a function of B_{spacer}.

B_{spacer}	R_6		R_1	
(T)	(m)	(in.)	(m)	(in.)
1.000	3.31589	130.547	1.96113	77.2097
1.100	3.28466	129.317	1.99235	78.4390
1.200	3.25864	128.293	2.01837	79.4634
1.300	3.23662	127.426	2.04039	80.3302
1.400	3.21775	126.683	2.05926	81.0732
1.500	3.20140	126.039	2.07562	81.7172

In order to keep the field in the spacers to approximately 1.2 T we choose for the *outer* spacer

Outer spacer outer dimension	=	R_6	=	3.25755 m	=	128.250 in.
Outer spacer field	=	B_{outer}	=	1.2046 T	=	12.046 kG

and for the *inner* spacer

Inner spacer inner dimension	=	R_1	=	2.01930 m	=	79.500 in.
Inner spacer field	=	B_{inner}	=	1.2039 T	=	12.039 kG

We now calculate the thicknesses of the top and bottom yokes. Their areas are simply their iron lengths multiplied by their thicknesses. Thus

$$B_{yoke}\, t_{yoke}\, l_i = \frac{\Phi}{2}$$

and for a field of $B_{yoke} = 1.204$ T we find $t_{yoke} = 0.285276$ m = 11.23 in. We take

Top and bottom yoke thicknesses = 11.250 in. = 0.28575 m.

We are now in a position to estimate the amount of iron required and generate the following table.

Item	Width		Length		Area	
	(m)	(in.)	(m)	(in.)	(m^2)	($in.^2$)
Top yoke	1.23825	48.750	1.13284	44.600	1.40274	2174.250
Bottom yoke	1.23825	48.750	1.13284	44.600	1.40274	2174.250
Top pole	0.34290	13.500	1.124595	49.053	0.42724	662.218
Bottom pole	0.34290	13.500	1.124595	49.053	0.42724	662.218
Outer spacer	(0.32198)[a]	(12.676)[a]			0.32301	500.671
Inner spacer	(0.23697)[b]	(9.330)[b]			0.32320	500.956
Total area					4.30617	6674.563

[a] Minimum width of outer spacer.
[b] Maximum width of inner spacer.

Then the volume and mass of iron required are obtained from the table on the next page.

Item	Thickness		Area		Volume	
	(m)	(in.)	(m^2)	(in.^{2})	(m^3)	(in.^{3})
Top yoke	0.28575	11.250	1.40274	2174.250	0.40083	24460.313
Bottom yoke	0.28575	11.250	1.40274	2174.250	0.40083	24460.313
Top pole	0.20701	8.150	0.42724	662.218	0.08782	5397.077
Bottom pole	0.20701	8.150	0.42724	662.218	0.08844	5397.077
Outer spacer	0.51562	20.300	0.32301	500.671	0.16655	10163.621
Inner spacer	0.51562	20.300	0.32320	500.956	0.16665	10169.407
Total volume					1.31050	80047.808

and the total mass of iron at 7900 kg/m^3 is 10.353×10^3 kg $= 22.828 \times 10^3$ lb. We take

$$\boxed{\text{Mass of iron} = 23.0 \times 10^3 \text{ lb} = 10.5 \times 10^3 \text{ kg.}}$$

4.7 Summary of arc-shaped magnet design parameters

A summary of the design parameters of this arc-shaped dipole is given on the next page as table 2.

5. Discussion of the vault dipole designs

Sections 3 and 4 of this report have presented two possible designs—an H-frame dipole and an arc-shaped dipole—for the 27.5° dipoles that are required in the vault section of beamline 2A. Both designs utilize the same conductor and coil configuration and have the same iron length. In most respects, the two designs are similar.

The H-frame design is the easiest to construct because all cuts are straight. It is estimated to require approximately two tons more of iron and 100 pounds more of copper than the arc-shaped dipole. The H-frame magnet also requires slightly more power to operate than does the arc-shaped dipole.

Because of its curved pole, the arc-shaped dipole would be expected to be more expensive to construct, although this may be mitigated by the slightly smaller requirements of raw materials. On the other hand, as noted above, an arc-shaped dipole would cost slightly in operating costs.

The decision of which, if either, of these designs is implemented will presumably be based on an engineering decision. In any event, the largest sections of either design—the top and bottom yokes—weigh less than 4 tons and are therefore capable of being lifted by the vault crane.

Table 1

Summary of H-magnet design parameters for Anaconda 0.5160 conductor

Yoke:	Iron length	44.600 in.	1132.8 mm
	Iron width	53.550 in.	1360.2 mm
	Iron thickness	12.000 in.	304.8 mm
	Coil-slot width	6.650 in.	168.9 mm
	Side-yoke height	20.400 in.	518.2 mm
Pole:	Width	16.250 in.	412.8 mm
	Height	8.200 in.	208.3 mm
	Chamfer at 45°	0.625 in.	15.9 mm
Iron:	Total mass	26.250×10^3 lb	11.910×10^3 kg
Dipole:	Overall width	53.550 in.	1360.2 mm
	Overall height	53.550 in.	1360.2 mm
	Overall length (incl. coil)	56.400 in.	1432.6 mm
Coil:	Anaconda conductor		
	Conductor OD	0.526 in.	13.1 mm
	Conductor ID	0.287 in.	7.3 mm
	Nominal coil width	5.000 in.	127.0 mm
	Nominal coil height	6.800 in.	172.7 mm
	Total coolant flow	4.167 USGPM	15.8 ℓ/min
	Turn configuration	9 wide $\times$ 12 high	
	Resistance (hot) per coil	62.0×10^{-3} Ω	
	Number of cooling circuits per coil	6	
Copper:	Total length per magnet	2880 ft	880 m
	Total mass per magnet	2160 lb	980 kg
	Total length to order	3170 ft	970 m
	Total mass to order	2380 lb	1080 kg
Power:	Total current	600.0 A	
	Total Voltage	82.5 V	
	Power	49.5 kW	

Table 2

Summary of Arc-shaped magnet design parameters for Anaconda 0.5160 conductor

Yoke:	Iron length	44.600 in.	1132.8 mm
	Iron width	48.750 in.	1238.3 mm
	Iron thickness	11.250 in.	285.8 mm
	Coil-slot width	6.625 in.	168.3 mm
	Side-yoke height	20.300 in.	515.6 mm
Pole:	Width	13.500 in.	342.9 mm
	Height	8.150 in.	207.0 mm
	Chamfer at 45°	0.625 in.	15.9 mm
Iron:	Total mass	23.0×10^3 lb	10.5×10^3 kg
Dipole:	Overall width	48.750 in.	1238.3 mm
	Overall height	43.300 in.	1099.8 mm
	Overall length (incl. coil)	56.350 in.	1431.3 mm
Coil:	Anaconda conductor		
	Conductor OD	0.526 in.	13.1 mm
	Conductor ID	0.287 in.	7.3 mm
	Nominal coil width	5.100 in.	129.5 mm
	Nominal coil height	6.800 in.	172.7 mm
	Total coolant flow	4.569 USGPM	17.3 ℓ/min
	Turn configuration	9 wide × 12 high	
	Resistance (hot) per coil	59.2×10^{-3} Ω	
	Number of cooling circuits per coil	6	
Copper:	Total length per magnet	2640 ft	810 m
	Total mass per magnet	1980 lb	900 kg
	Total length to order	2910 ft	900 m
	Total mass to order	2190 lb	995 kg
Power:	Total current	600.0	A
	Total Voltage	77.5	V
	Power	46.5	kW