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Subject A design for the quadrupoles for beam line 2A		

1. Introduction

Protons are to be extracted from extraction port 2A to deliver beam to an external ISAC facility. The current beamline design calls for quadrupoles of the 4Q9/8 type that were designed and manufactured by Alpha Magnetics. Because this design is of a proprietary nature, it was thought best to have a TRIUMF design available in order to assess costs. This report presents designs for a quadrupole that would be suitable for use on the beam line.

2. Design Parameters

TRANSPORT calculations require a quadrupole with an effective length of 0.260 m that is capable of producing a pole-tip field of 8.5 kG. We establish the following parameters for the quadrupole design.

Maximum pole-tip field	8.50 kG		
Full aperture	4.10 in.	=	0.1041 m
Effective length	10.24 in.	=	0.2600 m
Maximum current	310 A		

In this study several different sizes of conductor were considered. We illustrate the design procedure using an Anaconda 0.3648 in. square conductor. Results for the other conductor sizes are listed in table 1 at the end of this report.

3. Ampere-turns per Coil

Allowing 5% for stray fileds, we obtain the required Ampere-turns per pole from

$$NI \text{ per pole} = \frac{1.05}{2}(\text{half aperture})\frac{\text{Pole-tip field}}{4\,\pi\,10^{-7}} = 18,491$$

For a current of 310 Amperes we choose

$$\boxed{N = \text{number of turns per coil} = 60}$$

in the configuration illustrated on the next page. Following Banford[1)], we calculate iron parameters (with the nominal design parameters inbrackets) are

Yoke thickness (= 0.8 * aperture/2.)	1.650 in.	=	41.910 mm
Pole width (= 1.7 * aperture/2.)	3.500 in.	=	88.900 mm
Pole radius (= 1.15 * aperture/2.)	2.375 in.	=	60.325 mm
Pole length (= effective length - aperture/2.)	8.250 in.	=	209.550 mm

4. Coil Design

The copper parameters for the 0.3648 in. conductor are given in Anaconda data as follows.

OD	0.36480 in.	=	9.26592 mm
ID	0.20400 in.	=	5.18160 mm
Copper area	0.09730 in^2	=	62.77407 mm^2
Cooling area	0.03269 in^2	=	21.09028 mm^2
Mass	0.37597 lb/ft	=	0.55950 kg/m
Resistance at 20° C	83.70 $\mu\Omega$/ft	=	274.61 $\mu\Omega$/m
k (British units)	0.02320		

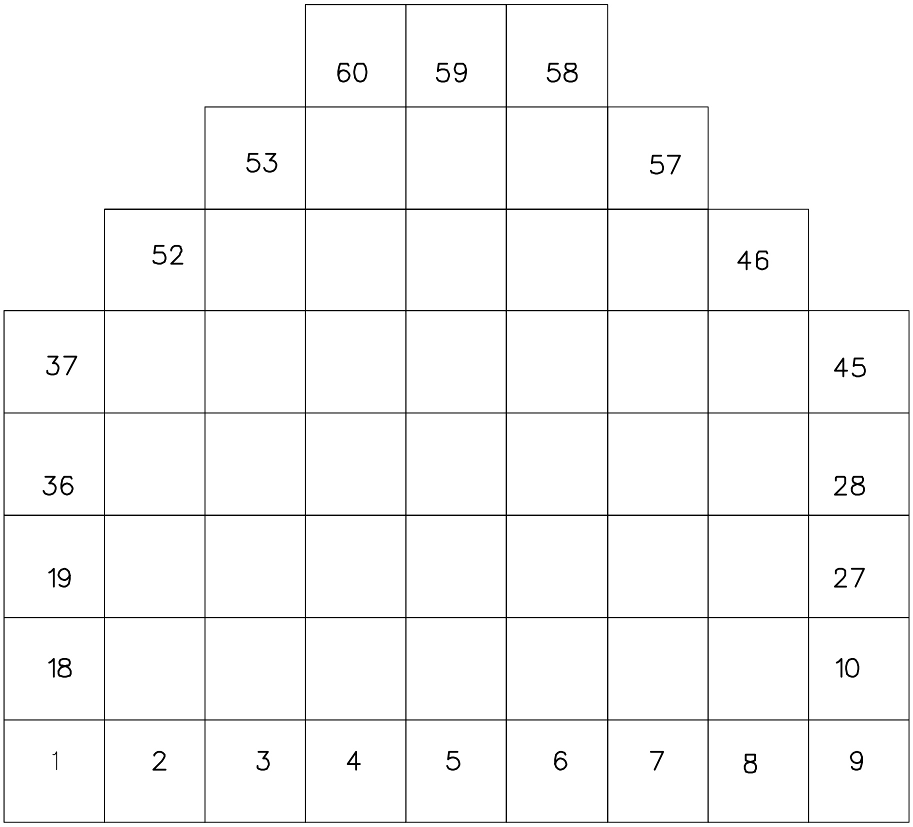

Fig. 1. Illustration of the coil configuration.

We assume that each conductor is double-wrapped with insulation 0.007 inch thickness such that the *total* insulation per conductor has:

Minimum thickness	=	0.022 in.	=	0.559 mm
Nominal thickness	=	0.028 in.	=	0.711 mm
Maximum thickness	=	0.034 in.	=	0.864 mm

The tolerance of the outer dimension of the conductor is listed as 0.003 in [= 0.076 mm] so that the dimensions of a *wrapped* conductor are:

Minimum	0.3838 in.	=	9.75 mm
Nominal	0.3928 in.	=	9.98 mm
Maximum	0.4018 in.	=	10.21 mm

We further allow

1. a gap between layers of 0.010 in. maximum.
2. for keystoning, assume 0.010 in.
3. a 4-turn ground wrap of 0.007 in. tape.

Then the length [in. (mm)] of the *long* side of the coil is

	Maximum	Minimum
Wrapped conductor	3.616 (91.851)	3.454 (87.737)
Gapping (8x0.010)	0.080 (2.032)	
Ground wrap (4x0.007x2)	0.056 (1.422)	0.056 (1.422)
Total (in)	3.752 (95.306)	3.510 (89.159)

The nominal coil width = 3.631 in. = 92.232 mm. We take

$$\boxed{\text{Coil width} = 3.650 \text{ in.} = 92.710 \text{ mm}}$$

The length [in (mm)] of the *short* side of the coil is obtained in a similar manner. We have

	Maximum	Minimum
Wrapped conductor	3.214 (81.646)	3.070 (77.988)
Gapping (7x0.010)	0.070 (1.778)	
Keystoning (8x0.010)	0.080 (2.032)	0.040 (1.016)
Ground wrap (4x0.007x2)	0.056 (1.422)	0.056 (1.422)
Total (in)	3.420 (86.878)	3.166 (80.427)

The nominal coil height = 3.293 in. = 83.652 mm. We take

$$\boxed{\text{Nominal coil height} = 3.350 \text{ in.} = 85.090 \text{ mm}}$$

The distance of the median turn from the coil boundary is found as follows.

$$\begin{aligned} S &= \sum [\text{number of turns/layer}][\text{layer number}] \\ &= 9(1) + 9(2) + 9(3) + 9(4) + 9(5) + 7(6) + 5(7) + 3(8) \\ &= 236 \end{aligned}$$

so that the mean turn is

$$< l >= \frac{S}{N} = \frac{236}{60} = 3.933$$

layers from the inner coil boundary. Assuming that the approximate width per conductor is equal to the maximum coil height divided by the number of layers, the distance of the median turn from the inner coil boundary is:

$$< x >= \frac{(3.933)(3.420)}{8} = 1.68 \text{ in} = 42.72 \text{ mm}$$

To calculate the amount of conductor needed, we allow 2x0.040 in. (2 mm) tolerance on the pole width and height and assume a 0.250 in. (6 mm) clearance between the coil and the pole. We further assume a minimum radius of curvature of the conductor of $R_{min} = 4$ conductor thicknesses and that the pole ends are rounded with a radius of $R_{pole} = R_{min} - 0.250$ in. Using the values given on page 1, we then find that he length the mean turn is

$$\begin{aligned} < L > &= 2[\text{ Pole length} + \text{pole width } - \ 4R_{pole}] + 2\,\pi\, R_{min}] \\ &= 2[8.25 + 0.080 + 3.50 + 0.080 - 4(1.4592 - 0.250)] + 2\,\pi(1.4592 + 1.6817) \\ &= 33.88 \text{ in} = 860.6 \text{ mm}. \end{aligned}$$

We take

$$\boxed{\text{Mean length per turn} = 35 \text{ in.} = 890 \text{ mm}}$$

so that the length per coil, L, is

$$\boxed{L = N < L > = 175 \text{ ft} = 53 \text{ m}}$$

Because 4 coils are required per quadrupole, then

Total length per quadrupole	700 ft	=	212 m
Allow 10% for winding losses	70 ft	=	21 m
Total	770 ft	=	233 m

Then order

$$\boxed{\text{Total length of 800 ft} = 245 \text{ m.}}$$

of conductor of mass 0.3760 lb/ft

$$\boxed{\text{Total mass of 310 lb} = 140 \text{ kg}}$$

6. Power Requirements

At 20°C, the resistance of the coil is:

$$R_{20°C} = (83.70 \times 10^{-6}\ \Omega/\text{ft})(175\ \text{ft}) = 0.01465\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40°C coolant temperature rise), then the mean coil temperature will be 50°C. With a 30°C rise above ambient of the coil we then have:

$$\begin{aligned} R_{hot} &= R_{20°C}[1 + (\text{Temp. coeff}/°\text{C})\times\Delta\text{T}(°\text{C})\ ] \\ &= 0.01485\times[1 + 0.00393\times 30] \\ &= 0.01637\ \Omega \text{ per coil} \end{aligned}$$

Thus at a current of 310 A we obtain

$$\text{Voltage per coil} = (0.01637\ \Omega)(310\ \text{A}) = 5.08\ \text{V}.$$

Therefore, allowing for lead loss, we choose a power supply that has:

$$\boxed{\begin{array}{ccr} \text{I} & = & 310.0 \text{ A minimum} \\ \text{V} & = & 25.0 \text{ V total} \\ \text{P} & = & 7.8 \text{ kW total} \end{array}}$$

7. Cooling Requirements

In these calculations we use the British system of units. The power required per coil is

$$\text{Power per coil} = I^2\, R_{hot} = (310)(310)(0.01637) = 1.573\ \text{kW}$$

The required flow rate is given by

$$v\ \text{ft/sec} = \frac{2.19}{\Delta T(°F)}\ \frac{P\ (\text{kW})}{\text{Cooling area (in}^2)}.$$

For $\Delta T = 72°\text{F} = 40°\text{C}$ and $\text{A} = 0.03269\ \text{in}^2 = 21.090\ \text{mm}^2$, we have

$$v = \frac{(2.19)(1.573)}{(72)(0.03269)} = 1.464\ \text{ft/sec}$$

so the required volume of flow is

$$\text{Volume/circuit} = 3.12247\, v\, (\text{ft/sec}) \times \text{Cooling area (in}^2)$$

so that

$$\begin{aligned} \text{Volume/circuit} &= 3.12247(1.464)(0.03269) \\ &= 0.1494\ \text{USGPM} = 0.5656\ \ell/\text{min}. \end{aligned}$$

The pressure drop is given by

$$\Delta p = k\, v^{1.79}\ \text{psi/ft}$$

with k a function of the cooling area. In our case, with $k = 0.0232$ we obtain

$$\Delta p = (0.0232)(1.464)^{1.79} = 0.0459\ \text{psi/ft} = 0.1506\ \text{psi/m}$$

and the total pressure drop across the coil is

$$\boxed{\text{Pressure drop per coil} = 8.03 \text{ psi.}}$$

8. Iron Dimensions

For the estimation of the iron dimensions, we assume that the perpendicular distance from 45° symmetry line and the angled side of the coil is twice the pole-coil separation—that is 0.500 in. = 12.7 mm. Then we take the vertical distance measured along the inner edge of the coil from the centre-line of the pole to the intersection with the 45° symmetry axis as

$$\begin{aligned} y_1 &= (\text{Pole width})/2 + (1+2\sqrt{2})(\text{Pole-coil separation}) \\ &\quad + (\text{Number of layers of maximum width})(\text{Maximum coil height})/(\text{Total number of layers}) \\ &= 1.750 \text{ in.} + (1+2\sqrt{2})(0.250) \text{ in.} + 5(3.420)/8 \\ &= 4.845 \text{ in.} = 123.1 \text{ mm.} \end{aligned}$$

This is also the horizontal distance, $x_1[= y_1]$, from the center of the quadrupole to the inner edge of the coil. We take the distance from the inner coil-edge to the inner yoke, x_2, to be

$$\begin{aligned} x_2 &= \text{Maximum coil width + pole-coil separation} \\ &= 3.752 \text{ in.} + 0.250 \text{ in.} \\ &= 4.002 \text{ in.} = 101.66 \text{ mm.} \end{aligned}$$

The curved section of the pole intersects the straight edge at a distance;

$$\begin{aligned} x_3 &= \frac{\text{Aperture}}{2} + (\text{pole radius}) - \sqrt{(\text{pole radius})^2 - \frac{(\text{pole width})^2}{4}} \\ &= 2.819 \text{ in.} = 71.61 \text{ mm.} \end{aligned}$$

From this we calculate the length of the straight edge of the pole, L_{sp}, as:

$$\begin{aligned} L_{sp} &= x_1 + x_2 - x_3 \\ &= 4.845 \text{ in.} + 4.002 \text{ in.} - 2.819 \text{ in.} \\ &= 6.028 \text{ in.} = 153.10 \text{ mm.} \end{aligned}$$

We take

$$\boxed{\text{Length of straight edge of pole} = L_{sp} = 6.031 \text{ in.} = 153.18 \text{ mm.}}$$

so that the distance from the center of the quadrupole to the inner edge of the yoke, L_{inner}, is

$$\begin{aligned} L_{inner} &= x_3 + L_{sp} \\ &= 2.819 \text{ in.} + 6.031 \text{ in.} \\ &= 8.850 \text{ in.} = 224.79 \text{ mm} \end{aligned}$$

and the overall length of the pole, L_{pole}, is

$$\begin{aligned} L_{pole} &= L_{inner} - (\text{Aperture})/2 \\ &= 8.850 \text{ in.} - 2.05 \text{ in.} \\ &= 6.800 \text{ in.} = 172.72 \text{ mm.} \end{aligned}$$

We then have the quadrupole half-width as

$$\begin{aligned}\text{Quadrupole half-width} &= (\text{Pole length}) + (\text{Aperture}/2) + (\text{Yoke thickness}) \\ &= 6.800 \text{ in.} + (4.100 \text{ in.})/2 + 1.750 \text{ in.} \\ &= 10.600 \text{ in.} = 269.24 \text{ mm.}\end{aligned}$$

$$\boxed{\text{Quadrupole half-width} = 10.600 \text{ in.} = 269.24 \text{ mm.}}$$

We estimate the half-length of the inner vertical yoke by assuming that the angled edge of the quadrupole begins at a vertical height, y_{ang}, of

$$\begin{aligned}y_{ang} &= y_1 - (2\sqrt{2} - 1)(\text{Pole-coil separation}) \\ &= 4.845 \text{ in.} + 0.457 \text{ in.}\end{aligned}$$

Thus

$$y_{ang} = 4.388 \text{ in.} = 111.45 \text{ mm.}$$

Then a line through $(x_{st} = L_{inner}, y_{st} = y_{ang}) = (\ 8.850 \text{ in.}, 4.388 \text{ in.})$ and perpendicular to the 45° symmetry line intersects the symmetry line at $(x_e, y_e) = (\ 6.619 \text{ in.}, 6.619 \text{ in.})$ so that the half-length of the angled section of the inner yoke becomes

$$\begin{aligned}\text{Half-length of inner angled yoke} = L_{angle} &= \sqrt{(6.619 - 8.850)^2 + (6.619 - 4.388)^2} \\ &= 3.155 \text{ in.} = 80.14 \text{ mm.}\end{aligned}$$

9. Iron mass

From the above data we calculate for *one octant* of the quadrupole:

$$\begin{aligned}\text{Area of one-half rectangular pole} &= (\text{Straight length of pole})\frac{(\text{Pole width})}{2} \\ &= 10.554 \text{ in}^2 = 6809.2 \text{ mm}^2\end{aligned}$$

and, defining

$$\Theta = \sin^{-1}\left[\frac{\text{polewidth}}{2(\text{pole radius})}\right] = 0.8284 \text{ rad} = 47.46°,$$

then

$$\begin{aligned}\text{Area of one-half curved pole} &= \frac{(\text{pole radius})^2}{4}[2\Theta - \sin(2\Theta)] \\ &= 0.9314 \text{ in}^2 = 600.9 \text{ mm}^2\end{aligned}$$

so that one-half of the pole area, $A_{p/2}$, is 11.486 in^2 = 7410.1 mm^2 and the total pole area, A_p, is

$$\text{Area of one pole} = A_p = 2\,A_{p/2} = 22.971 \text{ in}^2 = 14820.1 \text{ mm}^2 = 0.14820 \text{ m}^2.$$

Assuming that the vertical and angled section of the yoke are joined at an angle of 22.5°, the yoke area, A_y, is calculated from

$$\begin{aligned}\text{Yoke area} = A_y &= (\text{Yoke thickness})[\text{ inner vertical length} + \text{angled yoke length} \\ &\quad + (2(\text{yoke thickness})/2)\tan(22.5°)] \\ &= (1.750 \text{ in.}))(4.388 \text{ in.} + 3.155 \text{ in.} + (2(1.750 \text{ in.})/2)(0.41421) \\ &= 14.469 \text{ in}^2 = 9334.7 \text{ mm}^2\end{aligned}$$

Then the total volume of the octant is

$$\begin{aligned}\text{Volume of octant} &= (A_y + A_{p/2})(Polelength) \\ &= (25.955)(8.250) \\ &= 214.127 \text{ in.}^3 = 0.003509 \text{ m}^3\end{aligned}$$

so that the *total* mass of iron per quadrupole (at a density of 7900 kg/m^3) is 488.99 lb or 221.8 kg. We take

$$\boxed{\text{Total iron mass per quadrupole} = 500\ \text{lb} = 227\ \text{kg.}}$$

10. Discussion

This note has presented a design study for a quadrupole that could be used on beam line 2A. As noted earlier, several different conductor sizes were investigated. Table 1 following lists the relevant data for other conductor sizes.

From the data given in table 1, it is seen that all but the 0.2893 in. OD conductor would be suitable for the quadrupole. The smallest conductor considered is ruled out because of the coolant flow rate that is required, thus resulting in too large pressure drop across the coil. It is suggested that either the 0.3249 in. OD conductor or the 0.3648 in. OD conductor be used. Use of the 0.3249 in. OD conductor would require that no more than two cooling circuits could be run in series if a maximum pressure drop of 60 psi is allowed. THis is not a great restriction. Were the 0.3648 in. OD conductor used, the cooling circuits for the coils could be run in series. Use of the 0.4096 in. OD conductor would be overkill.

An overall cross section of the quadrupole is shown in figure 2. Figure 3 shows a dimensioned octant for the case of the conductor size used for the illustration of the method of calculation.

As shown in figure 2, the quadrupole was designed in an octagonal shape. For manufacturing purposes, however, it would be less expensive to make the yoke rectangular in shape—that is, as the standard 4Q14/8 quadrupoles are designed.

References

1. A. P. Banford, *The Transport of Charges Particle Beams*, Spon, 1966.

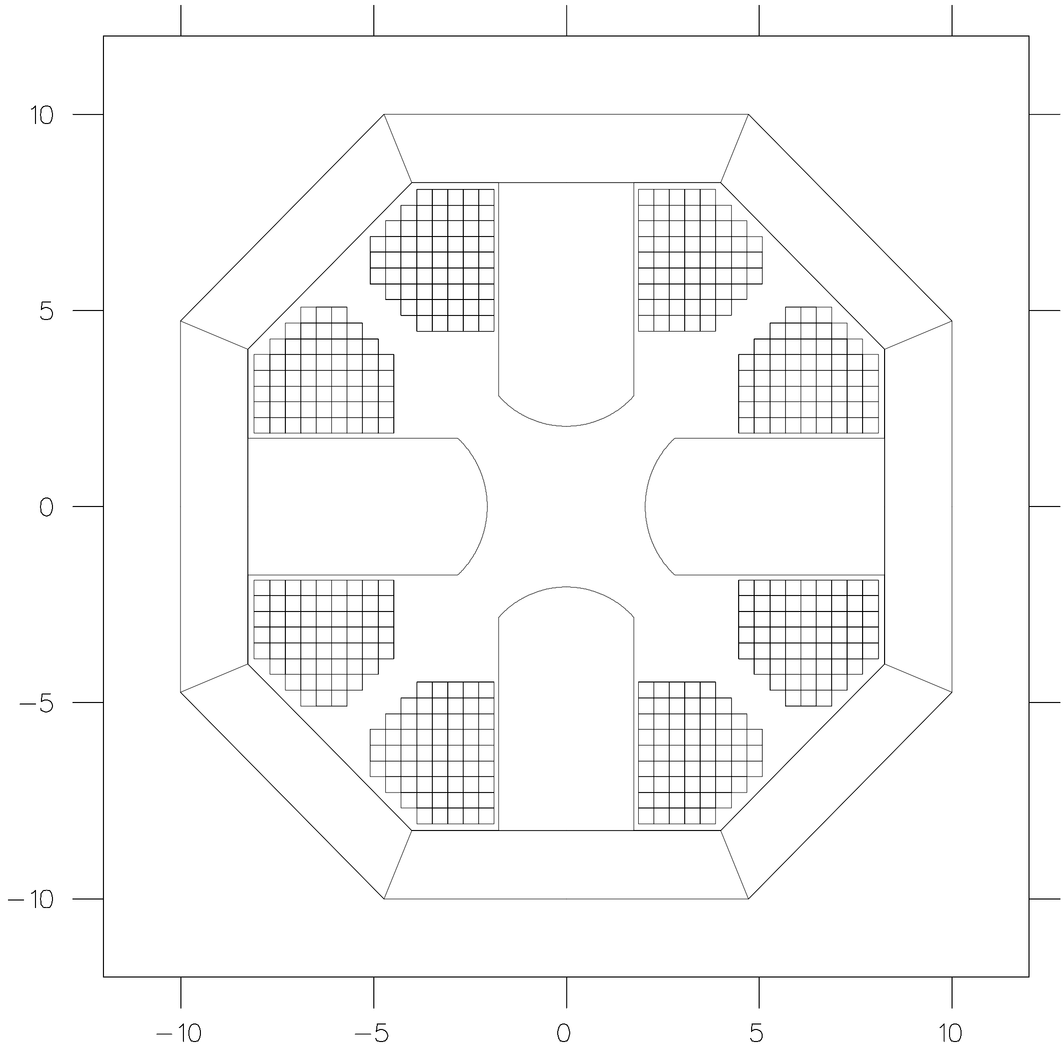

Fig. 2. A cross section of the completed quadrupole.

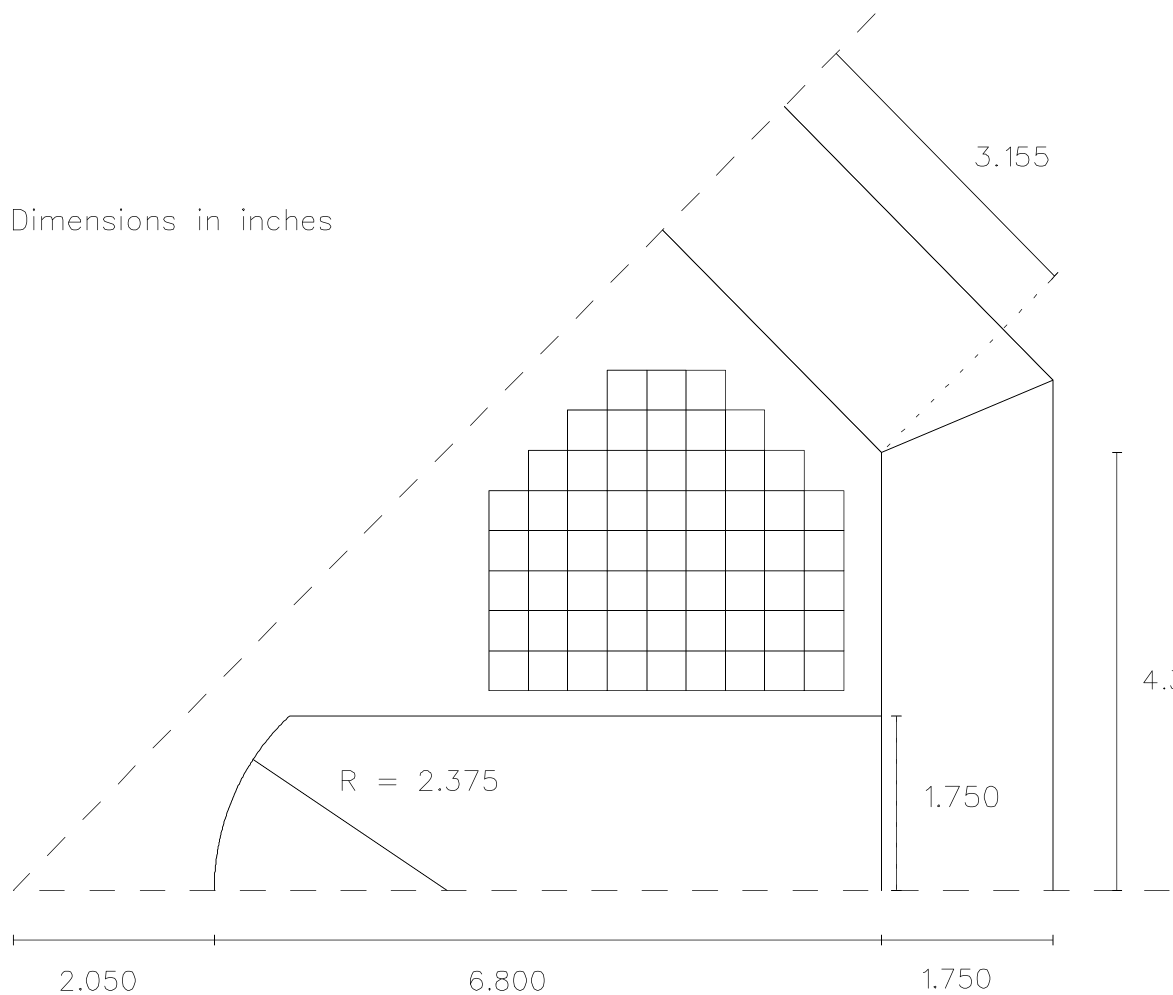

Fig. 3. A quadrant of the quadrupole. Dimensions are for the 0.3648 in. conductor.

Table 1
Summary of quadrupole parameters for various conductor dimension

Parameter		Anaconda outer dimension (in.)			
		0.2893	0.3249	0.3648	0.4096
Conductor:	ID (in.)	0.1610	0.1810	0.2040	0.2290
	Copper area (in.^{2}	0.0612	0.0772	0.0973	0.1232
	Cooling area (in.^{2})	0.0204	0.0257	0.0327	0.0412
	Weight (lb/ft)	0.2364	0.2984	0.3760	0.4760
	Resistance at 20° C ($\mu\Omega$/ft)	133.20	105.50	83.70	66.12
	k (British units)	0.0305	0.0267	0.0232	0.2000
Coil:	Number of turns	60	60	60	60
	Number of layers	8	8	8	8
	Nominal width (in.)	3.000	3.300	3.650	4.100
	Nominal height (in.)	2.750	3.000	3.350	3.750
	Length of mean turn (in.)	34.0	35.0	35.0	36.0
	Length per coil (ft)	170.0	175.0	175.0	180.0
	Weight per coil (lb)	40.2	52.2	65.8	85.7
	Weight per quadrupole (lb)	160.8	208.9	263.2	342.7
	Resistance per coil (hot) (mΩ)	25.31	20.64	16.37	13.30
	Coolant flow rate (ft/sec)	3.634	2.345	1.464	0.944
	Volume per coil (USGPM)	0.2310	0.1884	0.1494	0.1214
	Pressure drop per coil (psi)	52.23	21.48	8.03	3.25
Iron:	Yoke thickness (in.)	1.750	1.750	1.750	1.750
	Inner long yoke length (in.)	8.020	8.376	8.776	9.234
	Angled inner yoke length (in.)	5.360	5.814	6.310	6.976
	Overall length (in.)	8.250	8.250	8.250	8.250
	Pole radius (in.)	2.375	2.375	2.375	2.375
	Pole width (in.)	3.500	3.500	3.500	3.500
	Pole length (in.)	5.750	6.250	6.800	7.500
	Weight (lb)	430.	460.	500.	540.
Power:	Current (A)	310.	310.	310.	310.
	Voltage (V)	36.	30.	25.	20.
	Power (kW)	11.2	9.3	7.8	6.2
Quadrupole:	Aperture (in.)	4.100	4.100	4.100	4.100
	Width (in.)	19.100	20.100	21.200	22.600
	Height (in.)	19.100	20.100	21.200	22.600
	Length (in.)	14.250	15.750	15.450	16.250
	Assembled weight† (lb)	590.	670.	765.	885.

† Excluding power and coolant connections.