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Subject Conceptual design for the MEBT dipoles		

1. Introduction

For the charge selection system of the medium energy transport system (MEBT) the optimized solution requires two 45° dipole with a bending radius of 0.300 m and a maximum field of 11.1 kG[1]. This note presents a conceptual design for these dipoles and includes an analysis using the program POISSON[2].

2. Derived parameters

Given a required field of 11.1 kG, we design a magnet with a maximum field of 11.5 kG, a bending radius of 0.300 m and a 45° deflection angle, and we calculate the following.

$$\begin{aligned}
\text{Effective length} \quad &= l_{eff} \\
&= \text{(Radius of curvature)(Bend angle)} \\
&= (0.300\ \text{m})(45\,\pi\ \text{rad}/180) \\
&= 0.23562\ \text{m} = 9.2764\ \text{in.}
\end{aligned}$$

$$\begin{aligned}
\text{Magnetic rigidity} \quad &= (B\,\rho)_0 \\
&= (11.50\ \text{kG})(0.300\ \text{m}) \\
&= 3.450\ \text{kG-m}
\end{aligned}$$

$$\begin{aligned}
\text{Equivalent proton momentum} \quad &= (B\,\rho)_0\ /\ 33.356 \\
&= 3.450\ /\ 33.356 \\
&= 0.10343\ \text{GeV/c}
\end{aligned}$$

$$\begin{aligned}
\text{Equivalent proton energy} \quad &= E_0 \left[-1 + \sqrt{1 + \left(\frac{pc}{E_0} \right)^2} \right] \\
&= 0.93826 \left[-1 + \sqrt{1 + \left(\frac{0.10343}{0.93826} \right)^2} \right] \\
&= 5.67149 \times 10^{-3}\ \text{GeV} = 5.672\ \text{MeV}
\end{aligned}$$

We therefore design a dipole with basic properties:

Bend angle (θ)	45.00	degrees	
Maximum magnetic field (B_0)	11.50	kG	
Radius of curvature (ρ)	11.81102	in.	(0.300 m)
Magnetic rigidity [$(B\,\rho)_0$]	3.450	kG-m	
Full gap (g)	2.250	in.	(0.0572 m)
Effective length (l_{eff})	9.276	in.	(0.2356 m)
Maximum (proton) energy (E_{max})	5.672	MeV	
Maximum (proton) momentum (p_{max})	0.10343	GeV	

The gap specified does not include pole shims that reduce the air gap to 2 in. However, this should allow for a minimum air gap of 1.125 in.

3. Derived parameters

From the above we have the following derived quantities.

$$\begin{aligned}
\text{Straight-line length} &= 2\,\rho\sin(\theta/2) \\
&= 2(0.300)(0.38268) \\
&= 0.22961\text{ m} = 9.040\text{ in.} \\
\text{Iron length} &= \text{Straight-line length} - \text{full aperture} \\
&= 0.22961\text{ m} - 0.05080\text{ m} \\
&= 0.17881\text{ m} = 7.040\text{ in.}
\end{aligned}$$

where we have used the minimum aperture of 2.0 in. We take

$$\boxed{\begin{array}{lcccc} \text{Straight-line length} & = & 9.040\text{ in.} & = & 0.22961\text{ m} \\ \text{Iron length} = L_{iron} & = & 7.100\text{ in.} & = & 0.18034\text{ m} \end{array}}$$

4. Ampere-turns per coil

The required Ampere-turns per pole are calculated from the relation

$$NI\text{ per pole} = \frac{1}{2}\left[1.1\frac{B_0\,g}{4\,\pi\times10^{-7}}\right] = 28736\text{ A-t}$$

where we have allowed for a 10% flux leakage. We take

$$\boxed{\text{NI per pole} = 28{,}800\text{ A-t}}$$

and generate the following table

I (Amperes)	100	200	300	400	500	600	700	800	900	1000
N (Turns)	288	144	96	72	58	48	41	36	32	29

We choose

$$\boxed{\begin{array}{lccl} \text{Maximum current} & = & 600 & \text{Amperes} \\ \text{Number of turns per coil} & = & 48 & \end{array}}$$

in a coil configuration 4 turns wide and 12 layers high.

5. Pole parameters

We assume a 0.625 in. (15.9 mm) chamfer at 45 degrees on each edge of the pole and that the full-width of the beam is 1.26 in. = 32.0 mm. The deviation of the central trajectory from a line parallel to the entry and exit points is

$$\Delta = \rho(1-\cos(\theta/2)) = 0.89906\text{ in.} = 22.8\text{ mm}$$

Then the pole width is calculated as

$$\begin{aligned}
\text{Pole width} = W_{iron} &= 2(\text{Chamfer}) + \text{Beam width} + \Delta + 2(\text{gap}) \\
&= 2(0.625) + 1.260 + 0.899 + 2(2.250) \\
&= 7.909\text{ in.} = 201\text{ mm.}
\end{aligned}$$

Based on the POISSON results that follow, we take

$$\boxed{\text{Pole width at its base} = 11.000\text{ in.} = 279.4\text{ mm.}}$$

6. Coil design

We assume a current density of 3000 A/in.^{2} = 4.65 A/mm^2 and calculate the required conductor area from

$$\text{Conductor area} = \frac{600\ \text{A}}{3000\ \text{A/in.}^2} = 0.2000\ \text{in.}^2 = 129\ \text{mm}^2$$

The smallest Anaconda conductor that satisfies this criterion is 0.516 inch square conductors; its properties are listed as:

Conductor OD	0.5160 in.	13.106 mm
Conductor ID	0.2870 in.	7.290 mm
Copper area	0.1940 in.^{2}	125.161 mm^2
Cooling area	0.06469 in.^{2}	41.735 mm^2
Mass	0.7495 lb/ft	1.115 kg/m
Resistance at 20°C	41.99 $\mu\Omega$/ft	137.76 $\mu\Omega$/m
k (British units)	0.0152	0.0152

We assume that each conductor is double-wrapped with insulation such that the *total* insulation per conductor has:

Minimum thickness = 0.022 in. = 0.559 mm
Nominal thickness = 0.028 in. = 0.711 mm
Maximum thickness = 0.034 in. = 0.864 mm

The tolerance of the outer dimension of the conductor is listed as 0.004 in. = 0.102 mm so that the dimensions of a *wrapped* conductor are:

Minimum = 0.534 in. = 13.6 mm
Nominal = 0.544 in. = 13.8 mm
Maximum = 0.554 in. = 13.1 mm

We further allow

a) a gap between layers of 0.010 in. maximum,
b) for keystoning, assume 0.010 in. and
c) a 4-turn ground wrap of 0.007 in. tape.

Then the *widths* of the coils are

	Coil dimension			
	(in.)		(mm)	
	Maximum	Minimum	Maximum	Minimum
Wrapped conductor	2.216	2.136	56.29	54.25
Gapping (3x0.010)	0.030		0.76	
Ground wrap (4x0.007x2)	0.056	0.056	1.42	1.42
Total (in.)	2.302	2.192	58.47	55.67

We take

	Coil width	
	(in.)	(mm)
Maximum coil width	2.300	58.42
Nominal coil width	2.250	57.15

The *heights* of the coils are

	Coil dimension			
	(in.)		(mm)	
	Maximum	Minimum	Maximum	Minimum
Wrapped conductor	6.648	6.408	168.86	162.76
Gapping (11x0.010)	0.110		2.79	
Keystoning (12x0.010)	0.120	0.060	3.05	1.52
Ground wrap (4x0.007x2)	0.056	0.056	1.42	1.42
Total (in.)	6.934	6.524	176.12	165.70

We take

	Coil height	
	(in.)	(mm)
Maximum coil width	7.000	177.8
Nominal coil width	6.750	171.5

We take the conductor dimension D to be

$$\begin{aligned} D &= \text{Nominal dimension} + 4(\text{Insulation thickness}) + \text{Turn separation} \\ &= 0.516 + 0.028 + 0.010 \\ &= 0.554 \text{ in.} = 14.07 \text{ mm.} \end{aligned}$$

We now assume that the minimum bending radius of the conductor, R_{min}, is

$$R_{min} = 4(\text{Conductor dimension}) \approx 2.0 \text{ in.}$$

Because we wish to keep the overall length of the dipole as short as possible and maintain a reasonable pole-coil separation at the end corners of the pole, we calculate the pole-coil gap, G, along the sides of the pole as follows. Let g be the pole-coil separation at the *ends* of the magnet and let x be that at the corners of the pole. As indicated in the figure below, we locate the center of curvature of the coil a distance $2g$ from the longitudinal edge of the pole and a distance $R_{min} - g$ from the pole end.

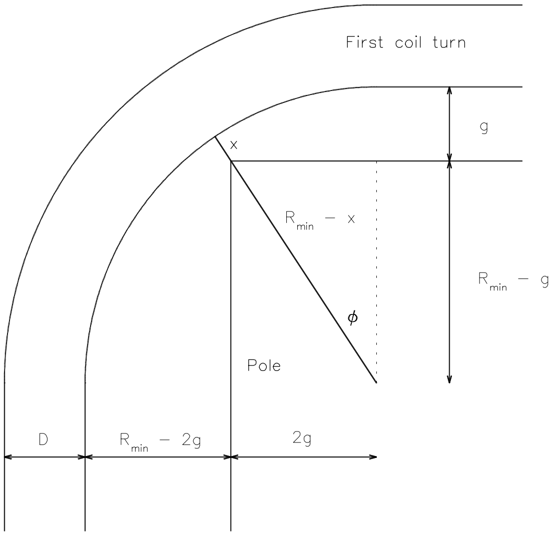


Fig. 1. The geometry used in the calculation of the coil location.

Then, if ϕ is the angle indicated in the figure above, we have

$$\tan\phi = \frac{2g}{R_{min} - g}$$

and we can write

$$R_{min} - x = \frac{2\,g}{\sin\phi} = \frac{R_{min} - g}{\cos\phi}$$

Thus, for $g = 0.5$ in. and $R_{min} = 2.0$ in. we have

$$\phi = \tan^{-1}\left[\frac{2(0.5)}{2.0 - 0.5}\right] = 33.690^\circ$$

and

$$x = 2.0 - \frac{2(0.5)}{\sin 33.690} = 0.197 \text{ in.}$$

Thus the coil-pole corner clearance is approximately 0.2 inch, a value that should be adequate. The pole-coil gap, G, along the sides of the pole is then

$$G = R_{min} - 2\,g = 2\,g = 1.0 \text{ inch.}$$

To calculate the length of the coil we note that the straight section of the n^{th} conductor along the longitudinal portion of the coil is a distance

$$D(n) = n\,D + G + (\text{pole width})/2$$

from the (longitudinal) centre-line of the pole. The (outer) radius of curvature of this n^{th} turn is

$$R(n) = R_{min} + G + n\,D = 4\,g + 2\,g + n\,D = 6\,g + n\,D$$

The length of the straight longitudinal section of the winding is

$$L_{length} = L_{iron} - 2(R_{min} - g) = L_{iron} - 2(4\,g - g) = L_{iron} - 6\,g$$

and that of the straight section along the pole-width is

$$L_{width} = W_{iron} - 2(2\,g) = W_{iron} - 4\,g$$

Thus the length of the n^{th} turn is

$$\begin{aligned} l(n) &= 2[L_{length} + L_{width}] + 2\,\pi\,R(n) \\ &= 2[L_{iron} + W_{iron} - 10\,g + 6\,\pi\,g] + 2\,\pi\,n\,D \end{aligned}$$

and the length of an N-turn layer is

$$L(N) = \sum l(n) = 2\,N\,[L_{iron} + W_{iron} - 10\,g + 6\,\pi\,g] + \pi\,N\,(N+1)\,D$$

Using the following parameters

	Value	
	in.	mm
L_{iron} (in.)	7.100	180.3
W_{iron} (in.)	11.000	279.4
g (in.)	0.500	12.7
D (in.)	0.554	14.1
R_{min} (in.)	2.000	50.8

we find that the length of a 4-turn layer of the 0.5160-inch square conductor is 215 in. (5461 mm). We take

$$\boxed{\text{Length of a 4-turn layer} = 20 \text{ ft} = 6.1 \text{ m.}}$$

The length of copper per coil is then

$$\boxed{\text{Length of copper per coil} = 240 \text{ ft} = 73.2 \text{ m.}}$$

Because two coils are required per dipole we then have

Total length per dipole	480 ft	[146 m]
Allow 10% for winding losses	48 ft	[14 m]
Total	528 ft	[160 m]

Then, for the 0.516 inch-square conductor with a weight of 0.7495 lb/ft, order for each dipole

Length of 0.5160 conductor	550 ft	168 m.
Weight of 0.5160 conductor	415 lb	188 kg.

7. Power requirements

At 20°C, the resistance of one of the coils, R_{20}, is

$$R_{20} = (41.99 \times 10^{-6}\ \Omega/\text{ft})(240 \text{ ft}) = 10.1 \times 10^{-3}\ \Omega$$

We assume an ambient temperature of 20°C, an inlet water temperature of 30°C and an outlet water temperature of 70°C (thus allowing a 40°C coolant temperature rise). Then the mean coil temperature will be 50°C. With a 30°C rise above ambient of the coil we then have

$$\begin{aligned} R_{hot} &= R_{20C}\,[1 + (\text{Temp. coeff}/^\circ\text{C})\Delta T(^\circ C)] \\ &= R_{20}\,[1 + 0.00393(30)] \\ &= (0.0101)(1.1179) = 0.01129\ \Omega \text{ per coil.} \end{aligned}$$

Thus at a current of 600 A the required voltage per coil is

$$V_{min} = I\,R_{hot} = (600)(0.01129) = 6.8 \text{ V per coil.}$$

Therefore, allowing for a 10% lead loss, we choose power supplies that have

I (A minimum)	600
V (V minimum)	15.0
P (kW minimum)	9.0

8. Cooling requirements

In these calculations we use the British system of units. The power required per coil is:

$$\text{Power per coil} = P = I^2\,R_{hot} = (600)(600)(0.0101) = 3.636 \text{ kW.}$$

and the required flow rate is given by

$$v \text{ (ft/sec)} = \frac{2.19}{\Delta T(^\circ\text{F})}\,\frac{P \text{ (kW)}}{\text{Cooling area (in}^2)} = 3.04167 \times 10^{-2} \times \frac{P \text{ (kW)}}{\text{Cooling area (in}^2)}$$

for $\Delta T = 72^\circ\text{F} = 40^\circ\text{C}$. Choosing $v = 2.50$ ft/sec to define the maximum power dissipation per water circuit we have for a cooling area of 0.06469 in.^{2}

$$P_{max} = \frac{(2.50)(72)[\text{Cooling area (in.}^2)]}{2.19} = (82.19178)(0.06469) = 5.317 \text{ kW/water circuit},$$

from which we calculate the number of cooling circuits per coil (excluding lead loss) as

P = Power per coil (kW)	3.636
P_{max} (kW/water circuit)	5.317
Number of circuits = P/P_{max}	0.684
Number of circuits to use	1
v = Flow rate (ft/sec)	1.710

The volume of flow required per circuit is

$$\begin{aligned} \text{Volume/circuit} &= 2.6\, v(\text{ft/min})\,[\text{Cooling area(in}^2)]\ \text{IGPM} \\ &= 3.12250\, v(\text{ft/min})\,[\text{Cooling area(in}^2)]\ \text{USGPM} \end{aligned}$$

Thus

Volume per cooling circuit (USGPM)	0.345
Volume per coil (USGPM)	0.345
Volume per magnet (USGPM)	0.691

9. Pressure drop

The pressure drop is given by

$$\Delta P = k\, v^{1.79} \text{ psi/ft}$$

with k a function of the cooling area. In our case, with $k = 0.0152$, we obtain

$$\Delta P = 0.0152(1.710)^{1.79} = 0.03971 \text{ psi/ft}$$

and the pressure drop across the (single) cooling circuit is

$$\Delta P \text{ (psi/coil} = (0.03971 \text{ psi/ft})(240 \text{ ft}) = 9.531 \text{ psi}$$

$$\boxed{\text{Pressure drop per coil} = 9.53 \text{ psi}}$$

10. Iron dimensions

The following relations are used to calculate the iron dimensions.

Coil-slot width = Maximum coil width + Inner pole-coil gap + Outer pole-coil gap
Dipole width = 2[Coil-slot width + Yoke thickness] + Pole width
Dipole length = 2[End pole-coil separation + Maximum coil width] + Pole length
Pole height = Maximum coil height + Chamfer + 0.59 in. [15mm]
Length of side yoke = 2[Pole height] + Gap

For the 0.516 in. conductor, we take the following values for some of these parameters. The inner pole-coil gap has been taken as $G = 2g = 1.000$ in. We take the outer pole-coil gap to be $g = 0.500$ in.

		Calculated as (in.)	Value used in.	Value used mm
Coil-slot width	=	2.300 + 1.000 + 0.500	3.800	96.5
Dipole length	=	2(0.500 + 2.300) + 7.1	12.700	322.6
Pole height	=	7.000 + 0.625 + 0.50	7.875	200.0
Length of side yoke	=	2(7.875) + 2.250	18.000	457.2

In order to complete the calculation we must determine the yoke thickness. To do this we assume that the field rises linearly from 0 at the inner edge of the yoke to 0.6(field in the gap) at the beginning of the flat portion of the pole and that the flux divides equally between the side yokes. We note here that in the pole profile used in the POISSON calculations (see figure 2b) the horizontal distance from the end of the pole flat to the base of the pole is 2 in. This distance arises from pole-edge contouring and a 0.500 in. taper on the pole side. In what follows, this distance is termed 'pole contour'.

Then, if B_y and t are the yoke flux and thickness, respectively, and $B_g = 11.500$ kG is the field in the gap, we have, with dimensions in inches,

$$\begin{aligned}
B_y\,t &= \frac{1}{2}\,\frac{[2(\text{Coil-slot width} + \text{Pole contour})](0.6\,B_g)}{2} + \frac{1}{2}[\text{Pole width} - 2(\text{Pole contour})]B_g \\
&= \frac{1}{2}\,\frac{[2(3.800 + 2.000)](0.6\,B_g)}{2} + \frac{1}{2}[(11.000 - 2(2.000)]B_g \\
&= \frac{1}{2}[3.480 + 7.000]B_g \\
&= 5.240\,B_g \\
&= 60.26 \text{ kG-in.} = 0.15306 \text{ T-m}
\end{aligned}$$

We then have the following table.

B_y (kG)	6.0	7.0	8.0	9.0	10.0	11.0
t (in.)	10.043	8.609	7.533	6.696	6.026	5.478

POISSON calculations show that a yoke thickness of 7.000 in. is required to keep pole and yoke saturation down at 11.5 kG. Consequently, we take

$$\boxed{\begin{aligned} t &= 7.000 \text{ in.} = 0.178 \text{ m} \\ B_y &= 8.609 \text{ kG} = 0.861 \text{ T} \end{aligned}}$$

Then the dipole width is calculated as

$$\begin{aligned}
\text{Dipole width} &= 2[\text{Coil-slot width} + \text{Yoke thickness}] + \text{Pole width} \\
&= 2(3.800 \text{ in.} + 7.000 \text{ in.}) + 11.000 \text{ in.} \\
&= 32.600 \text{ in.} = 0.828 \text{ m.}
\end{aligned}$$

11. Iron mass

Treating the pole as rectangular, from the preceding data we calculate the iron areas. These are listed on the next page. The areas of the yokes are twice that calculated in the table; the volume of iron is its area multiplied by the pole length (7.10 in.). We then have, using a density of 0.2833 lb/in.^{3} for iron and a conversion factor of 2.205 lb/kg, the data listed in the second table of the next page.

Calculation of the yoke area

	British			Metric		
	Height (in.)	Width (in.)	Area (in.^{2})	Height (mm)	Width (mm)	Area (mm^2)
Top yoke	7.000	32.600	228.2	177.8	828.0	147,225.5
Top pole	7.875	11.000	86.6	200.0	279.4	55,887.0
Side yoke	18.000	7.000	126.0	457.2	177.8	81,290.2
Total			440.8			284,402.7

Calculation of the iron mass

	British	Metric
Total yoke area	881.6 in.^{2}	0.5688 m^2
Volume of iron	6259.4 in.^{3}	0.1026 m^3
Weight of iron	1773.3 lb	804.2 kg
Estimated iron weight	1800.0 lb	816.3 kg
Estimated copper weight	360.0 lb	163.3 kg
Estimated dipole weight	2160.0 lb	979.6 kg

As a result of the POISSON calculations that are described below, a 1-in. thick field clamp was added at the entry and exit ends of the pole. This clamp adds an extra 2×4.3 in. = 8.6 in. to the overall length of the dipole.

Table 1 presents a summary of the calculations presented here.

12. POISSON calculations

Following the conceptual design given above, the program POISSON 2), modified by Rodenz 3), was used in a study of the magnetic fields in both the yoke and the air gap. This modified version of POISSON allows a simpler input of geometric data and automatically produces input for the AUTOMESH routine of the program.

POISSON was used to determine both the pole profile that would provide the required field uniformity ($\leq$0.1% over a ±1.5-in. wide band centered about the longitudinal center-line of the pole) and to estimate the effective length of the dipole.

12.1. Transverse field calculations

In the following calculations, the coil was made of the 0.516-in. square conductor. A mesh size of 0.125 in. was used in each of the horizontal and vertical directions. Table 2 lists an input file for the transverse calculations.

Based on the results given in ref[4)], the pole was shaped with a 0.25 in. horizontal by 0.125 in. vertical slope that began 3.5 in. from the center of the pole. There followed a 0.625 in. horizontal flat and a 0.625 in. taper at 45°, the latter ending 5.0 in. from the pole center. The step and taper were added to alleviate saturation at the pole edges. The side of the pole was sloped to the coil slot by a taper ending 5.5 in. from the horizontal pole center and 7.875 in. vertically below the major flat portion of the pole. Thus, exclusive of the step on the pole, the pole was made 7.875 in. deep by 2(5.5) = 11 in. wide at its base and 2(4.375) = 8.75 in. wide at its top.

The yoke and pole dimensions as detailed above are shown in the upper and lower portions, respectively, of figure 2.

Preliminary POISSON calculations indicated that 26,700 Ampere-turns were required to produce a field of 11.506 kG at the center of the pole on the mid-plane. At that excitation, the POISSON results showed no saturation in the yoke but indicated that saturation was near in the corner of the pole where the step was located.

The program PLOTDATA [5)] was used to plot the data obtained from the POISSON runs. The upper portion of figure 3 shows the POISSON results at an excitation of 26,700 A-t as a function of distance above the mid-plane. The curves are labelled according to the height in eighths of an inch above the mid-plane. Thus the curve labelled '0' is on the mid-plane, that labeled '4' is 4/8 in. = 0.5 in. above the mid-plane, etc. The lower portion of figure 3 shows this data on an expanded scale. From this latter figure it is seen that over the region $x = \pm 1.5$ in. the field is predicted to be (11,506.5 ± 4) G. Thus the field uniformity is predicted to be within ±0.035% over a 3 in. band centered about the longitudinal centerline of the pole.

PLOTDATA was also used to obtain contour plots of the field in the yoke and in the gap at an excitation of 26,700 A-t. Figure 4 shows the field contours in the yoke and pole. Shown in figure 5 are contours of field in the gap, B_y. From this figure it is seen that the field in the gap may be characterized as B_y = (11,506 ± 2) G in a region bounded by $x \approx \pm 1.5$ in. horizontally about the pole center and $y \approx \pm 1.0$ in. about the mid-plane.

Further calculations at 20%, 40%, 60% and 80% of that excitation were made. Figure 6a shows a plot of the predicted field as a function of excitation. This data is also listed in table 3(a).

12.2 End-effect calculations – Effective length

Following ref[6)], a C-shaped dipole was used to estimate the effective length of the dipole. A vertical yoke 10 in. wide by 20 in. high and a (flat) pole width of 8 in. were used in this POISSON run. Mid-plane symmetry and horizontal and vertical mesh sizes of 0.125 in. were used. In addition, a 1 in. thick, 4.3 in. wide field clamp was incorporated.

The upper portion of figure 7 shows the configuration used. In the lower portion is a plot of the calculated field profile on the mid-plane at an excitation of 28,000 A-t. This excitation provided a central field of 11.52 kG. The input file for this calculation is listed in table 4.

A more detailed plot of the predicted fields at 28,000 A-t is shown in figure 8. This figure shows the predicted fields from the center of the pole outward as a function of distance above the mid-plane. In addition, calculations at 20%, 40%, 60% and 80% of that excitation were made. Listed in table 3(b) are the values of the central, mid-plane fields obtained from these calculations.

In the geometry used, the center of the pole is approximately 7.25 inches from the inner edge of the yoke. We use PLOTDATA to integrate from the pole center to a point well outside the magnet (30 in.) and divide by the field at the pole center, $B_y(0)$, to obtain the effective length of the half-magnet. This we double to obtain the effective length of the dipole. Thus

$$l_{eff} = \frac{2}{B_y(0)} \int_{7.5}^{30} B_y(x)\, dx$$

This technique was applied for both the mid-plane ($y = 0$ in.) calculations and a series of eight horizontal planes, 0.125 in. apart, above the mid-plane to a final height of $y = 1.0$ in. A similar approach was taken for the other excitations that were run. In each calculation, the field $B_y(0)$ was taken as the field at the pole center of the plane in question.

Values extracted for the effective length of the dipole as a function of excitation are plotted in figure 9. In that figure, the lower of the three curves represents the calculated effective lengths at distances $y \leq 0.750$ in. above the mid-plane. The middle curve is for $y = 0.875$ in. above the mid-plane and the upper is for $y = 1.000$ in. above the mid-plane.

From these data it is seen that the effective length of the C-magnet is predicted to vary from approximately 5.055 in. at an excitation of 5,600 A-t to approximately 5.02 in. at an excitation of 28,000 A-t. Thus the effective length is predicted to vary approximately 0.7% over the complete range of excitation. However, the effective length is constant at the higher value at and below excitations of approximately 17,000 A-t.

Table 5 lists the values calculated for the effective lengths. The listed effective lengths are those of the C-magnet. Also tabulated is *twice* that value because we are dealing with an H-magnet. We note that the calculated (straight-line) H-magnet effective length, 10.04 in. at 28,000 A-t, is 1.0 in. longer than that calculated in section 3. Consequently, the effective edge of the magnet is located approximately $(10.04 - 7.10)/2 = 1.47$ in. beyond the iron edge of the pole—that is, approximately at the center of the coil.

As in section 2, we take the effective length to be the iron length, $l_{C,i}$, plus a fringe-field length, $l_{C,ff}$. In the model used we calculate the iron length from

$$\begin{aligned} l_{C,i} &= \text{outer edge of yoke} - \text{pole center} \\ &= 11.300 \text{ in.} - 7.300 \text{ in.} = 4.000 \text{ in.} \end{aligned}$$

and obtain on the mid-plane at 28,000 A-t

$$\begin{aligned} l_{C,ff} &= l_{C,eff}/2 - l_{C,i} \\ &= 5.020 \text{ in.} - 4.000 \text{ in.} \\ &= 1.020 \text{ in.} \end{aligned}$$

Because only one end of the magnet has been modeled with the C-magnet, we have the iron length for the H-frame magnet, $l_{H,i}$,

$$\begin{aligned} l_{H,i} &= l_{H,eff} - 2\, l_{C,ff} \\ &= 9.040 \text{ in.} - 2(1.020 \text{ in.}) \\ &= 7.000 \text{ in.} \end{aligned}$$

where $l_{H,eff}$ is the straight-line effective length of the magnet, l_e, of section 2. In that section the iron length of the H-magnet, l_i, was taken as $l_i = l_e - g = 7.100$ in. and is in good agreement with that obtained from the POISSON calculation. We take

$$\boxed{\text{Iron length of } 45^\circ \text{ MEBT magnet} = 7.100 \text{ in.}}$$

12.3 Calculation of K1

$K1$ is an integral that takes into account fringe-field effects in a dipole and is used in correction of the transverse focal length when the finite extent of the fringing fields are included. It is defined by

$$K1 = \int_{-\infty}^{+\infty} K1(x) dx = \frac{1}{g\, B_0^2} \int_{-\infty}^{+\infty} B_y(x)[B_0 - B_y(x)] dx.$$

where $B_y(x)$ is the magnitude of the fringing field on the magnetic mid-plane at a position x and x is the perpendicular distance measured from the entrance (or exit) face of the magnet to the point in question. B_0 is the asymptotic value of $B_y(x)$ well inside the magnet entrance. Typical values of $K1$ for real magnets range from 0.3 to 1.0 depending on the detailed shape of the magnet profile and the location of the coils.

The upper portion of figure 10 show the function

$$K1(x) = B_y(x)[B_0 - B_y(x)]/(g\,B_0^2)$$

calculated as a function of distance from the mid-plane for the C-magnet at an excitation of 28,000 A-t. The lower portion shows similar data for $K1$ calculated as

$$K1 = \int_{7.25}^{30} K1(x)dx = \frac{1}{g\,B_0^2}\int_{7.25}^{30} B_y(x)[B_0 - B_y(x)]dx.$$

Note that the lower limit of the range of integration was the center of the pole (x = 7.25 in.) and the upper limit was at x = 30 in. Because we assume that the H-magnet will be symmetrical, the value for $K1$ will be twice the values shown in figure 10(b). (We point out that, strictly speaking, $K1$ is defined only on the mid-plane. However, as noted above, calculations were also made off the mid-plane. In those cases, the values of B_0 used were those at the center of the plane in question.)

Shown in figure 11 are the values of $K1$ that were obtained plotted as a function of excitation and as a function of distance above the mid-plane. These data are also tabulated in table 6.

Brown et al.[7)] give the following table for values of $K1$ depending of the type of the fringing-field boundary and its drop-off.

Model	$K1$
Linear	1/6
Clamped “Rogowski”	0.4
Unclamped “Rogowski”	0.7
Square-edged	0.45

From table 6 and figure 11 we note that on the mid-plane the value of $K1 \approx 0.55$ is approximately mid-way between that expected of a clamped and unclamped Rogowski contour. This slowly falls to $K1 \approx 0.40$ at 0.75 in. above the mid-plane, a value quoted as a clamped Rogowski contour. Given that the anticipated vertical beam size in the dipole is ±5mm = ±0.20 in., we suggest that a value of $K1 = 0.55$ be used in TRANSPORT calculations.

13. Discussion

The program POISSON has been used to decide the final iron dimensions of the 45° H-frame dipoles for the charge selection section of the ISAC MEBT beam line. A final design is proposed utilizing a coil formed from a conductor with an outer dimension that is 0.516 in. square.

At the time of writing this report it is undecided as to the final shape of the dipoles—that is, whether they would be rectangular or would have angled pole edges. However, the POISSON results presented here would be applicable regardless of which type is chosen.

References

1. R. E. Laxdal, *Suggested MEBT Optimization*, TRIUMF Report TRI-DN-ISAC-23, November, 1966.
2. M. T. Menzel and H. K. Stokes, *User's Guide for the POISSON/SUPERFISH Group of Codes*, Los Alamos National Laboratory Report LA-UR-87-115, January, 1987.
3. G. W. Rodenz, *User's Guide to the Program FRONT*, Los Alamos Accelerator Code Group, June, 1992.
4. G. M. Stinson, *A revised conceptual design for the 15° dipoles on beamline 2*A, TRIUMF Report TRI-DNA-96-10, December, 1966.
5. J. L. Chuma, *PLOTDATA Command Reference Manual*, TRIUMF Report, TRI-CD-87-03b, August, 1989. 1996.
6. G. S. Clark, *Concept Design for the Beam Line 2A B1/B2 Dipole, Revision 3*, TRIUMF Report, TRI-DN-96-24, November, 1996.
7. K. L. Brown, F. Rothacker, D. C. Carey and Ch. Iselin, *TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems*, CERN Report CERN 80-04, CERN, 1980.

Table 1

Summary of the calculations for the 45° MEBT dipole

		British	Metric
Yoke:	Iron length	7.100 in.	180.3 mm
	Iron width	32.600 in.	828.0 mm
	Iron thickness	7.000 in.	177.8 mm
	Coil-slot width	3.800 in.	96.5 mm
	Side-yoke height	18.000 in.	457.2 mm
Pole:	Width at base	11.000 in.	279.40 mm
	Height	7.875 in.	200.03 mm
	Chamfer at 45°	0.625 in.	15.88 mm
Iron:	Total mass	1,800 lb	820 kg
Dipole:	Overall width	32.600 in.	828.3 mm
	Overall height	32.000 in.	812.8 mm
	Overall length (including coil)	12.700 in.	322.6 mm
	Overall length (including clamp)	21.300 in.	541.0 mm
Coil:	Conductor OD	0.516 in.	13.11 mm
	Conductor ID	0.287 in.	7.29 mm
	Nominal coil width	2.250 in.	57.15 mm
	Nominal coil height	6.750 in.	171.45 mm
	Total coolant flow	0.691 USGPM	2.62 ℓ
	Turn configuration	4 wide × 12 high	4 wide × 12 high
	Resistance (hot) per coil	$11.29 \times 10^{-3}\ \Omega$	$11.29 \times 10^{-3}\ \Omega$
	Number of cooling circuits per coil	1	1
Copper:	Total length per magnet	480 ft	146 m
	Total mass per magnet	360 lb	165 kg
	Total length to order	550 ft	160 m
	Total mass to order	415 lb	190 kg
Power:	Total current	600.0 A	600.0 A
	Total Voltage	15.0 V	15.0 V
	Power	9.0 kW	9.0 kW

Table 2

Input file for POISSON for the calculation of the transverse field of the 45° MEBT dipole

```
title
45 deg dipole for ISAC MEBT - 7 in. pole, 7 in. yoke, 0.5 in taper, 2.25 in gap and 26,700 A-t
run
pois
mode        0
xmax = 16.25
ymax = 16.0
xmesh       0.125
ymesh       0.125
symm = 6
nseg        4
conv = 2.54
zseg       0.0      16.25    16.25     0.0
rseg       0.0       0.0     16.0     16.0
matpro      1
nseg       13
conv = 2.54
zseg       0.0      3.5      3.75     4.375   5.0     5.5   6.5   8.8   9.25   9.25   16.25   16.25   0.0
rseg       1.125    1.125    1.0      1.0     1.625   9.0   9.0   9.0   9.0    0.0     0.0    16.0    16.0
matpro      2
nseg       4.
conv = 2.54
zseg       6.5      8.8      8.8      6.5
rseg       9.0      9.0      8.875    8.875
matpro      1
nseg       4.
conv = 2.54
zseg       6.5      8.8      8.8      6.5
rseg       8.875    8.875    1.875    1.875
matpro      1
current = 26700.
kbot = 1
lbot = 1
ltop = 9
fieldmap    2
begin
end
```

Table 3(a)
Calculated fields as a function of excitation of the MEBT 45° dipole

Transverse calculation

Ampere-turns	Field (G)
26,700	11,506.5
21,360	9,274.3
16,020	6,971.6
10,680	4,649.5
5,340	2,320.1

Table 3(b)
Calculated fields as a function of excitation of the MEBT 45° dipole

End-effect calculation

Ampere-turns	Field (G)
28,000	11,521.3
22,400	9,629.0
16,800	7,282.7
11,200	4,864.2
5,600	2,428.7

Table 4

Input file for POISSON for the calculation of the end-effect calculations of the 45° MEBT dipole

```
title
45 deg dipole for ISAC MEBT - 8 in. pole, 1 in. clamp, 2.25 in gap and 28,000 A-t
run
pois
mode        0
xmax = 30.0
xmin = −10.0
ymax = 20.0
ymin = 0.0
xregion    15.0
yregion    12.0
xmesh       0.125
ymesh       0.125
symm = 2
nseg        4
conv = 2.54
zseg      −10.0    30.0    30.0    −10.0
rseg        0.0     0.0    20.0     20.0
matpro      1
nseg        8
conv = 2.54
zseg      −10.0     0.0     0.0      3.3    3.3     11.3    11.3  −10.0
rseg        0.0     0.0     9.0      9.0    1.125    1.125  20.0   20.0
matpro      2
nseg        6.
conv = 2.54
zseg       11.3    14.6    14.6     15.6   15.6    11.3
rseg        9.5     9.5     1.125    1.125 10.5    10.5
matpro      2
nseg        4.
conv = 2.54
zseg        0.5     2.8     2.8      0.5
rseg        9.0     9.0     8.875    8.875
matpro      1
nseg        4.
conv = 2.54
zseg        0.5     2.8     2.8      0.5
rseg        8.875   8.875   1.875    1.875
matpro      1
current = 28000.
nseg        4.
conv = 2.54
zseg       11.8    14.1    14.1     11.8
rseg        8.875   8.875   1.875    1.875
matpro      1
current = −28000.
kbot = 1
lbot = 1
ltop = 9
fieldmap    2
begin
end
```

Table 5

Calculated effective length of the MEBT 45° dipole as a function of excitation and distance above the mid-plane

Ampere-turns	Height above mid-plane (in.)	$B_y(0)$ (G)	$\int B_y dx$ (G-in.)	l_{eff} C-magnet (in.)	l_{eff} H-magnet (in.)
28000.	0.000	11521.30	57837.87	5.020	10.040
	0.125	11521.33	57836.45	5.020	10.040
	0.250	11521.43	57838.31	5.020	10.040
	0.375	11521.57	57837.36	5.020	10.040
	0.500	11521.76	57836.78	5.020	10.040
	0.625	11521.94	57839.68	5.020	10.040
	0.750	11522.27	57836.36	5.020	10.039
	0.875	11522.57	57855.85	5.021	10.042
	1.000	11522.82	57876.75	5.023	10.046
22400.	0.000	9629.05	48554.68	5.043	10.085
	0.125	9629.06	48553.56	5.042	10.085
	0.250	9629.09	48555.18	5.043	10.085
	0.375	9629.17	48554.25	5.042	10.085
	0.500	9629.27	48553.64	5.042	10.085
	0.625	9629.32	48556.37	5.043	10.085
	0.750	9629.48	48553.13	5.042	10.084
	0.875	9629.63	48572.36	5.044	10.088
	1.000	9629.69	48591.83	5.046	10.092
16800.	0.000	7282.74	36815.77	5.055	10.110
	0.125	7282.75	36814.96	5.055	10.110
	0.250	7282.75	36816.21	5.055	10.111
	0.375	7282.75	36815.49	5.055	10.110
	0.500	7282.72	36814.94	5.055	10.110
	0.625	7282.69	36817.21	5.055	10.111
	0.750	7282.56	36814.19	5.055	10.110
	0.875	7282.50	36830.34	5.057	10.115
	1.000	7282.32	36850.25	5.060	10.120
11200.	0.000	4864.17	24599.20	5.057	10.114
	0.125	4864.18	24598.65	5.057	10.114
	0.250	4864.19	24599.47	5.057	10.115
	0.375	4864.21	24598.98	5.057	10.114
	0.500	4864.23	24598.59	5.057	10.114
	0.625	4864.25	24600.16	5.057	10.115
	0.750	4864.30	24598.16	5.057	10.114
	0.875	4864.34	24608.87	5.059	10.118
	1.000	4864.34	24622.10	5.062	10.124
5600.	0.000	2428.75	12277.32	5.055	10.110
	0.125	2428.75	12277.04	5.055	10.110
	0.250	2428.75	12277.45	5.055	10.110
	0.375	2428.77	12277.20	5.055	10.110
	0.500	2428.74	12277.03	5.055	10.110
	0.625	2428.79	12277.83	5.055	10.110
	0.750	2428.81	12276.76	5.055	10.109
	0.875	2428.83	12282.13	5.057	10.114
	1.000	2428.83	12288.87	5.060	10.119

Table 6

Calculated $K1$ values for the C-magnet as a function of excitation and distance above the mid-plane

Ampere-turns	Height above mid-plane (in.)	$K1$
28000.	0.000	0.5682
	0.125	0.5647
	0.250	0.5543
	0.375	0.5106
	0.500	0.5363
	0.625	0.4748
	0.750	0.4287
	0.875	0.3647
	1.000	0.2734
22400.	0.000	0.5631
	0.125	0.5596
	0.250	0.5490
	0.375	0.5307
	0.500	0.5045
	0.625	0.4680
	0.750	0.4207
	0.875	0.3541
	1.000	0.2570
16800.	0.000	0.5619
	0.125	0.5583
	0.250	0.5477
	0.375	0.5293
	0.500	0.5029
	0.625	0.4660
	0.750	0.4181
	0.875	0.3501
	1.000	0.2487
11200.	0.000	0.5620
	0.125	0.5585
	0.250	0.5478
	0.375	0.5294
	0.500	0.5030
	0.625	0.4653
	0.750	0.4181
	0.875	0.3499
	1.000	0.2482
5600.	0.000	0.5603
	0.125	0.5567
	0.250	0.5461
	0.375	0.5277
	0.500	0.5012
	0.625	0.4643
	0.750	0.4164
	0.875	0.3481
	1.000	0.2462

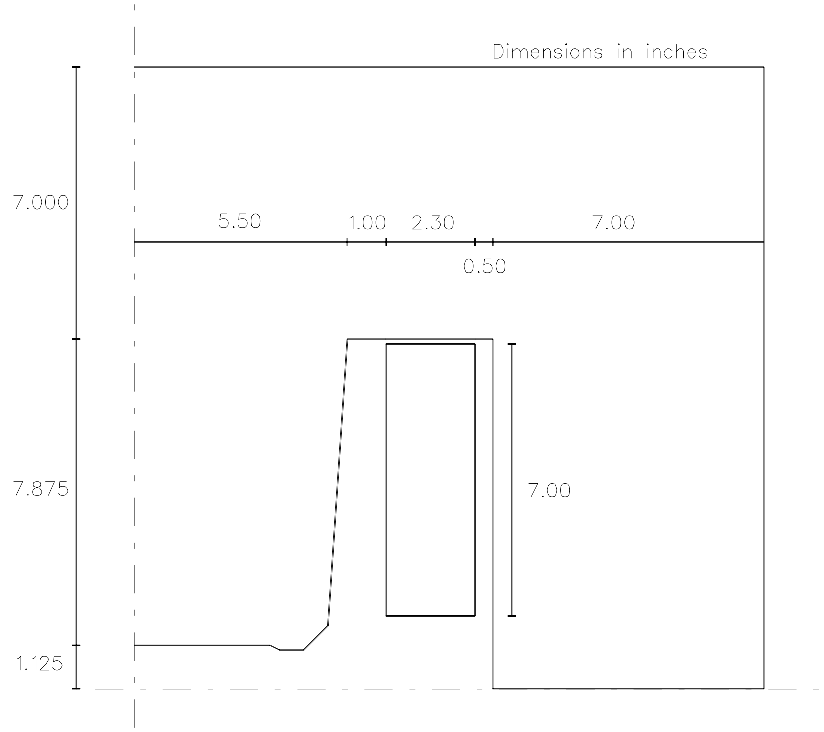


Fig. 2(a). Geometry used for the yoke of the MEBT 45° dipole.

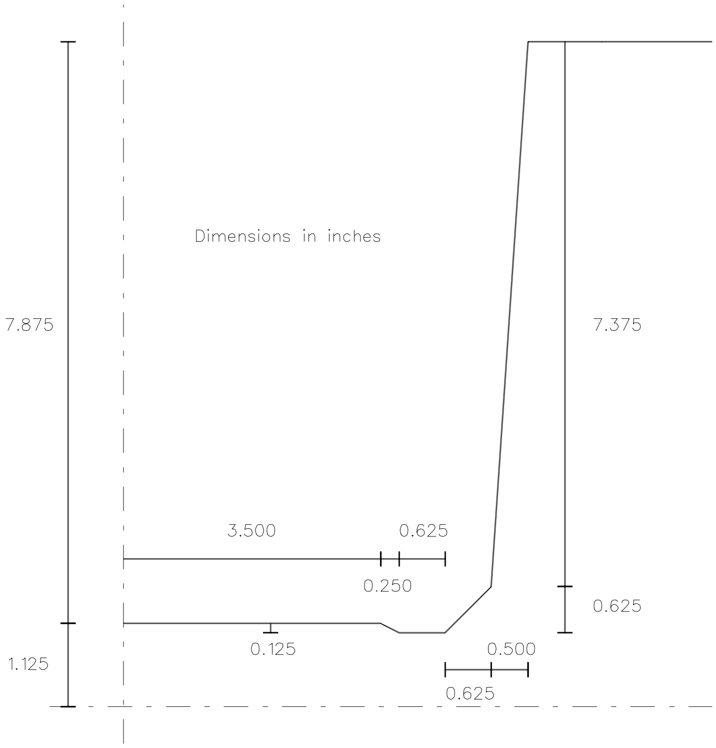


Fig. 2(b). Detailed geometry used for the pole of the MEBT 45° dipole.

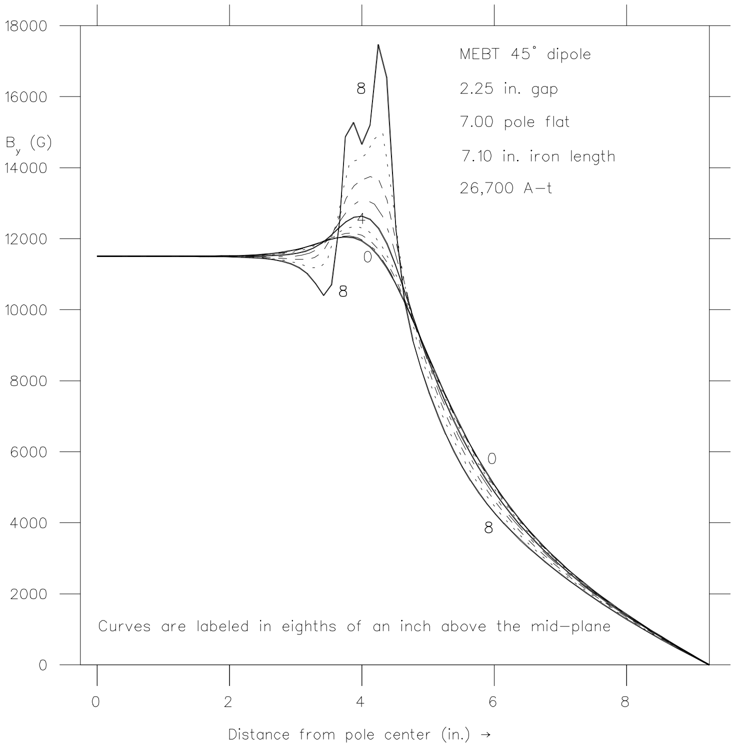


Fig. 3(a). Field at 26,700 A-t in the gap of the MEBT 45° dipole as a function of height.

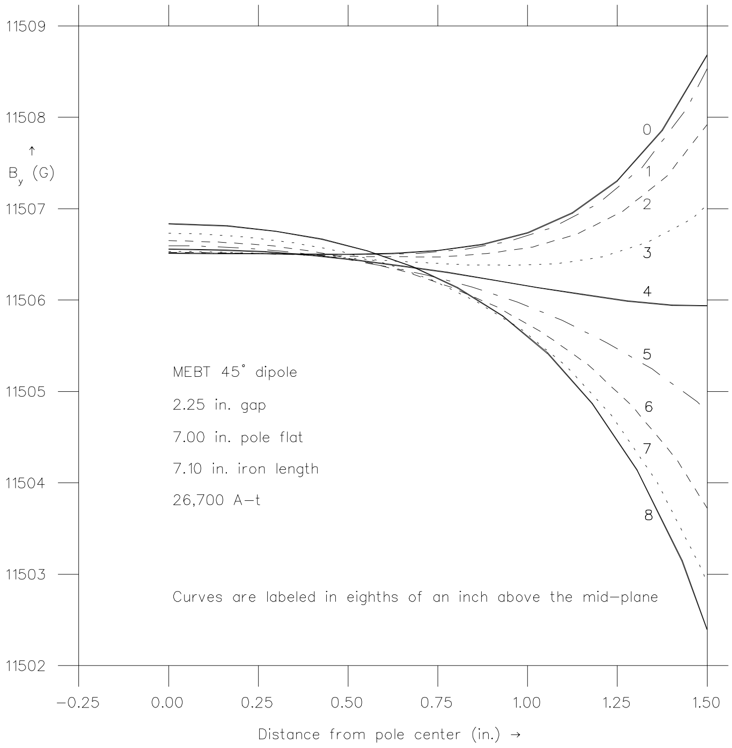


Fig. 3(b). Uniform field at 26,700 A-t in the gap of the MEBT 45° dipole as a function of height.

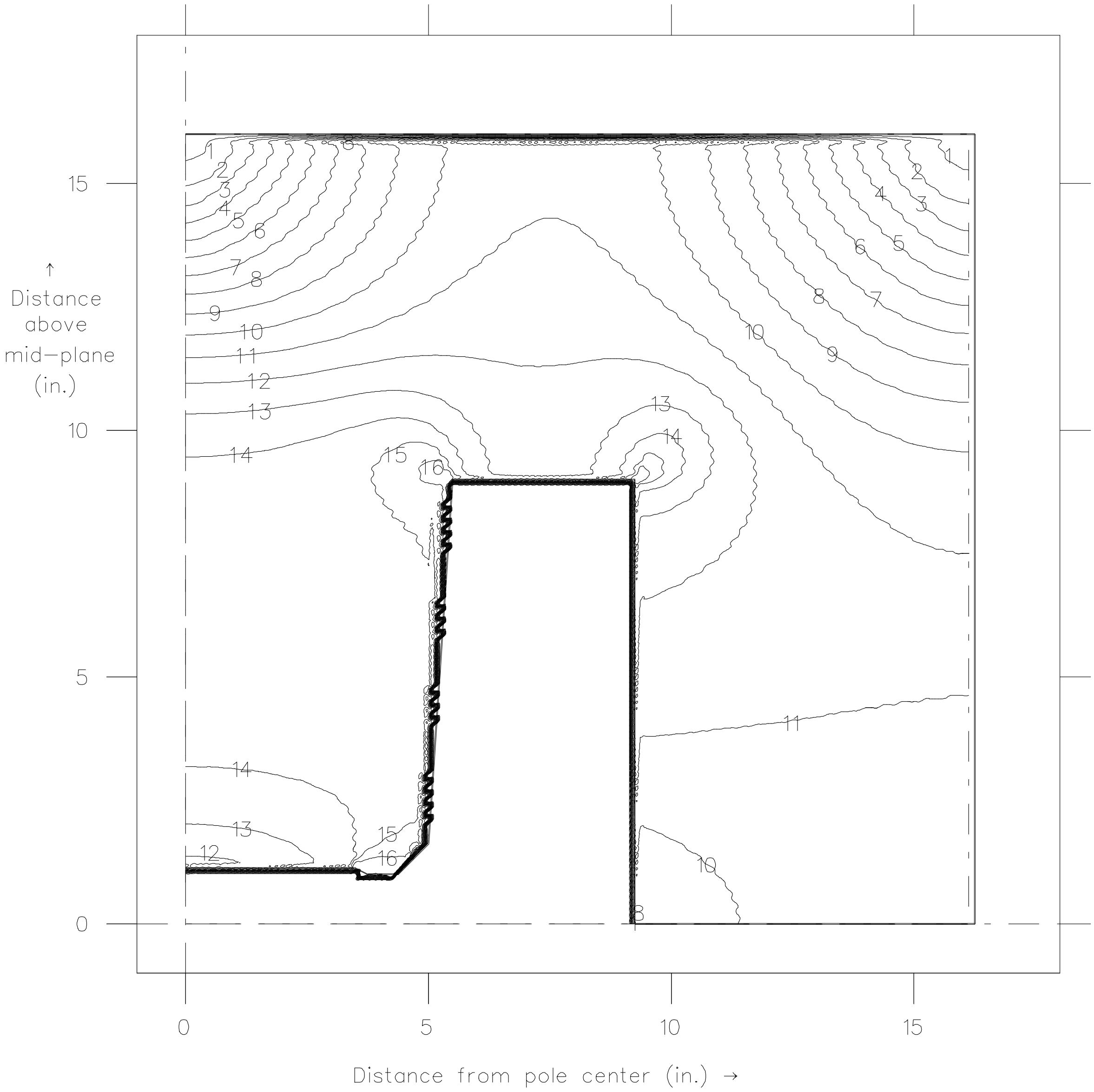


Fig. 4. Contours of the field distribution in the yoke taken from POISSON results at 26,700 A-t.

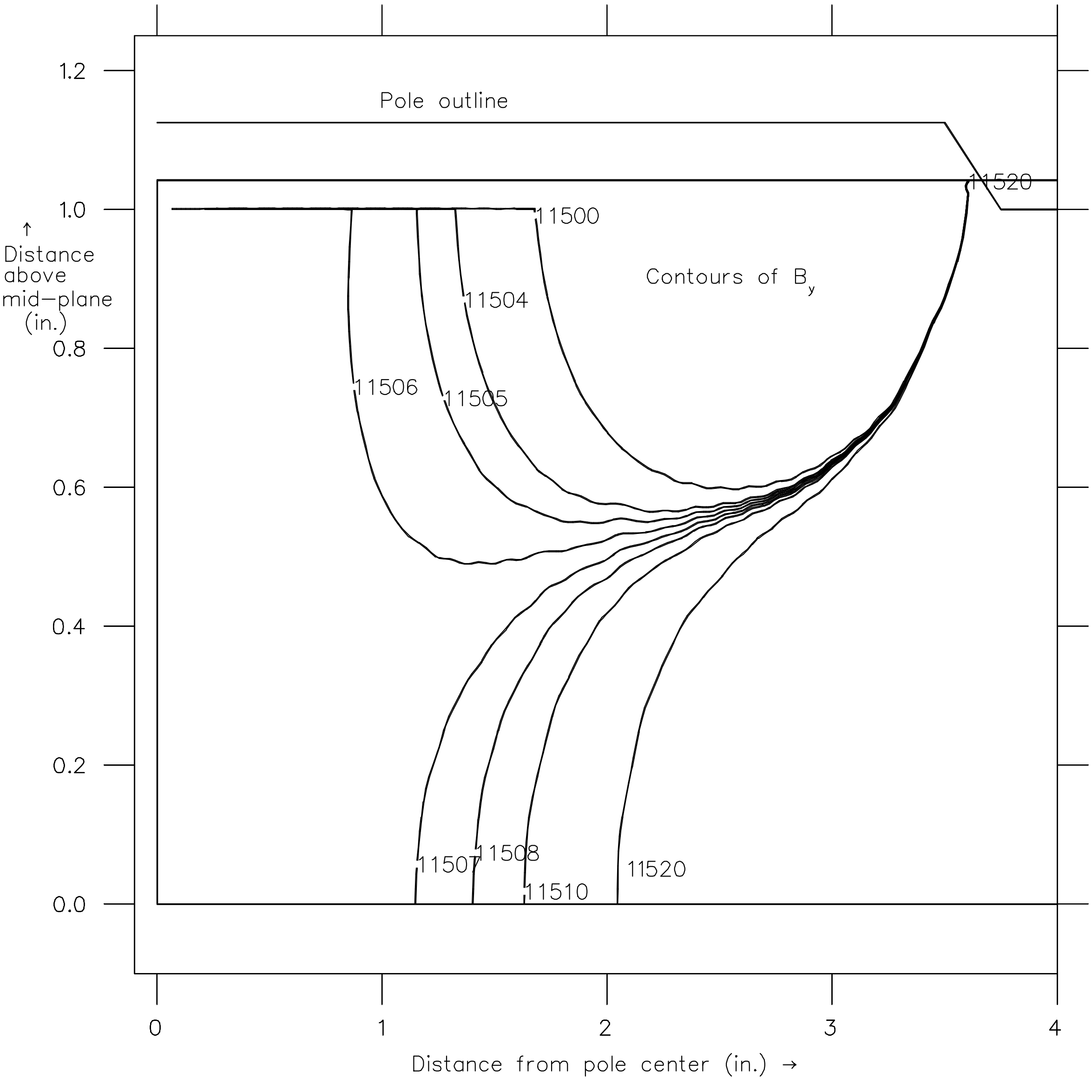


Fig. 5. Contours of the field distribution in the gap taken from POISSON results at 26,700 A-t.

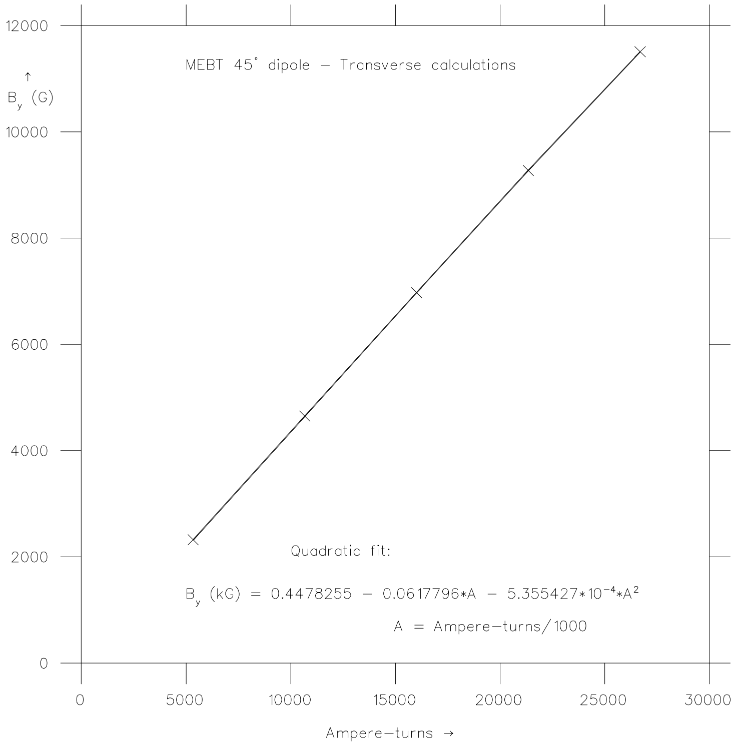


Fig. 6(a). B_y as a function of excitation for the MEBT 45° dipole – transverse calculation.

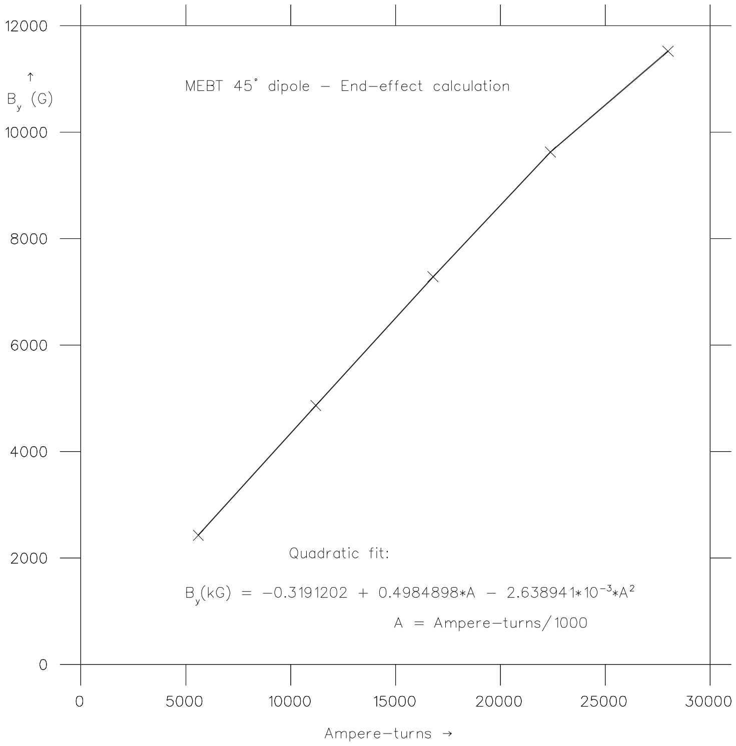


Fig. 6(b). B_y as a function of excitation for the MEBT 45° dipole – end-effect calculation.

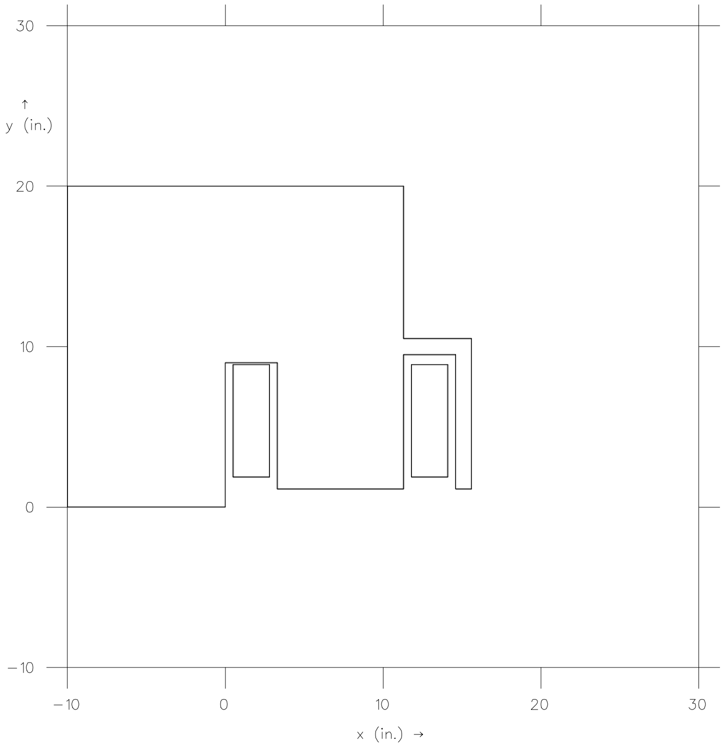


Fig. 7(a). Geometry used for the end-effect calculation.

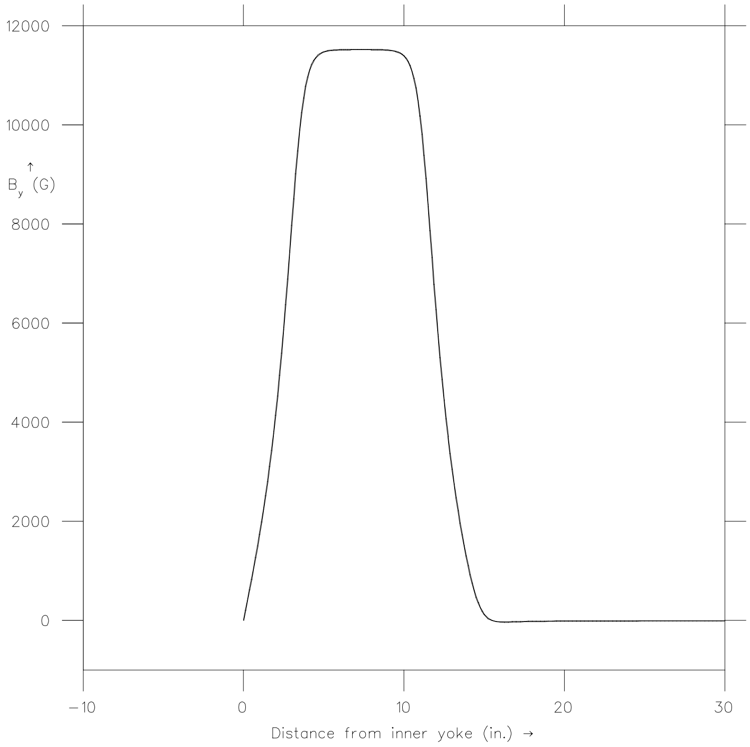


Fig. 7(b). B_y on the mid-plane for the MEBT 45° dipole – end-effect calculation.

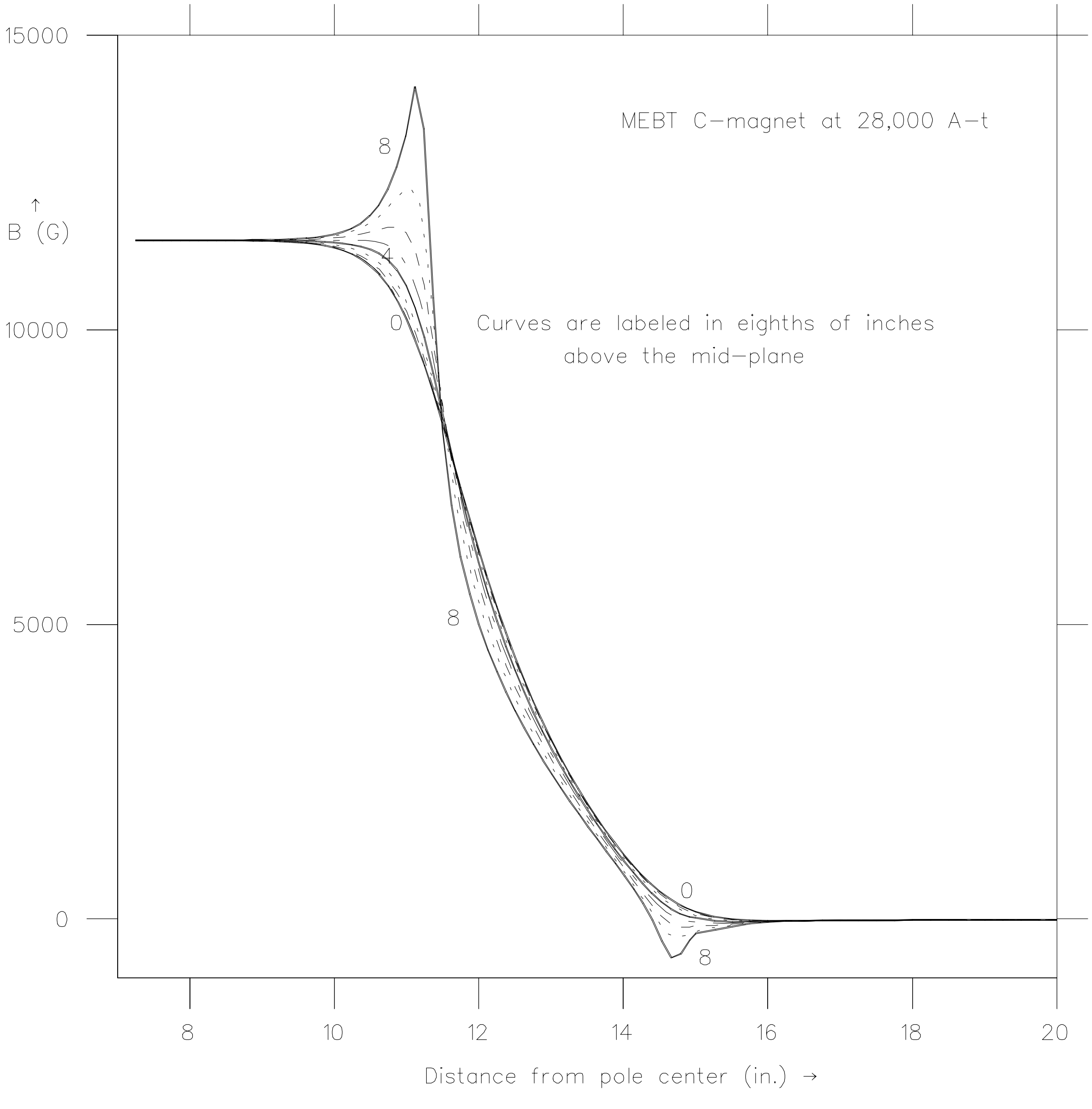


Fig. 8. Predicted field profiles at 28,000 A-t of the MEBT C-magnet as a function of distance above the mid-plane.

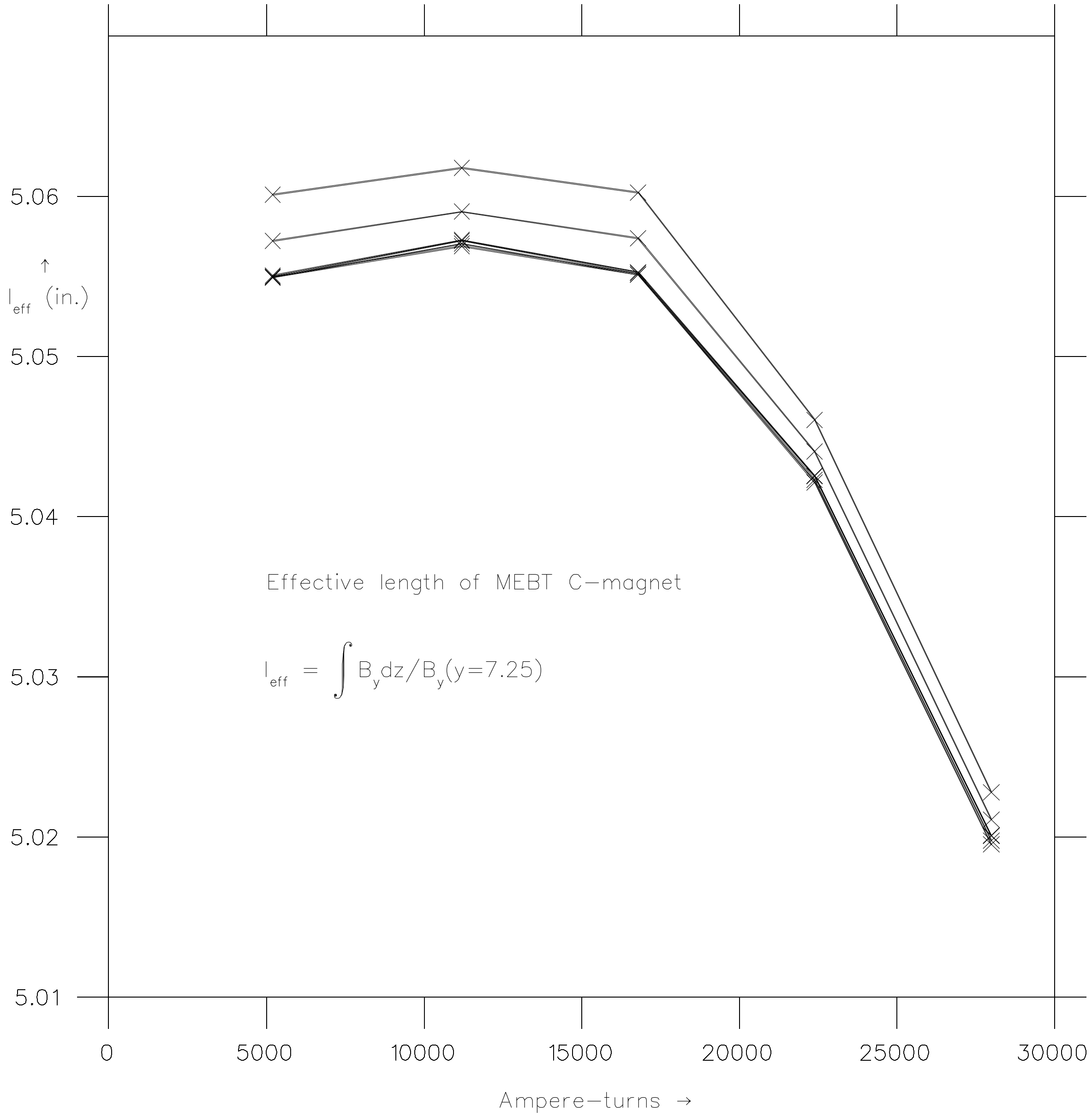


Fig. 9. Calculated effective lengths of the MEBT C-magnet as a function of excitation and distance above the mid-plane.

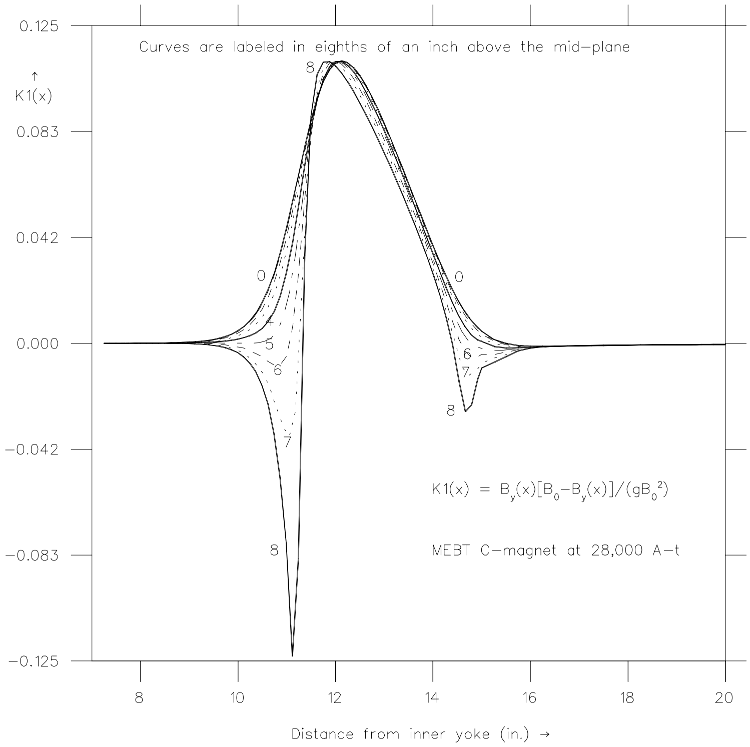


Fig. 10(a). The function $K1(x)$ of the C-magnet at 28,000 A-t.

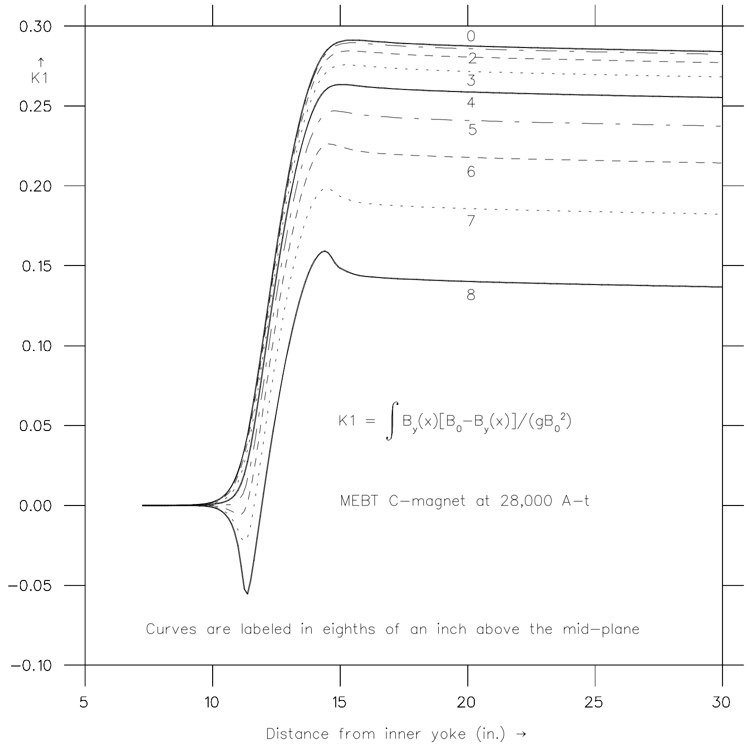


Fig. 10(b). The value of $K1 = \int K1(x)dx$ as a function of distance above the mid-plane for the C-magnet at 28,000 A-t.

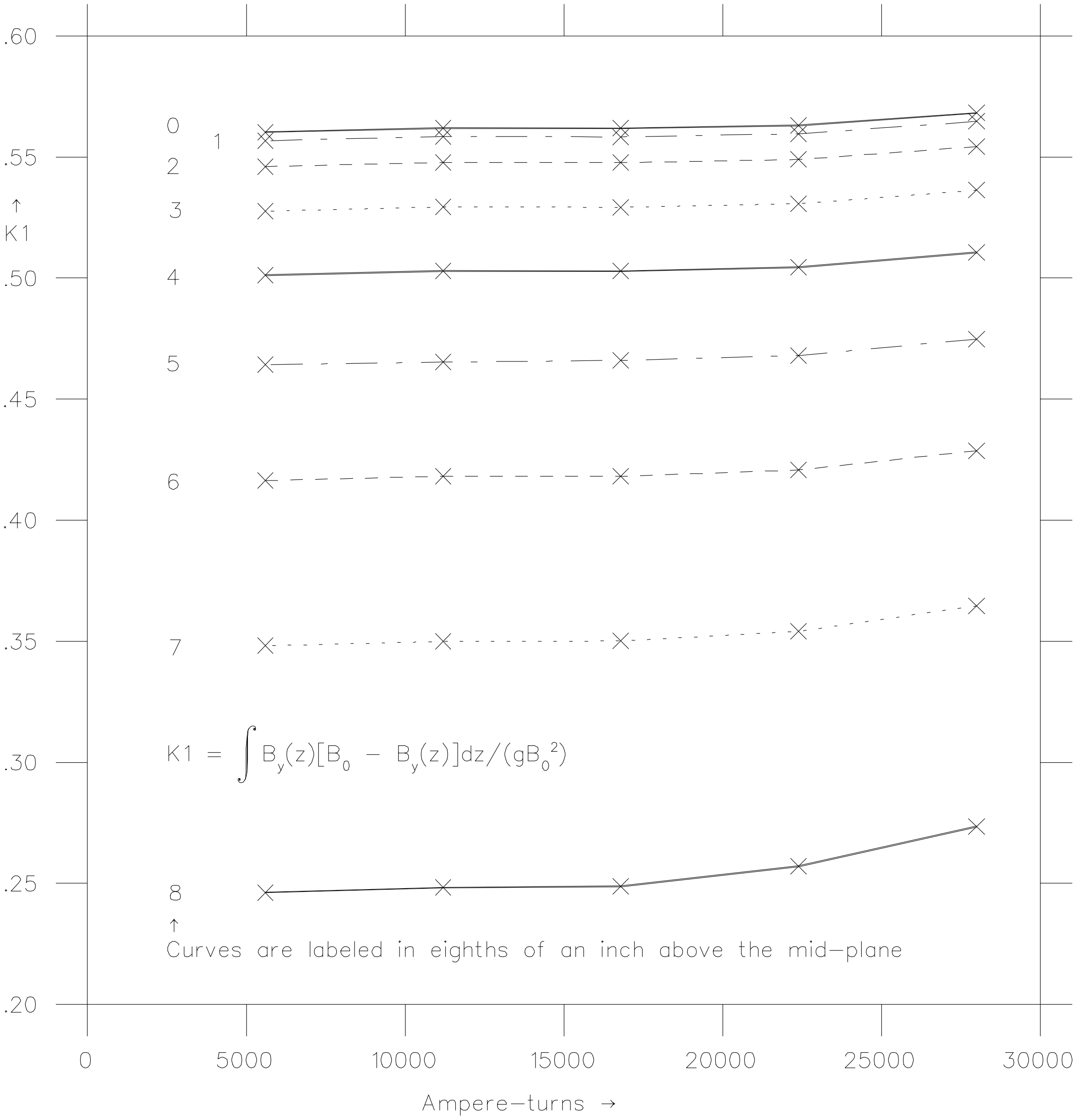
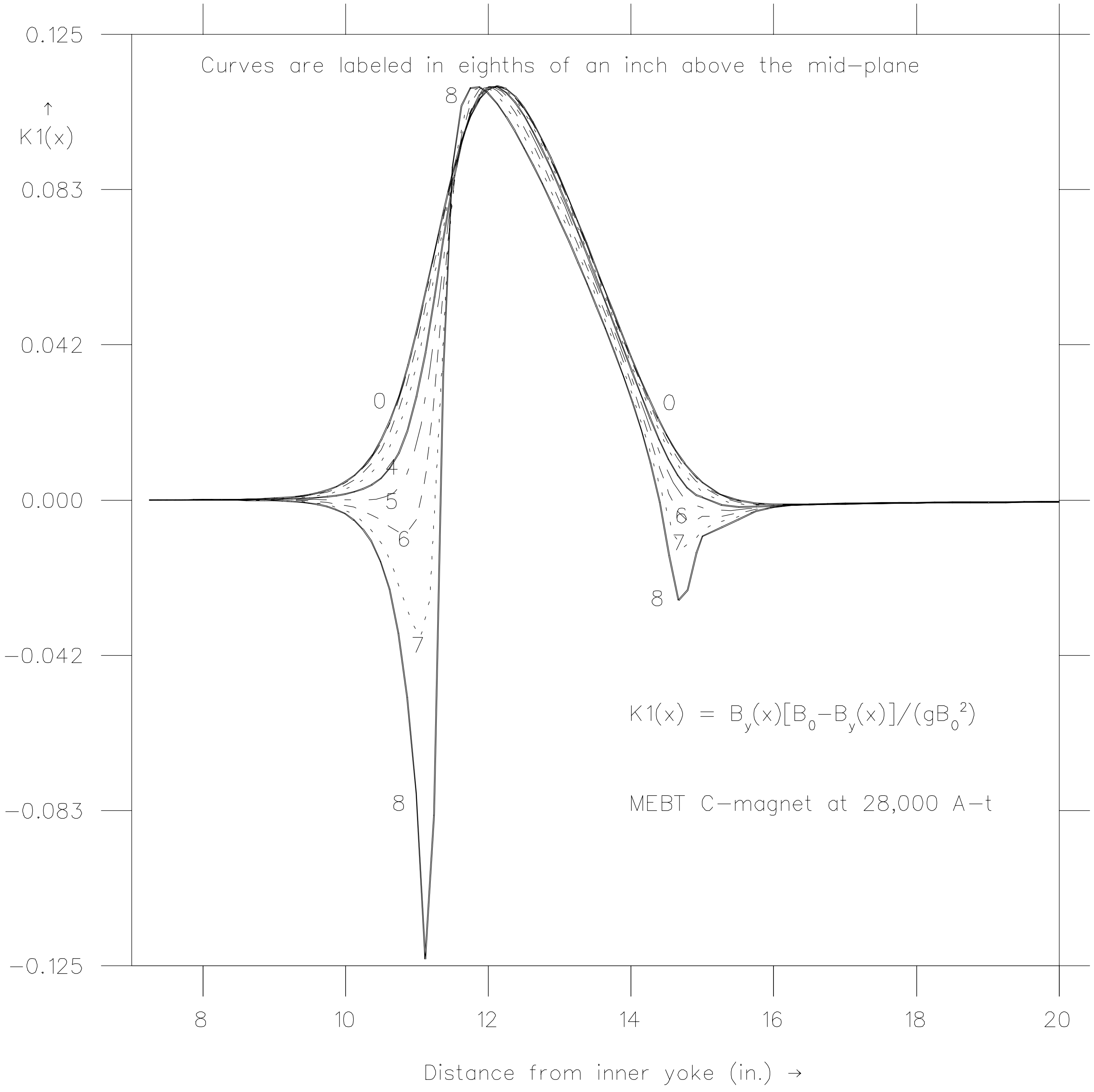
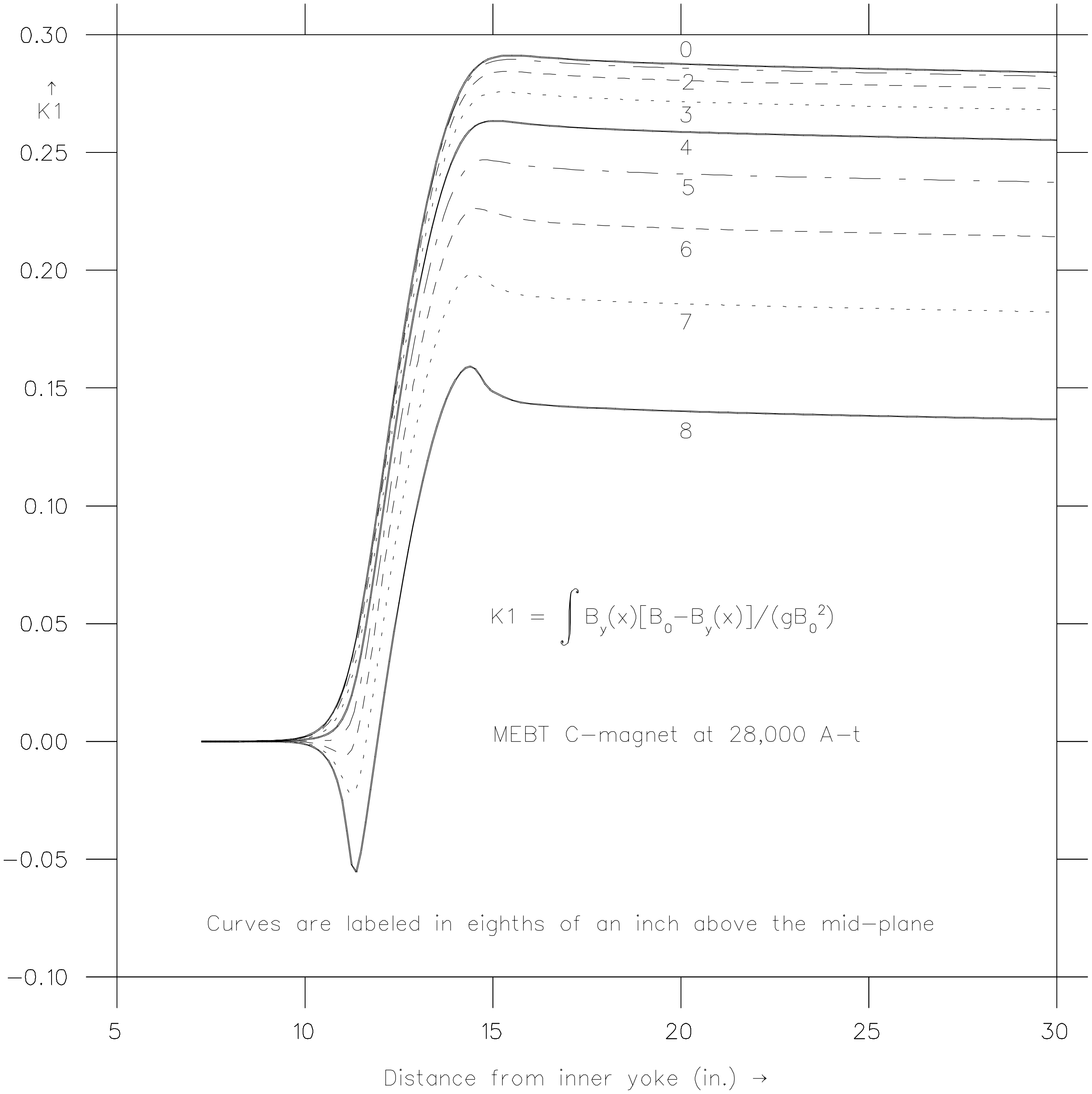
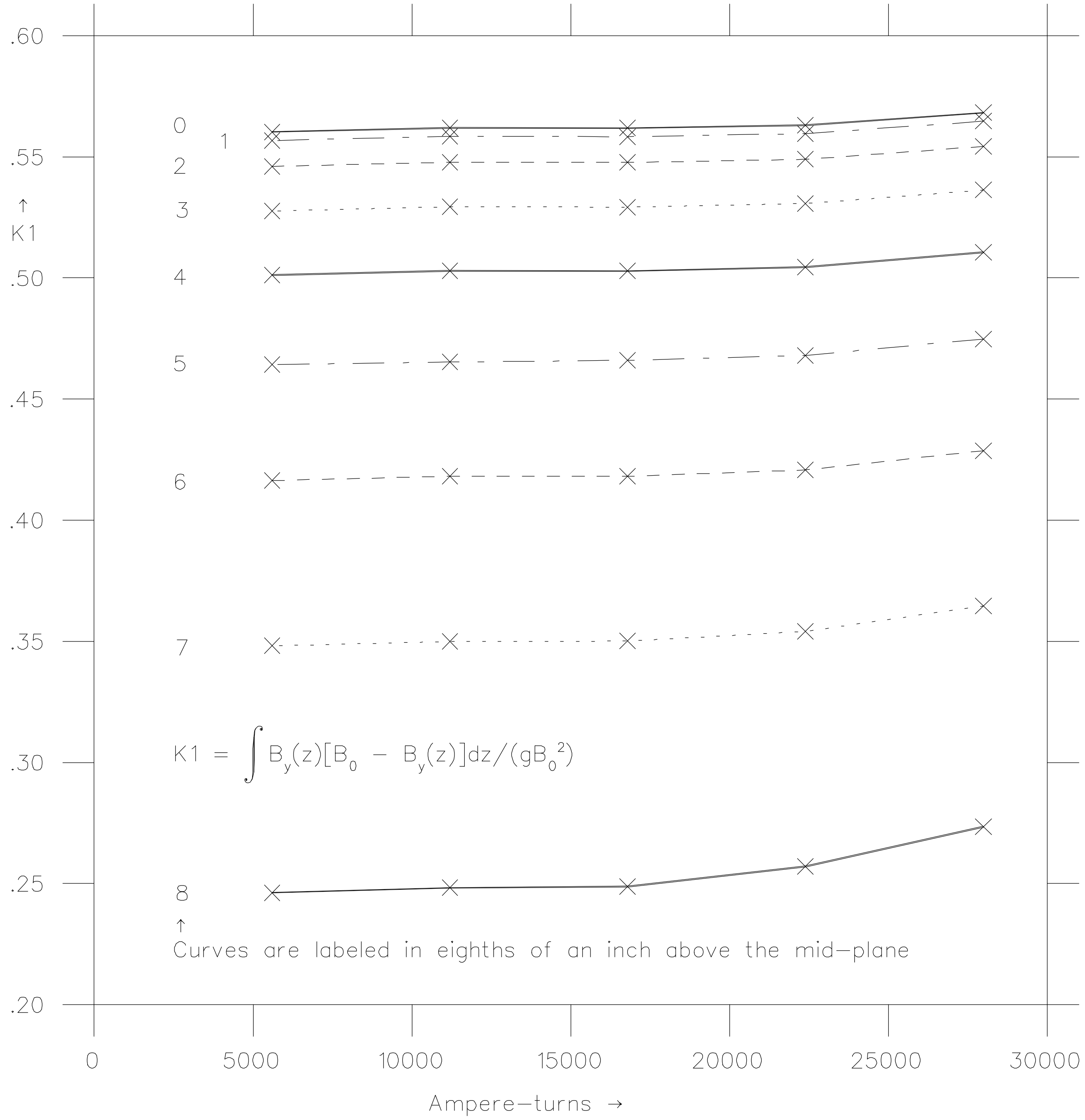


Fig. 11. Values of $K1$ of the MEBT C-magnet as a function of excitation and distance above the mid-plane.

Curves are labeled in eighths of an inch above the mid-plane
↑
K1(x)
0.125
0.083
0.042
0.000
−0.042
−0.083
−0.125
8
10
12
14
16
18
20
Distance from inner yoke (in.) →
0
4
5
6
7
8
0
6
7
8
8
K1(x) = By(x)[B0−By(x)]/(gB0²)
MEBT C−magnet at 28,000 A−t



0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
↑ K1
5
10
15
20
25
30
Distance from inner yoke (in.) →
0
2
3
4
5
6
7
8
K1 = ∫ B_y(x)[B_0−B_y(x)]/(gB_0^2)
MEBT C−magnet at 28,000 A−t
Curves are labeled in eighths of an inch above the mid−plane



$$K1 = \int B_y(z)[B_0 - B_y(z)]dz/(gB_0^2)$$

↑
Curves are labeled in eighths of an inch above the mid-plane