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Subject A conceptual design for 15° switching dipole for beam line 2A		

1. Introduction

Protons are to be extracted from extraction port 2A to deliver beam to an external ISAC facility. Earlier reports[1,2] presented designs for the 27.5° vault dipoles and for a 30° switching magnet. It has been proposed that the 30° switching magnet could be replaced with a 15° switching magnet and two 15° dipoles—one on each line leading to each target. This report presents a design for the ±15° switching dipole that would be required on the beam line.

2. Design parameters for the switching magnet

TRANSPORT calculations for the beam line require a magnet with an effective length 0.681 m that is capable of producing a field of 14.0 kG for 500 MeV protons. We design the magnet for a maximum energy of 520 MeV and field of 14.307 kG. We also add the following additional parameters.

B_0	=	Maximum magnetic field	=	14.307 kG
g	=	Maximum air gap	=	10.16 cm
θ	=	Maximum bend angle	=	15.0°
s	=	Length of the central trajectory	=	68.139 cm

We first calculate the basic parameters of the magnet.

$$\rho_0 = \text{radius of curvature of the central trajectory} = \frac{s}{\theta} = \frac{(180.0)(0.68139)}{(15.0)(\pi)} = 2.60272 \text{ m} = 104.469 \text{ in.}$$

$$\boxed{\text{Radius of curvature of the central trajectory} = \rho_0 = 2.603 \text{ m} = 102.5 \text{ in.}}$$

We take the effective straight-line length of the magnet to be

$$l_e = 2\,\rho_0 \sin\frac{\theta}{2} = 2(2.60272)(0.13053) = 0.67945 \text{ m} = 26.750 \text{ in.}$$

$$\boxed{\text{Straight-line effective length of the magnet} = l_e = 0.680 \text{ m} = 26.75 \text{ in.}}$$

and assume that the the iron length, l_i, is obtained from

$$l_e = l_i + g$$

so that

$$l_i = l_e - g = 0.67945 - 0.1016 = 0.57785 \text{ m} = 22.74985 \text{ in.}$$

$$\boxed{\text{Iron length of the magnet} = l_i = 0.578 \text{ m} = 22.75 \text{ in.}}$$

3. Pole width and geometry

We assume a pole geometry as indicated in the figure on the next page. The beam enters at the mid-point of the pole (O in the diagram) and exits at an angle of ±15° at the points C and D in the diagram. The (straight-line) lengths $\overline{\text{OC}}$ and $\overline{\text{OF}}$ are then each 22.75 inches in length, as is the radius of curvature, R, of

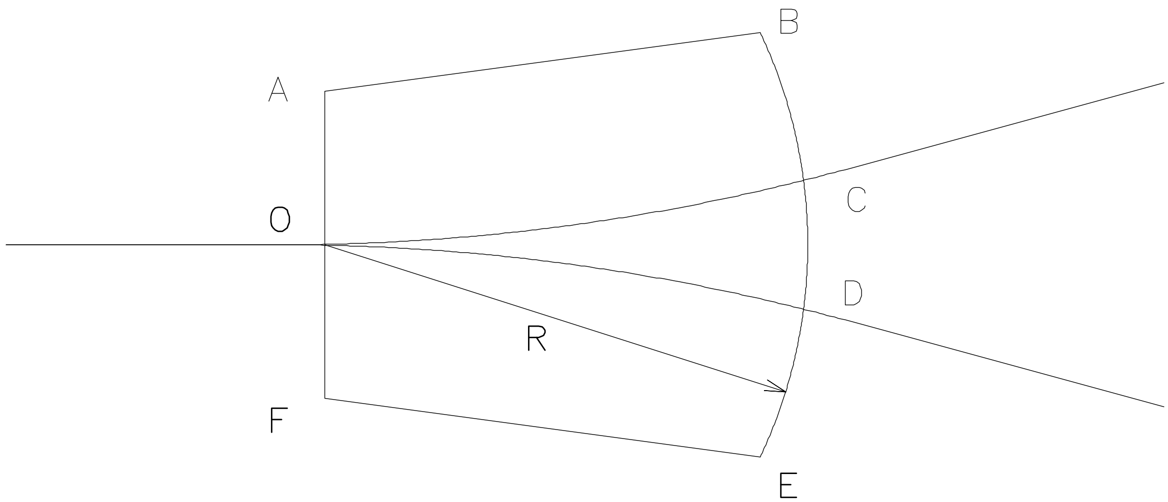

Fig. 1. Geometry of the pole of the switching magnet.

the exit pole face.

The maximum width of the beam at the entrance of the dipole is $x \approx \pm\, 1$ in. We assume a 0.625 in. (16 mm) chamfer, c, on the pole edges and take the width of the entrance pole end, W_{iron}, to be

$$W_{iron} = 2.5\,g + x + 2\,c = 2.5(4.000)\text{ in.} + 2.000\text{ in.} + 2(0.625)\text{ in.} = 13.250\text{ in.}$$

$$\boxed{\text{Pole width} = W_{iron} = \overline{\text{AH}} = \overline{\text{BD}} = \overline{\text{EG}} = 13.250\text{ in.} = 336.6\text{ mm.}}$$

The lengths of the sides $\overline{AB}$ and $\overline{EF}$ are determined from the intersection of the pole edge with the circle defining the exit face of the pole. Taking the point O to be the origin of a Cartesian (x, y) coordinate system, the the equation of the line through the points A and B is

$$y = x\tan\frac{\theta}{2} + \frac{W_{iron}}{2}$$

and that of the exit pole face is

$$x^2 + y^2 = R^2.$$

The line and the circle intersect at (positive values) of x and y of

$$x_{int} = \frac{\sqrt{4\,R^2[1+\tan^2(\theta/2)] - W_{iron}^2} - W_{iron}\tan(\theta/2)}{2[1+\tan^2(\theta/2)]}$$

$$y_{int} = \frac{\tan(\theta/2)\,\sqrt{4\,R^2[1+\tan^2(\theta/2)] - W_{iron}^2} + W_{iron}}{2[1+\tan^2(\theta/2)]}$$

Inserting the appropriate values for θ, W_{iron} and R we obtain

$$x_{int} = 20.73750\text{ in.} \qquad \text{and} \qquad y_{int} = 9.35514\text{ in.}$$

and the lengths $\overline{\text{AB}}$ and $\overline{\text{FE}}$ become

$$\overline{\text{AB}} = \overline{\text{FE}} = \sqrt{(20.73750)^2 + (9.35514 - 6.625)^2} = 20.91644\text{ in.}$$

$$\boxed{\overline{\text{AB}} = \overline{\text{FE}} = 20.92\text{ in.}}$$

For future reference we note that the distance $\overline{\mathrm{BE}}$ is

$$\overline{\mathrm{BE}} = 2\, y_{int} = 18.71028 \text{ in.}$$

$$\boxed{\overline{\mathrm{BE}} = 18.71 \text{ in.}}$$

[Alternately, the distance $\overline{\mathrm{AB}}$ can be calculated directly from the cosine law

$$R^2 = (\overline{\mathrm{AB}})^2 + (W_{iron}/2)^2 - 2\,\overline{\mathrm{AB}}\,(W_{iron}/2)\cos((\pi+\theta)/2)$$

that has the solutions, writing $\Phi = (\pi+\theta)/2$,

$$\overline{\mathrm{AB}} = \overline{\mathrm{HG}} = \frac{W_{iron}}{2}\cos\Phi \pm \sqrt{\frac{W_{iron}^2}{4}[\cos^2\Phi - 1] + R^2}$$

and the positive sign taken.]

4. Ampere-turns per coil

The required Ampere-turns per coil are calculated from the relation

$$NI \text{ per coil} = \frac{1}{2}\left[1.1\,\frac{B_0\, g}{\mu_0}\right] = \frac{1}{2}\,\frac{(1.1)(1.43066)(0.1016)}{4\pi\times 10^{-7}} = 63,618 \text{ A-t}$$

where we have allowed for a 10% flux leakage. We take

$$\boxed{NI \text{ per coil} = 64{,}000 \text{ Ampere-turns}}$$

and generate the following table

I (Amperes)	100	200	300	400	500	600	700	800	900	1000
N (turns)	640	320	213	160	128	107	92	80	71	64

Because an inexpensive 600 A, 100 V power supply is available, we choose

I	=	600 Amperes
Coil configuration		9 turns wide by 12 turns high

5. Coil design

We assume a current density of 3000 A/in^2 = 4.65 A/mm^2 and calculate the required conductor area from

$$\text{Conductor area} = \frac{600\ A}{3000\ A/\text{in}^2} = 0.2000 \text{ in.}^2 = 129.03 \text{ mm}^2$$

This is satisfied within 10% by Ananconda 0.4600 and 0.5160 in.-square conductors; their parameters are given in the table below.

	Anaconda 0.4600		Anaconda 0.5160	
OD	0.4600 in.	[11.684 mm]	0.5160 in.	[13.106 mm]
ID	0.2550 in.	[6.477 mm]	0.2870 in.	[7.290 mm]
Copper area	0.1529 in.^{2}	[98.645 mm^2]	0.1940 in.^{2}	[125.161 mm^2]
Cooling area	0.1529 in.^{2}	[98.645 mm^2]	0.06469 in.^{2}	[41.735 mm^2]
Mass	0.5910 lb/ft	[0.880 kg/m]	0.7495 lb/ft	[1.115 kg/m]
Resistance at 20°C	53.25 $\mu\Omega$/ft	[174.70 $\mu\Omega$/m]	41.99 $\mu\Omega$/ft	[137.762 $\mu\Omega$/m]
k (British units)	0.01760		0.01520	

We assume that each conductor is double-wrapped with insulation that is $t_i = 0.007$ in. (0.178 mm) thick with a tolerance of 0.0015 in. (0.038 mm). Then the *total* insulation per conductor has:

Minimum thickness	$4(0.007 - 0.0015)$ in.	=	0.022 in. = 0.559 mm
Nominal thickness	$4(0.007)$ in.	=	0.028 in. = 0.711 mm
Maximum thickness	$4(0.007 + 0.0015)$ in.	=	0.034 in. = 0.864 mm

The tolerance of the outer dimension of the conductor is listed as 0.004 in. = 0.100 mm so that the dimensions of a *wrapped* conductor are:

Minimum	Conductor dimension + 0.022 in. − 0.004 in.	=	Conductor dimension + 0.018 in.
Nominal	Conductor dimension + 0.028 in.		
Maximum	Conductor dimension + 0.034 in. + 0.004 in.	=	Conductor dimension + 0.038 in.

We further allow

a) a gap between layers of 0.010 in. (0.254 mm) maximum
b) for keystoning, assume 0.010 in. (0.254 mm)
c) a 4-turn ground wrap of 0.007 in. (0.178 mm) tape

Then the *width* of the coil is obtained from

	Anaconda 0.4600		Anaconda 0.5160	
	Maximum	Minimum	Maximum	Minimum
Wrapped conductor	4.482 in.	4.302 in.	4.986 in.	4.806 in.
Gapping (8x0.10)	0.080 in.		0.080 in.	
Ground wrap (4x0.178x2)	0.056 in.	0.056 in.	0.056 in.	0.056 in.
Total (in.)	4.618 in.	4.358 in.	5.122 in.	4.862 in.

The average coil width is 4.488 in. [113.995 mm] for the 0.4600 in. conductor and 4.992 in. [126.797 mm] for the 0.5160 in. one. We take

	Anaconda 0.4600		Anaconda 0.5160	
Maximum coil width	4.625 in.	[117.5 mm]	5.250 in.	[133.4 mm]
Nominal coil width	4.500 in.	[114.3 mm]	5.000 in.	[127.0 mm]

The *height* of the coil is

	Anaconda 0.4600		Anaconda 0.5160	
	Maximum	Minimum	Maximum	Minimum
Wrapped conductor	5.976 in.	5.736 in.	6.648 in.	6.408 in.
Gapping (11x0.10)	0.110 in.		0.080 in.	
Keystoning (12x0.010)	0.120 in.	0.060 in.	0.120 in.	0.060 in.
Ground wrap (4x0.178x2)	0.056 in.	0.056 in.	0.056 in.	0.056 in.
Total	6.262 in.	5.852 in.	6.934 in.	6.524 in.

The average coil height is 6.057 in. [153.848 mm] for the 0.4600 in. conductor and 6.729 in. [176.124 mm] for the 0.5160 in. one. We take

	Anaconda 0.4600		Anaconda 0.5160	
Maximum coil height	6.300 in.	[160.0 mm]	7.000 in.	[177.8 mm]
Nominal coil height	6.100 in.	[154.9 mm]	6.800 in.	[172.7 mm]

We take the conductor dimension D to be

$$D = \text{Nominal dimension} + 4(\text{Insulation thickness}) + \text{Turn separation}$$

so that we then have for the 0.4600 in. conductor

$$\begin{aligned} D &= 0.460 \text{ in.} + 0.028 \text{ in.} + 0.010 \text{ in.} \\ &= 0.498 \text{ in. } [12.65 \text{ mm}] \end{aligned}$$

and for the 0.5160 in. conductor

$$\begin{aligned} D &= 0.516 \text{ in.} + 0.028 \text{ in.} + 0.010 \text{ in.} \\ &= 0.554 \text{ in. } [14.07 \text{ mm}]. \end{aligned}$$

Assuming a pole-coil gap of $G = 0.75$ in. [19 mm], then the *outer* edge of the n^{th} conductor is a distance

$$D_n = G + nD + 4t_i$$

perpendicular to the pole edge.

In order to calculate the length of the coil, we assume that it is wound as illustrated in the diagram below.

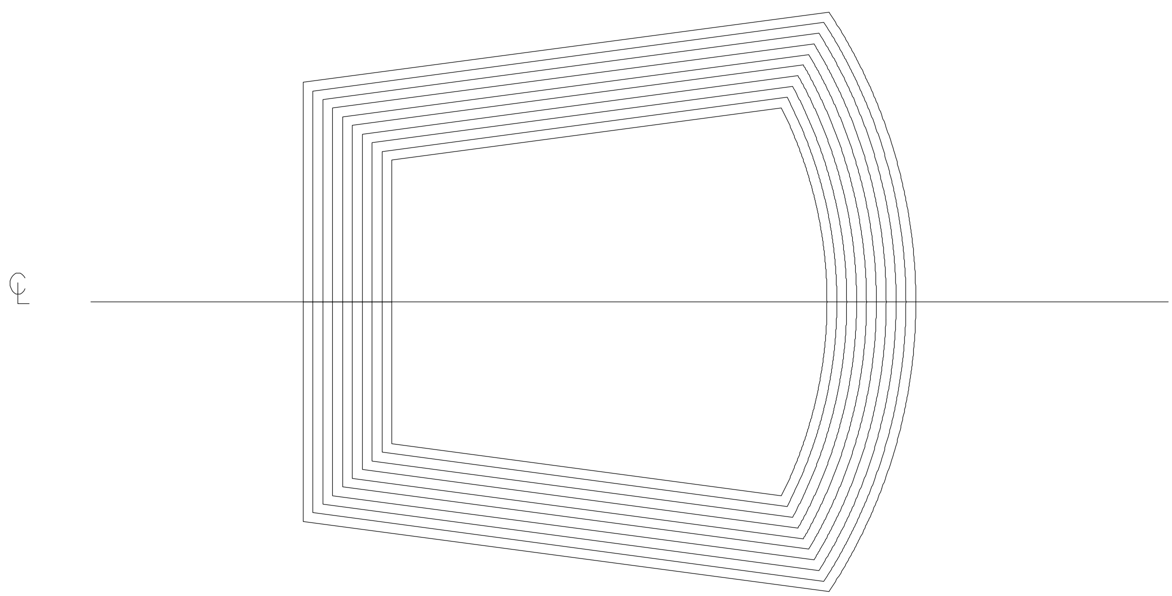

Fig. 2. Assumed method of coil winding

The half-length of the coil on the entrance edge of the dipole is the value of y at the intersection of the lines given by the equations

$$\begin{aligned} x &= -D_n \\ y &= x \tan\frac{\theta}{2} + \frac{W_{iron}}{2} + \frac{D_n}{\cos(\theta/2)} \end{aligned}$$

The point of intersection of these two lines is

$$\begin{aligned} x_1 &= -D_n \\ y_1 &= x_1 \tan\frac{\theta}{2} + \frac{W_{iron}}{2} + \frac{D_n}{\cos(\theta/2)} \end{aligned}$$

and thus the length of the n^{th} turn along the entrance side of the magnet is

$$\begin{aligned} l_{in} &= 2\, y_1 \\ &= W_{iron} + 2\, D_n \left[\frac{1 - \sin(\theta/2)}{\cos(\theta/2)}\right] \end{aligned}$$

The length of the straight longitudinal section of the winding is obtained in a similar manner; it is determined by the above point of intersection and that of the curves given by the following two equations.

$$\begin{aligned} x^2 + y^2 &= R_n^2 \\ y &= x \tan\frac{\theta}{2} + K_n \end{aligned}$$

where we have written

$$\begin{aligned} R_n &= R + D_n \\ K_n &= \frac{W_{iron}}{2} + \frac{D_n}{\cos(\theta/2)} \end{aligned}$$

These curves intersect at positive values of x and y of

$$\begin{aligned} x_2 &= \frac{\sqrt{R_n^2[1+\tan^2(\theta/2)] - K_n^2} - K_n \tan(\theta/2)}{1+\tan^2(\theta/2)} \\ y_2 &= \frac{\tan(\theta/2)\sqrt{R_n^2[1+\tan^2(\theta/2)] - K_n^2} + K_n}{1+\tan^2(\theta/2)} \end{aligned}$$

and the length of the n^{th} section along the angled pole-edge is

$$l_{str} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The length of the curved section of the n^{th} turn is

$$l_{curv} = 2\, R_n\, \Theta_n$$

where

$$\Theta_n = \tan^{-1}\frac{y_2}{x_2}$$

Thus the length of the n^{th} turn is

$$l_n = l_{in} + 2\, l_{str} + l_{curv}$$

and the length of an N-turn layer is simply

$$L_N = \sum_{n=1}^{N} l_n$$

These relationships are easily programmed. When this is done and the appropriate values inserted we find the following for a 9-turn layer.

Parameter	0.4600 in.		0.5160 in.	
W_{iron}	13.250	in.	13.250	in.
R	22.750	in.	22.750	in.
D	0.498	in.	0.554	in.
l_9	885.	in.	905.	in.

Because the lengths are only slightly different for the two conductors, we take

$$\boxed{\text{Length of 9-turn layer of either conductor} = 924 \text{ in.} = 77 \text{ ft} \approx 23.5 \text{ m.}}$$

where extra has been allowed for additional bends. The length per coil becomes

$$\boxed{\text{Length per coil of either conductor} = 925 \text{ ft} \approx 282 \text{ m.}}$$

Because two coils are required per dipole, then

Total length per dipole	1,850 ft	$\approx$	564 m
Allow 10% for winding losses	185 ft	$\approx$	56 m
Total	2,035 ft	$\approx$	620 m

Then order

$$\boxed{\text{Total length of copper} = 2{,}100 \text{ ft} \approx 640 \text{ m}}$$

of conductor. The required mass of the 0.4600 conductor at 0.5910 lb/ft is 1,241 lb [563 kg] and that of the 0.5160 conductor at 0.7495 lb/ft is 1,574 lb [714 kg].

Total mass of 0.4600 conductor	=	1,250 lb $\approx$ 570 kg.
Total mass of 0.5160 conductor	=	1,600 lb $\approx$ 725 kg.

6. Power requirements

At 20°C, the resistance of a coil of the 0.4600 in. conductor is

$$R_{20^\circ} = 53.25 \times 10^{-6}\ \Omega/\text{ft} \times 925 \text{ ft} = 0.04926\ \Omega$$

and that of a coil of the 0.5160 in. conductor is

$$R_{20^\circ} = 41.99 \times 10^{-6}\ \Omega/\text{ft} \times 925 \text{ ft} = 0.03884\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40° C coolant temperature rise). Then the mean coil temperature will be 50°C.

With a 30°C rise above ambient of the coil we then have:

$$R_{hot} = R_{20^\circ}[1 + (\text{Temp. coeff}/^\circ\text{C})dT(^\circ\text{C})]$$

so that for the coil made of the 0.4600 in. conductor

$$R_{hot} = 0.04926[1 + (0.00393)(30)] = 0.05506\ \Omega \text{ per coil}$$

and that for the coil made of the 0.5160 in. conductor

$$R_{hot} = 0.03884[1 + (0.00393)(30)] = 0.04342\ \Omega \text{ per coil}$$

Thus, at a current of 600 A, we obtain

$$\text{Voltage per coil} = 33.04 \text{ V for the 0.4600 in. conductor}$$
$$\text{Voltage per coil} = 26.05 \text{ V for the 0.5160 in. conductor}$$

Therefore, allowing for a 10% lead loss, we choose a power supply that has

	Anaconda 0.4600	Anaconda 0.5160
I (A minimum)	600	600
V (V minimum)	75	60
P (kW minimum)	45	33

7. Cooling requirements

In these calculations we use the British system of units.

The power required per coil of the 0.4600 in. conductor is

$$\text{Power per coil} = I^2 R_{hot} = (600)(600)(0.05506) = 19.82 \text{ kW}.$$

and that required per coil of the 0.5160 in. conductor is

$$\text{Power per coil} = I^2 R_{hot} = (600)(600)(0.04342) = 15.63 \text{ kW}.$$

The required flow rate is given by:

$$\begin{aligned} v \text{ (ft/sec)} &= \frac{2.19}{\Delta T(^\circ \text{ F})} \times \frac{P(\text{kW})}{\text{Cooling area (in}^2)} \\ &= 0.0304167 \times \frac{P(\text{kW})}{A_c \text{ (in.}^2)} \end{aligned}$$

for $\Delta T = 72°\text{F} = 40°\text{C}$. The 0.4600 in. conductor has a cooling area of $A_c = 0.05107$ in.^{2} [32.948 mm^2] and the 0.5160 in. conductor one of $A_c = 0.06469$ in^2 [41.735 mm^2]. Choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit we have:

$$\begin{aligned} P_{max} &= \frac{(2.50)(72)(0.05107)}{2.19} = 4.198 \text{ kW/water circuit [0.4600 in. conductor]} \\ &= \frac{(2.50)(72)(0.06469)}{2.19} = 5.317 \text{ kW/water circuit [0.5160 in. conductor]} \end{aligned}$$

from which we calculate the number of cooling circuits per coil (excluding lead loss) as

				Anaconda 0.4600	Anaconda 0.5160
P	=	Total power per coil		19.82 kW	15.63 kW
Number of circuits	=	P / P_{max}	=	4.72	2.94

Thus we take

Number of cooling circuits per coil	=	6 for the 0.4600 in. conductor
	=	3 for the 0.5160 in. conductor

This requires a flow rate of $v = 1.965$ ft/sec per water circuit of the 0.4600 in. conductor and a flow rate of $v = 2.450$ ft/sec per water circuit of the 0.5160 in. conductor. The volume of flow required per circuit is

$$\begin{aligned} \text{Volume/circuit} &= v \, \frac{\text{ft}}{\text{sec}} \times A_{H_2O} \, (\text{in}^2) \times 60 \, \frac{\text{sec}}{\text{min}} \times \frac{1}{144} \, \frac{\text{ft}^2}{\text{in}^2} \times 62.4 \, \frac{\text{lb}}{\text{ft}^3} \times \frac{1}{10} \, \frac{\text{IG}}{\text{lb}} \times 1.20095 \, \frac{\text{USG}}{\text{IG}} \\ &= 3.1225 \, v \, (\text{ft/sec}) \times \text{Cooling area (in}^2) \text{ USGPM} \end{aligned}$$

Thus we have the following volumes of flow.

	0.4600 in. conductor		0.5160 in. conductor	
Volume per cooling circuit	0.313 USGPM	1.186 ℓ/min	0.495 USGPM	1.873 ℓ/min
Volume per coil	1.880 USGPM	7.118 ℓ/min	1.485 USGPM	5.619 ℓ/min
Volume per magnet	3.761 USGPM	14.235 ℓ/min	2.969 USGPM	11.238 ℓ/min

8. Pressure drop

The pressure drop is given by

$$dP = k \, v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case, for the 0.4600 in. conductor with $k = 0.0176$ and $v = 1.965$ ft/sec we obtain

$$\Delta P = (0.0176)(1.965)^{1.79} = 0.0590 \text{ psi/ft} = 0.1935 \text{ psi/m}.$$

For the 0.51400 in. conductor with $k = 0.0152$ and $v = 2.450$ ft/sec we obtain

$$\Delta P = (0.0152)(2.450)^{1.79} = 0.0756 \text{ psi/ft} = 0.2480 \text{ psi/m}.$$

and the total pressure drop across one cooling circuit is:

$$\begin{aligned}\text{Pressure drop per cooling circuit} &= 0.0590\ \text{psi/ft} \times (925/6)\ \text{ft} = 9.10\ \text{psi}\ [0.4600\ \text{in. conductor}] \\ &= 0.0756\ \text{psi/ft} \times (975/3)\ \text{ft} = 24.56\ \text{psi}\ [0.5160\ \text{in. conductor}]\end{aligned}$$

9. Iron dimensions

We propose to construct the top and bottom yokes of pieces of iron that completely cover the pole as illustrated below. Also indicated is the assumed field distribution at the mid-point of the pole.

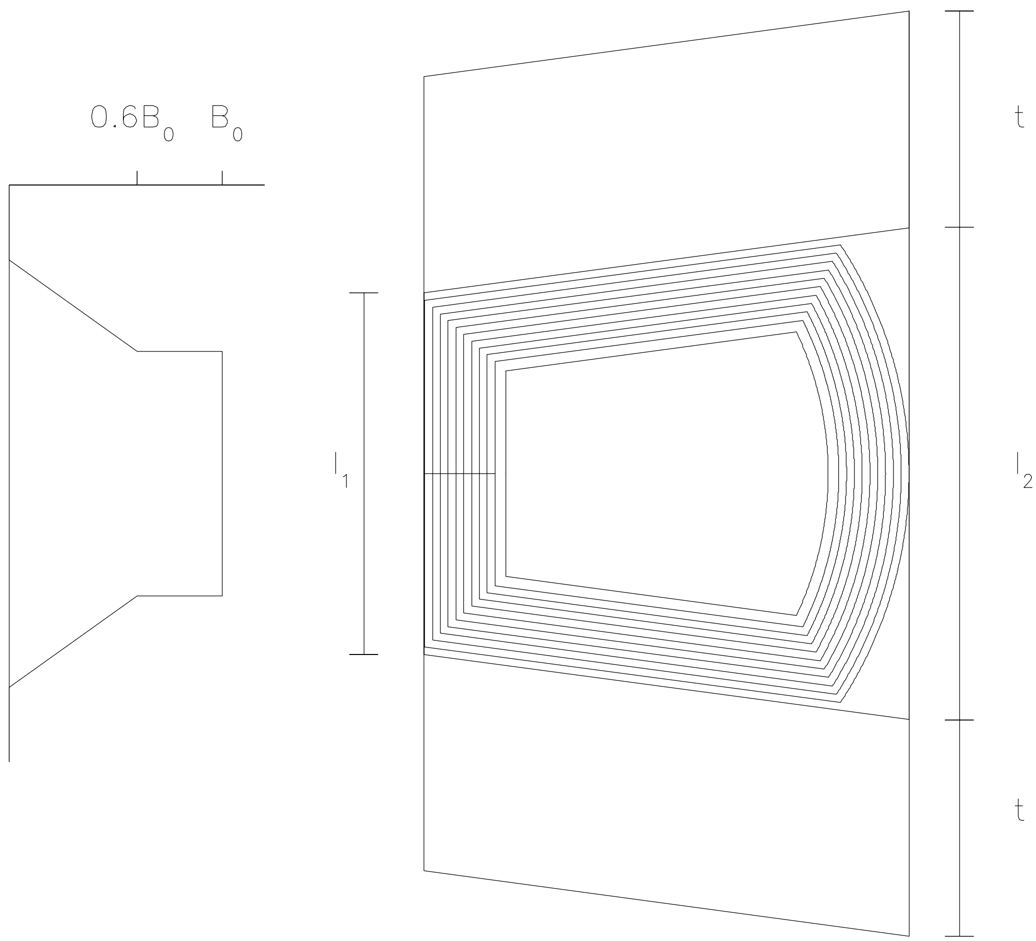

Fig. 3. Cross section of the magnet and the assumed field profile.

Thus the top and bottom yokes are trapezoids whose parallel sides are of lengths $l_1 + 2t$ and $l_2 + 2t$. The spacers are parallelograms with one side of length t. The length t is to be determined from the consideration that we want the flux to distribute itself equally between the two spacers.

We begin by calculating the flux through the pole face. Using the notation from page 2, the pole area is given by

$$A_{pole} = \frac{1}{2}(\overline{\mathrm{AF}} + \overline{\mathrm{BE}})x_{int} + \frac{1}{2}R^2(\Theta - \sin\Theta)$$

where

$$\Theta = 2\tan^{-1}\left[\frac{y_{int}}{x_{int}}\right]$$

In this instance, $\overline{\mathrm{AF}} = 13.250$ in., $\overline{\mathrm{BE}} = 18.710$ in., $x_{int} = 20.738$ in. and $y_{int} = 9.355$ in. Thus

$$\Theta = 2\tan^{-1}\left[\frac{9.355}{20.738}\right] = 48.561^\circ = 0.84754 \text{ radian}$$

and

$$\begin{aligned} A_{pole}\ (\text{in.}^2) &= [(13.250 + 18.710)(20.738) + (22.750)^2(0.84754 - 0.74966)]/2 \\ &= [662.786 + 50.6611]/2 \\ &= 356.724 \text{ in.}^2 \end{aligned}$$

Then the magnetic flux through the pole is

$$\Phi_{pole} = (BA)_{pole} = (14.307 \text{ kG})(356.724 \text{ in.}^2) = 5,105 \text{ kG-in.}^2$$

$$\boxed{\Phi_{pole} = 5{,}110 \text{ kG-in.}^2 = 0.329 \text{ T-m}^2}$$

To this we add the flux through the coil slot. As is illustrated in the previous figure, the magnetic field profile has been assumed to rise linearly from zero at the inside edge of the yoke to a value of $0.6B_0$ at the flat part of the pole. At that point the field rises to the full value in the gap B_0 and remains at that value to the outer edge of the pole. At that point the field is assumed to abruptly drop to a value of $0.6B_g$ and to fall linearly to zero at the outer edge of the outer coil.

Thus the average field in the coil slot is $0.3B_0$.

The average length of the coil-slot along the slanted pole-edge is taken as the length of the fifth turn—that is, the length of the line defined by

$$y = x\tan(\theta/2) + W_{iron}/2 + (G + 4t_i + 5D)/\cos(\theta/2)$$

and the lines

$$x_1 = -(G + 4t_i + 5D) \qquad \text{and} \qquad x_2 = R + G + 4t_i + 5D$$

where, again, t_i is the insulation thickness. The points of intersection are found to be

$$\begin{array}{lll} x_1 = -3.268 \text{ in.} & \text{and} & y_1 = 9.491 \text{ in.} \\ x_2 = 26.018 \text{ in.} & \text{and} & y_2 = 13.347 \text{ in.} \end{array}$$

for the 0.4600 in. conductor and

$$\begin{array}{lll} x_1 = -3.548 \text{ in.} & \text{and} & y_1 = 9.737 \text{ in.} \\ x_2 = 26.298 \text{ in.} & \text{and} & y_2 = 13.666 \text{ in.} \end{array}$$

for the 0.5160 in. conductor. Then the (average) lengths of the side coil-slots, l_{side}, are

$$\begin{aligned} l_{side} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \begin{cases} \sqrt{(26.018 + 3.268)^2 + (13.347 - 9.491)^2} = 29.539 \text{ in. [0.4600 in. case].} \\ \\ \sqrt{(26.298 + 3.548)^2 + (13.666 - 9.737)^2} = 30.104 \text{ in. [0.5160 in. case].} \end{cases} \end{aligned}$$

The width of the side coil-slot is calculated from

$$\begin{aligned} \text{Side coil-slot width} &= \text{Maximum coil width} + 2(\text{Pole-coil separation}) \\ &= \begin{cases} 4.625 \text{ in.} + 2(0.750) \text{ in.} = 6.125 \text{ in. [0.4600 in. case].} \\ 5.250 \text{ in.} + 2(0.750) \text{ in.} = 6.750 \text{ in. [0.5160 in. case].} \end{cases} \end{aligned}$$

and the total flux through the side coil-slots becomes

$$\Phi_{sides} = 2(\text{Length of slot})(\text{Width of slot})(0.3\,B_0)$$
$$= \begin{cases} 2(29.539\text{ in.})(6.125\text{ in.})(0.3(14.307\text{ kG})) = 1553.1\text{ kG-in.}^2\ [0.4600\text{ in. case}]. \\ 2(30.104\text{ in.})(6.750\text{ in.})(0.3(14.307\text{ kG})) = 1744.3\text{ kG-in.}^2\ [0.5160\text{ in. case}]. \end{cases}$$

The average length of the entrance coil-slot is taken as

$$l_{entry} = [2y_1 + W_{iron}]/2$$
$$\begin{cases} [2(9.491\text{ in.}) + 13.250\text{ in.}]/2 = 16.116\text{ in. } [0.4600\text{ in. case}] \\ [2(9.737\text{ in.}) + 13.250\text{ in.}]/2 = 16.362\text{ in. } [0.5160\text{ in. case}] \end{cases}$$

and that of the exit coil-slot is taken as

$$l_{exit} = [2y_2 + \overline{\mathrm{BE}}]/2$$
$$\begin{cases} [2(13.347\text{ in.}) + 18.710\text{ in.}]/2 = 22.702\text{ in. } [0.4600\text{ in. case}]. \\ [2(13.666\text{ in.}) + 18.710\text{ in.}]/2 = 23.021\text{ in. } [0.5160\text{ in. case}]. \end{cases}$$

The widths of the coil slots on entrance and exit faces are taken as

$$\text{Entry and exit coil-slot widths} = 9D + G + 8t_i$$
$$= \begin{cases} 9(0.498)\text{ in.} + 0.750\text{ in.} + 8(0.007) = 5.288\text{ in. } [0.4600\text{ in. case}]. \\ 9(0.554)\text{ in.} + 0.750\text{ in.} + 8(0.007) = 5.792\text{ in. } [0.5160\text{ in. case}]. \end{cases}$$

Then the total flux through the entry coil-slot becomes

$$\Phi_{entry} = (\text{Length of slot})(\text{Width of slot})(0.3\,B_0)$$
$$= \begin{cases} (16.116\text{ in.})(5.288\text{ in.})(0.3(14.307\text{ kG})) = 365.8\text{ kG-in.}^2\ [0.4600\text{ in. case}] \\ (16.362\text{ in.})(5.792\text{ in.})(0.3(14.307\text{ kG})) = 406.8\text{ kG-in.}^2\ [0.5160\text{ in. case}] \end{cases}$$

and that through the exit coil-slot becomes

$$\Phi_{entry} = (\text{Length of slot})(\text{Width of slot})(0.3\,B_0)$$
$$= \begin{cases} (22.702\text{ in.})(5.288\text{ in.})(0.3(14.307\text{ kG})) = 515.3\text{ kG-in.}^2\ [0.4600\text{ in. case}]. \\ (23.021\text{ in.})(5.792\text{ in.})(0.3(14.307\text{ kG})) = 572.3\text{ kG-in.}^2\ [0.5160\text{ in. case}]. \end{cases}$$

The total flux through the coil slots is obtained from the following table.

Flux in	0.4600 in. conductor	0.5160 in. conductor
Side slots (kG-in.^{2})	1553.1	1744.3
Entrance slot (kG-in.^{2})	365.8	406.8
Exit slot (kG-in.^{2})	515.3	572.3
Total flux (kG-in.^{2})	2434.2	2723.4

Thus the total flux through the pole and coil slots, Φ_{total}, is estimated to be

$$\Phi_{total} = \text{Flux through pole} + \text{flux through coil slots}$$
$$= \begin{cases} 5110\text{ kG-in.}^2 + 2450\text{ kG-in.}^2 = 7560\text{ kG-in.}^2 & [0.4600\text{ in. case}]. \\ 5110\text{ kG-in.}^2 + 2740\text{ kG-in.}^2 = 7850\text{ kG-in.}^2 & [0.5160\text{ in. case}]. \end{cases}$$

We take

$$\boxed{\begin{aligned} \Phi_{total} &= 7{,}600\text{ kG-in.}^2 \quad [0.4600\text{ in. case}] \\ &= 7{,}900\text{ kG-in.}^2 \quad [0.5160\text{ in. case}]. \end{aligned}}$$

The length of the dipole along the longitudinal symmetry axis has been taken as

$$l_{axis} = R + 2(9D + G + 8t_i)$$

so that

$$l_{axis} = \begin{cases} 22.750 + 2[9(0.498) + 0.750 + 8(.007) = 33.326 \text{ in. for the } 0.4600 \text{ in. conductor.} \\ 22.750 + 2[9(0.554) + 0.750 + 8(.007) = 34.334 \text{ in. for the } 0.5160 \text{ in. conductor.} \end{cases}$$

The length along the outer edge of the dipole is

$$\begin{aligned} l_{outer} &= l_{axis}/\cos(\theta/2) \\ &= \begin{cases} 33.326/0.99144 \text{ in.} = 33.614 \text{ in. for the } 0.4600 \text{ in. conductor.} \\ 34.334/0.99144 \text{ in.} = 34.630 \text{ in. for the } 0.5160 \text{ in. conductor.} \end{cases} \end{aligned}$$

We average these and set the length of the cross section of the dipole to be

$$l_{av} = \begin{cases} 33.470 \text{ in. for the } 0.4600 \text{ in. conductor.} \\ 34.482 \text{ in. for the } 0.5160 \text{ in. conductor.} \end{cases}$$

With the requirement that the flux divide evenly between the side yokes and with t_y the thickness of the top yoke and B_y the field in it, we have

$$(\text{Cross-sectional area of yoke})(\text{Field in yoke}) = l_{av}\, t_y\, B_y = \begin{cases} 7,600/2 \text{ kG-in.}^2 \text{ [0.4600 in. case]} \\ 7,900/2 \text{ kG-in.}^2 \text{ [0.5160 in. case]} \end{cases}$$

or

$$t_y\, B_y = \begin{cases} 113.535 \text{ kG-in. [0.4600 in. case]} \\ 114.553 \text{ kG-in. [0.5160 in. case]} \end{cases}$$

The thicknesses of the spacers is determined in a similar manner. With t_s the width of the spacer, B_s the field in it and l_{outer} the length of each spacer we have

$$(\text{Area of spacer})(\text{Field in spacer}) = \begin{cases} (33.614\, t_s \sin([\pi - \theta]/2) B_s = 7,600/2 \text{ kG-in.}^2 \text{ [0.4600 in. case].} \\ (34.630\, t_s \sin[(\pi - \theta]/2) B_s = 7,900/2 \text{ kG-in.}^2 \text{ [0.5160 in. case].} \end{cases}$$

or

$$t_s\, B_s = \begin{cases} 114.024 \text{ kG-in. [0.4600 in. case]} \\ 115.047 \text{ kG-in. [0.4600 in. case]} \end{cases}$$

Because the field-thickness product is essentially the same for the top and bottom yokes and the spacers, we choose to make them of equal thicknesses. Taking the product as 120 kG-in. for either case, we make the following table.

B_y (kG)	10.	11.	12.	13.
t_y (in.)	12.00	10.91	10.00	9.23

We choose

Yoke field	B_y	10.435 kG	1.044 T
Yoke thickness	t	11.500 in.	0.279 m

The lengths l_1 and l_2 of figure 3 are found by considering the intersection of the line

$$y = x\tan(\theta/2) + W_{iron}/2 + [2(G + 4t_i) + \text{maximum coil width}]/\cos(\theta/2)$$

with the lines

$$x_1 = -(G + 8t_i + 9D) \qquad \text{and} \qquad x_2 = R + G + 8t_i + 9D$$

Those points are found to be

$$\begin{aligned} x_{in} &= -5.288 \text{ in.} & \text{and} \quad y_{in} &= 12.163 \text{ in.} \\ x_{exit} &= 28.038 \text{ in.} & \text{and} \quad y_{exit} &= 16.551 \text{ in.} \end{aligned}$$

in the case 0f the 0.4600 in. conductor and

$$x_{in} = -5.792 \text{ in.} \quad \text{and} \quad y_{in} = 12.671 \text{ in.}$$
$$x_{exit} = 28.542 \text{ in.} \quad \text{and} \quad y_{exit} = 17.191 \text{ in.}$$

in the case 0f the 0.5160 in. conductor. The lengths of the parallel sides of the upper and lower yokes are then, with t their thicknesses,

$$\begin{aligned} l_1 &= 2y_{in} \\ &= \begin{cases} 2(12.163) \text{ in.} = 24.326 \text{ in. } [0.4600 \text{ in. case}]. \\ 2(12.671) \text{ in.} = 25.342 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

$$\begin{aligned} l_2 &= 2[y_{exit} + t] \\ &= \begin{cases} 2(16.551) \text{ in.} = 33.102 \text{ in. } [0.4600 \text{ in. case}]. \\ 2(17.191) \text{ in.} = 34.382 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

Then the maximum width of the dipole is

$$\begin{aligned} \text{Maximum width} &= l_2 + 2t \\ &= \begin{cases} 33.102 + 2(11.500) = 56.102 \text{ in. } [0.4600 \text{ in. case}]. \\ 34.382 + 2(11.500) = 57.382 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

and, for future reference, the minimum width of the dipole is

$$\begin{aligned} \text{Minimum width} &= l_1 + 2t \\ &= \begin{cases} 24.326 + 2(11.500) = 47.326 \text{ in. } [0.4600 \text{ in. case}]. \\ 25.342 + 2(11.500) = 48.342 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

We take

	0.4600 in.	0.5160 in.
Maximum dipole width	56.100 in.	57.500 in.
Minimum dipole width	47.400 in.	48.500 in.

The overall length of the dipole has been calculated as

$$\begin{aligned} \text{Dipole length} &= R + 2(9D + G + 8t_i) \\ &= \begin{cases} 22.750 + 2[9(0.498) + 0.750 + 8(.007) = 33.326 \text{ in. for the } 0.4600 \text{ in. conductor.} \\ 22.750 + 2[9(0.554) + 0.750 + 8(.007) = 34.334 \text{ in. for the } 0.5160 \text{ in. conductor.} \end{cases} \end{aligned}$$

and the pole-height is obtained from

$$\begin{aligned} \text{Pole height} &= \text{Maximum coil height} + \text{Chamfer} + 15 \text{ mm} \\ &= \begin{cases} 6.300 \text{ in.} + 0.625 \text{ in.} + 0.591 \text{ in.} = 7.516 \text{ in. } [0.4600 \text{ in. case}]. \\ 7.000 \text{ in.} + 0.625 \text{ in.} + 0.591 \text{ in.} = 8.216 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

The height of the side yokes is

$$\begin{aligned} \text{Side-yoke height} &= 2(\text{Pole height}) + \text{Gap} \\ &= \begin{cases} 2(7.500 \text{ in.}) + 4.000 \text{ in.} = 19.000 \text{ in. } [0.4600 \text{ in. case}]. \\ 2(8.200 \text{ in.}) + 4.000 \text{ in.} = 20.400 \text{ in. } [0.5160 \text{ in. case}]. \end{cases} \end{aligned}$$

where the pole height has taken as 7.500 in. [190.5 mm] in the case of the 0.4600 in. conductor and 8.200 in. [208.3 mm] in the case of the 0.5160 in. conductor. We take

	0.4600 in. conductor		0.5160 in. conductor	
Coil-slot width	6.125 in.	155.6 mm	6.750 in.	171.5 mm
Pole height	7.500 in.	190.5 mm	8.200 in.	208.3 mm
Side-yoke height	19.000 in.	482.6 mm	20.400 in.	518.2 mm
Maximum dipole width	56.100 in.	1424.9 mm	57.500 in.	1460.5 mm
Minimum dipole width	47.400 in.	1204.0 mm	48.500 in.	1231.9 mm
Dipole length	33.400 in.	848.4 mm	34.400 in.	873.8 mm

10. Iron weight

We are now in a position to estimate the amount of iron required. We have

$$\begin{aligned}\text{Area of top or bottom yoke} &= \text{(Average length of parallel sides)(Longitudinal length)}\\ &= \begin{cases}(56.100+47.400)(33.350)/2 = 1,728.5\text{ in.}^2\ [0.4600\text{ in. case}].\\ (57.500+48.500)(34.400)/2 = 1,823.2\text{ in.}^2\ [0.5160\text{ in. case}].\end{cases}\end{aligned}$$

We take

$$\boxed{\begin{aligned}\text{Area of top or bottom yoke} &= 1750\text{ in.}^2 = 1.1290\text{ m}^2\ \ [0.4600\text{ in. case}].\\ &= 1850\text{ in.}^2 = 1.1935\text{ m}^2\ \ [0.5160\text{ in. case}].\end{aligned}}$$

The area of each spacer is obtained in a similar manner

$$\begin{aligned}\text{Area of spacer} &= \text{(Short side)(Long side)sin(Contained angle)}\\ &= \begin{cases}(11.500\text{ in.})(33.614\text{ in.})\sin(82.5) = 383.254\text{ in.}^2\ [0.4600\text{ in. case}].\\ (11.500\text{ in.})(34.630\text{ in.})\sin(82.5) = 394.838\text{ in.}^2\ [0.5160\text{ in. case}].\end{cases}\end{aligned}$$

We take

$$\boxed{\begin{aligned}\text{Area of spacer} &= 385\text{ in.}^2 = 0.2484\text{ m}^2\ \ [0.4600\text{ in. case}].\\ &= 400\text{ in.}^2 = 0.2581\text{ m}^2\ \ [0.4600\text{ in. case}].\end{aligned}}$$

Thus we have

Section	0.4600 in. conductor			0.5160 in. conductor		
	Area (in.^{2})	Height (in.)	Volume (in.^{3})	Area (in.^{2})	Height (in.)	Volume (in.^{3})
Top yoke	1,750.0	11.5	20,125.0	1850.0	11.5	21,275.0
Bottom yoke	1,750.0	11.5	20,125.0	1850.0	11.5	21,275.0
Vertical Yoke	385.0	19.0	7,315.0	400.0	20.4	8,160.0
Vertical Yoke	385.0	19.0	7,315.0	400.0	20.4	8,160.0
Top pole	356.8	7.5	2,676.0	356.8	8.2	2,925.8
Bottom pole	356.8	7.5	2,676.0	356.8	8.2	2,925.8
Total volume			60,232.0			64,721.5

We take the total volume of iron required to be

$$\text{Total volume of iron} = \begin{cases}60,500\text{ in.}^3 = 35.012\text{ ft}^3 = 0.9914\text{ m}^3\ [0.4160\text{ in. case}].\\ 65,000\text{ in.}^3 = 37.616\text{ ft}^3 = 1.0652\text{ m}^3\ [0.5160\text{ in. case}].\end{cases}$$

and the iron mass is obtained from

$$\text{Iron mass} = \text{(Iron volume)(Density)}$$

or, at a density of 7,900 kg/m^3,

$$\text{Iron mass} = \begin{cases} (0.9914\ \text{m}^3)(7,900\ \text{kg/m}^3) = 7.832\times 10^3\ \text{kg} = 17.270\times 10^3\ \text{lb [0.4600 in. case]}. \\ (1.0652\ \text{m}^3)(7,900\ \text{kg/m}^3) = 8.415\times 10^3\ \text{kg} = 18.555\times 10^3\ \text{lb [0.5160 in. case]}. \end{cases}$$

We take

$$\boxed{\begin{aligned} \text{Iron mass} &= 17.5\times 10^3\ \text{lb} = 7.94\times 10^3\ \text{kg [0.4600 in. case]} \\ &= 18.8\times 10^3\ \text{lb} = 8.53\times 10^3\ \text{kg [0.5160 in. case]} \end{aligned}}$$

11. Discussion

This hote has presented a conceptual design for a $\pm 15°$ switching magnet for use on beam line 2A. In this concept, the possibility of the use of one of two conductor sizes has been considered.

Use of the smaller, 0.4600 in. conductor would result in a slightly smaller total weight of iron and copper. Relative to the larger 0.5160 in. conductor, it is estimated that approximately 350 fewer pounds of copper and 1,300 fewer pounds of iron are necessary for a magnet made with the smaller conductor. Assuming a raw material cost pf $10/lb for copper and $1/lb for iron, this would reflect a manufacturing cost of approximately $5,000 less for a magnet made with the smaller conductor.

On the other hand, a magnet made with the larger conductor is estimated to require approximately 12 kW less power when operating. Thus, in the long term it may be less expensive to use the larger conductor.

Further, the larger conductor has been chosen for the coils of the vault dipoles because the smaller conductor cannot be used with the power supplies under consideration. Consequently, it is recommended that the switching magnet be constructed with a coil made of the 0.5160 in. conductor in order that a better price be obtained for the copper.

References

1. GM Stinson, *TRI-DNA-96-6*, March, 1996.
1. GM Stinson, *TRI-DNA-96-3*, January, 1996.

Table 1

Summary of $\pm 15°$ switching magnet design parameters

		0.460 in. conductor		0.516 in. conductor	
Yoke:	Iron length	33.400 in.	[848.4 mm]	34.400 in.	[873.8 mm]
	Iron width (max.)	56.100 in.	[1424.9 mm]	57.500 in.	[1460.5 mm]
	Iron width (min.)	47.400 in.	[1204.0 mm]	48.500 in.	[1231.9 mm]
	Iron thickness	11.500 in.	[292.1 mm]	11.500 in.	[292.1 mm]
	Coil-slot width	6.125 in.	[155.6 mm]	6.750 in.	[171.5 mm]
	Side-yoke height	19.000 in.	[482.6 mm]	20.400 in.	[518.2 mm]
Pole:	Width at entrance	13.250 in.	[336.6 mm]	13.250 in.	[336.6 mm]
	Height	7.500 in.	[190.5 mm]	8.200 in.	[208.3 mm]
	Chamfer at 45°	0.625 in.	]15.9 mm]	0.625 in.	[15.9 mm]
Iron:	Total mass	17.50×10^3 lb	[7.94×10^3 kg]	18.80×10^3 lb	[8.53×10^3 kg]
Dipole:	Overall width (max.)	56.100 in.	[1424.9 mm]	57.500 in.	[1460.5 mm]
	Overall height	42.000 in.	[1066.8 mm]	43.400 in.	[1102.4 mm]
	Overall length (incl. coil)	33.400 in.	[848.4 mm]	34.400 in.	[873.8 mm]
Coil:	Conductor OD	0.460 in.	[11.7 mm]	0.516 in.	[13.1 mm]
	Conductor ID	0.255 in.	[6.5 mm]	0.287 in.	[7.3 mm]
	Nominal coil width	4.500 in.	[114.3 mm]	5.000 in.	[127.0 mm]
	Nominal coil height	6.100 in.	[154.9 mm]	6.800 in.	[172.7 mm]
	Total coolant flow	3.761 USGPM	[14.2 ℓ/min]	2.969 USGPM	[11.2 ℓ/min]
	Turn configuration	9 wide $\times$ 12 high		9 wide $\times$ 12 high	
	Resistance (hot) per coil	55.06×10^{-3} Ω		43.42×10^{-3} Ω	
	Cooling circuits per coil	6		3	
Copper:	Total length per magnet	1,850 ft	[564 m]	1,850 ft	[564 m]
	Total mass per magnet	1100 lb	[500 kg]	1100 lb	[500 kg]
	Total length to order	2,100 ft	[640 m]	2,100 ft	[640 m]
	Total mass to order	1,250 lb	[570 kg]	1,600 lb	[725 kg]
Power:	Total current	600.0	A	600.0	A
	Total Voltage	75.0	V	60.0	V
	Power	45.0	kW	33.0	kW