TRIUMF	UNIVERSITY OF ALBERTA EDMONTON, ALBERTA	
	Date 1998/10/23	*File No.* TRI-DNA-98-6
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Subject A further study of the design of quadrupoles for the DRAGON facility		

1. Introduction

Previous notes [1,2] presented a conceptual design for the 4-inch and 6-inch diameter quadrupoles for the DRAGON facility. Since those reports were issued a question has arisen regarding the linearity of the gradient of these quadrupoles. Indeed, discussion has centered around the question of what linearity of the gradient is required. This report presents a study using the program POISSON [3] of the variation of the (calculated) gradient along the x-axis ($y = 0$) as a function of distance from the quadrupole center (x) for various shapes of the pole face of the quadrupoles.

2. General approach

The basic design described in ref [1,2] has been maintained in all of the calculations presented here. As in ref [1], a yoke thickness of 1.70 inches, a 5×5×4×4×3 coil configuration of 0.3648 inch-square conductor and an excitation of 6,500 A-t was used for the 4-inch bore quadrupoles; a 0.05 inch mesh was used in each plane. For the 6-inch bore quadrupoles, a 0.075 inch mesh, a yoke thickness of 2.50 inches, a $7 \times 6 \times 6 \times 5 \times 5 \times 4$ configuration of the same conductor and an excitation of 10,500 A-t has been used as in ref [2].

However, in this study the pole width of each type of quadrupole has been increased in order to accommodate the investigation of true hyperbolic profiles for the pole faces.

3. Studies of the 4-inch bore quadrupole

In this report the following pole-face profiles were considered.

1. A true hyperbolic pole profile generated by the equation $2\,x\,y \;=\; a^2$ with a being the half-aperture of the quadrupole.
2. An approximation to a true hyperbola generated by a series of seven (7) straight cuts per half pole. This arises because, in POISSON calculations, quadrupole symmetry allows the input of one-eighth of the quadrupole geometry. Thus seven cuts per half pole implies that the actual pole profile would be composed of a total of fourteen (14) linear cuts. In addition, the pole face at the 45° symmetry point would have a shallow V-shaped profile.
3. An approximation to a true hyperbola generated by a series of five (5) straight cuts per half pole. Thus the actual pole profile would have a total of ten (10) linear cuts and, again, would have a shallow V-shaped profile at the 45° symmetry point.
4. An approximation to a true hyperbola generated by a series of three (3) straight cuts per half pole. In this case the half-pole profile is generated by one flat cut and two angled cuts. Thus the the actual pole profile would consist of five (5) linear cuts with a flat cut at the 45° symmetry point. This was done to give a (rough) approximation to the pole shapes of the Chalk River quadrupoles.
5. An approximation to a true hyperbola generated by a circular pole of the Banford radius (where $R_{pole} \;=\; 1.15\,a$) to the intersection of the circle and the true hyperbola, followed by two straight cuts to approximate the remainder of the hyperbola. The intersection of the circle and the hyperbola is most easily found if both the circle and the hyperbola that represent the pole are symmetric about the positive y-axis. Then the equation of the circular portion of the pole is

$$x^2 \,+\, (y \,-\, y_0)^2 \;=\; R_{pole}^2$$

where $y_0 = a + R_{pole}$. That of the hyperbolic pole is

$$y^2 - x^2 = a^2$$

These two curves intersect at $(x_1, y_1) = (0, a)$ and at $(x_2, y_2) = (\pm\sqrt{R_{pole}^2 - a^2}, R_{pole})$. We are interested in these coordinates relative to a coordinate system in which the pole is symmetric about the 45° symmetry line of the coordinate system. Designating the latter coordinates with a subscript '45', their relation to the above coordinates is given by

$$\begin{aligned} x_{45} &= x\cos\theta - y\sin\theta = (x + y)/\sqrt{2} \\ y_{45} &= x\sin\theta + y\cos\theta = (y - x)/\sqrt{2} \end{aligned}$$

where $\theta = -45°$. Thus, in our case with $R_{pole} = 2.450$ in. and $a = 2.125$ in., the required intersection points are $(x_{45}, y_{45}) = (1.503$ in., 1.503 in.$)$ and $(x_{45}, y_{45}) = (2.595$ in., 0.870 in.$)$.

6. A pole face generated with the Banford radius so as to cover a full 45° angle. Thus the pole contour is a full semi-circle. This we refer to as the quarter circle approximation; in this case the pole width is 4.888 in.

7. Finally, for comparison, the pole face design generated as is ref [1]. We refer to this as the truncated circle approximation.

In all cases that involved straight cuts, their location was determined by simply 'eye-balling' a fit to the hyperbolic curve. We note again that in the cases of the 7-cut and 5-cut approximations the pole face at the 45° symmetry point had a shallow V-shaped form and that for the 3-cut approximation the pole face at that point was made flat so as to (roughly) simulate the shape of the Chalk River quadrupoles.

Also, in all cases, the pole sides were *not* chamfered but were kept straight. Thus the pole sides were parallel to the 45° symmetry line. For each of the hyperbolic, 7-cut, 5-cut and 3-cut pole configurations the pole width was taken to be 4.4507 inches. In the quarter circle approximation the pole width used was 4.8875 inches. These are to be compared with a pole width of 3.6125 inches that was used in the original design of ref [1].

3.1 General results

The results of all of the above studies for a quadrupole with bore of 4.25 inches are shown in figure 1. There we plot the gradient calculated by POISSON along the x-axis (y = 0) as a function of the distance from the quadrupole center ($x = 0$). As in ref [1], the mesh used was 0.05 inch in each of the vertical and horizontal planes. In its relaxation calculations POISSON uses the six nearest neighbors of a point. Consequently, at small values of x—that is, close to the 45° symmetry line—interpolation is poorer and the gradient calculation becomes inexact. Another way of stating this is that the field itself is calculated from vector potential differences and the gradient is then calculated from field differences. It is for this reason that results are shown in figure 1 only for values of x greater than 0.4 in.

In what follows, when we talk of the predicted gradient of any configuration we are referring to the its *absolute value*. The nominal design gradient for these 4-inch quadrupoles is 500 G/cm and, as shown in figure 1, all of the pole configurations considered here produce that value. It is seen that the predicted gradient variation of the 7-cut and 5-cut approximations reasonably reproduce that for a hyperbolic pole contour. All other pole profiles are predicted to have a reasonably flat gradient to approximately one-half of the quadrupole aperture. Beyond that point a decrease of the gradient is predicted.

We consider these two groups separately.

3.2 Results of the hyperbolic pole and of the 7-cut and 5-cut approximations

Figure 2 shows, on an expanded scale, the calculated gradients for a hyperbolic pole face and for the 7-cut and 5-cut approximations. The predicted value of the gradient for the hyperbolic contour is seen to decrease monotonically as the distance from the quadrupole center increases. Those of the 7-cut and 5-cut approximations are seen to oscillate slightly.

In order to produce a useful (and, possibly, doubtful) comparison of these results we take a simple average of the predicted maximum and minimum values over the full aperture, 0.4 in. $\leq x \leq$ 2.0 in., and consider the percentage differences from those average values. Thus we have

Maximum and minimum values of $|\partial B_y/\partial x|$ over 0.4 in. $\leq x \leq$ 2.0 in.

	Hyperbolic	7-cut approx.	5-cut approx.
Maximum (G/cm)	556.45	554.84	553.44
Minimum (G/cm)	555.40	553.83	552.58
Average (G/cm)	555.93	554.34	553.01
Deviation from average (%)	$\pm$0.094	$\pm$0.091	$\pm$0.078

This comparison shows that each of the above pole profiles is predicted to produce, on average, a variation of less than $\pm$0.1% in the gradient over the full aperture of the quadrupole. This variation should be acceptable. However, it is debatable as to whether the 7-cut approximation (that requires a total of fourteen linear cuts per pole) or the 5-cut approximation (that requires a total of ten linear cuts per pole) would be less expensive to manufacture than would a pole with a true hyperbolic face.

A similar comparison over 80% of the aperture, 0.4 in. $\leq x \leq$ 1.6 in., produces the following.

Maximum and minimum values of $|\partial B_y/\partial x|$ over 0.4 in. $\leq x \leq$ 1.6 in.

	Hyperbolic	7-cut approx.	5-cut approx.
Maximum (G/cm)	556.07	554.84	553.29
Minimum (G/cm)	555.45	554.66	552.58
Average (G/cm)	555.76	554.75	552.94
Deviation from average (%)	$\pm$0.056	$\pm$0.016	$\pm$0.064

Thus the predicted variation of the gradient of a quadrupole with a hyperbolic pole over its 80% aperture is roughly 60% of that over its full aperture. The small variation of the 7-cut approximation belies the predicted oscillatory behavior of the gradient. A similar comment applies to the 5-cut approximation for which that behavior is much more evident.

3.3 Quarter circle and 3-cut approximations

Figure 3 is a plot of the predicted variation of the quadrupole gradients of the quarter circle and 3-cut approximations to a hyperbolic pole. For these we consider only the range 0.4 in. $\leq x \leq$ 1.6 in.—that is, over 80% of the aperture. Clearly, the predicted variation is more extreme for these cases than for those considered above. However, if we again perform a simple averaging we find the following.

Maximum and minimum values of $|\partial B_y/\partial x|$ over 0.4 in. $\leq x \leq$ 1.6 in.

	Quarter circle approx.	3-cut approx.
Maximum (G/cm)	560.15	561.04
Minimum (G/cm)	558.88	557.23
Average (G/cm)	559.52	559.14
Deviation from average (%)	$\pm$0.113	$\pm$0.341

Thus, of these two cases, the quarter circle approximation is predicted to provide a smaller variation of the quadrupole gradient over 80% of its aperture. The larger deviation of the 3-cut approximation is, of course, caused by the monotonic decrease predicted of the gradient. The smaller deviation predicted for the quarter circle approximation is a result of its predicted oscillatory behavior. Such comparison, however, may be invalid because no particular care was taken (in this study) to optimize the locations of the three cuts.

3.4 Discussion of the 4-inch quadrupole results

A study of the effect of various pole shapes on the gradient of the 4-inch quadrupole design for the DRAGON facility has been undertaken. The conclusion is that a hyperbolic pole is best but that poles shapes formed by seven or five straight cuts are good approximations to that shape. However, the question as to whether the latter would be more economical must be answered.

It is also shown that the use of a semi-circular pole is predicted to produce less variation in the gradient than does one formed with three straight cuts. This conclusion is subject to the caveat noted in section 3.3 above. Regardless, these shapes would be suitable if a variation in gradient of the order of $\pm 0.5\%$is deemed satisfactory.

Before the design of these quadrupoles is finalized it is necessary that the optical requirements for the linearity of the gradient be established.

4. Studies of the 6-inch bore quadrupole

Similar studies to the above were carried out for the 6-inch quadrupoles for the DRAGON facility. However, on the assumption that results for 6-inch quadrupoles with straight cuts would be similar to those of the studies above, only a quadrupole as designed in ref [2)] and one with a hyperbolic pole profile were investigated.

The pole width of the 6-inch quadrupole of the nominal design of ref [2)] was 5.300 inches. Two quadrupoles with hyperbolic pole faces, one with a pole width of 6.371 inches and one with a pole width of 6.850 inches were studied in order to ascertain the effect of an increased extension of the hyperbolic profile. The nominal design we term Circular; the narrower hyperbolic-pole quadrupole is designated Hyperbolic #1 and the wider hyperbolic-pole quadrupole is called Hyperbolic #2.

Figure 4 shows the computed variation of the gradient along the x-axis for the three cases. The variation over the full aperture is shown to illustrate the rapid variation of the calculated gradient alluded to above. It is seen that the calculated gradient of the original design is reasonably flat over roughly the half-aperture of the quadrupole. Those of the two hyperbolic pole contours are seen to be much more uniform over the full aperture.

Figure 5 is a plot, on an expanded scale, of the calculated gradients of the two hyperbolic-contoured pole faces. The implication of this plot is that there is little to be gained by increasing the pole width beyond 6.4 inches.

Again, we compare these three calculations by taking a simple average of the predicted maximum and minimum values over the full aperture, 0.52 in. $\leq x \leq$ 3.07 in., and consider the percentage differences from those average values. Thus we have

Maximum and minimum values of $|\partial B_y/\partial x|$ over 0.52 in. $\leq x \leq$ 3.07 in.

	Hyperbolic #1	Hyperbolic #2	Circular
Maximum (G/cm)	416.36	416.30	417.86
Minimum (G/cm)	414.81	415.05	387.09
Average (G/cm)	415.59	415.68	402.48
Deviation from average (%)	$\pm$0.187	$\pm$0.155	$\pm$3.822

From this data it is clear that a quadrupole design with a hyperbolic pole profile is clearly better when one considers the variation of the gradient over the entire aperture of the quadrupole.

We may repeat the above averaging process over 80% of the quadrupole aperture—that is, for 0.52 in. $\leq x \leq$ 2.47 in.—and again consider the percentage differences from those average values. Thus we obtain

Maximum and minimum values of $|\partial B_y/\partial x|$ over 0.52 in. $\leq x \leq$ 2.47 in.

	Hyperbolic #1	Hyperbolic #2	Circular
Maximum (G/cm)	416.36	416.30	417.86
Minimum (G/cm)	415.80	415.36	410.67
Average (G/cm)	416.08	415.83	414.27
Deviation from average (%)	±0.067	±0.113	±0.868

Again it is seen that even over 80% of the quadrupole aperture, a hyperbolic pole profile is preferable to a circular one.

5. Discussion

This report has presented a study of the effects of various pole-face profiles on the linearity of the gradient along the x-axis for both 4-inch and 6-inch diameter quadrupoles. Not surprisingly, the conclusion in both instances is that a hyperbolic pole profile produces the most linear gradient.

It is pointed out, however, that other profiles could be viable if a gradient linearity of better than ±0.1% is not required and/or if only one-half of the quadrupole aperture is filled. Consequently, for the DRAGON quadrupoles in particular, further studies of these criteria are necessary before its quadrupole designs are finalized.

For completeness, figures 6 and 7 show the dimensions of the 4-in. bore quadrupole as designed with a hyperbolic pole contour. Figures 8 and 9 show similar data for the 6-in. bore quadrupole with a hyperbolic pole contour.

References

1. G.M. Stinson, *A conceptual design for the 4-inch diameter quadrupoles for the* DRAGON *facility*, TRIUMF Report TRI-DNA-98-4, July, 1998.

2. G.M. Stinson, *A conceptual design for the 6-inch diameter quadrupoles for the* DRAGON *facility*, TRIUMF Report TRI-DNA-98-5, July, 1998.

3. M. T. Menzel and H. K. Stokes, *User's Guide for the POISSON/SUPERFISH Group of Codes*, Los Alamos National Laboratory Report LA-UR-87-115, January, 1987.

Note added in proof: As this study was completed it was realized that with alternate coil configurations the transverse dimensions of these quadrupoles could be reduced. Figure 10 shows a 4-in. quadrupole with a hyperbolic pole and a $6 \times 5 \times 4 \times 3 \times 2$ coil configuration. It is seen that the pole depth is decreased from 3.926 in. shown in figure 6 to 3.547 in. with the revised coil configuration. Also the 3.826 in. dimension shown that figure is reduced to 3.447 in. This reduction is achieved by removing one layer of coil parallel to the yoke. Thus the dimension of 13.802 in. shown in figure 7 is reduced to 13.044 in. (=2(2.225 in. + 3.447 in.) + 1.700 in.). Similarly, as shown in figure 11, if a $6 \times 6 \times 5 \times 5 \times 4 \times 3 \times 2 \times 1$ coil configuration is used for the 6-in. quadrupole, the pole depth is decreased from the 4.646 in. dimension of figure 8 to 4.294 in. and the 4.585 in. dimension of that figure is reduced to 4.234 in. Again, this reduction is achieved by removing one layer of coil parallel to the yoke. The 18.042 in. dimension shown in figure 9 is thus reduced to 17.339 in. These comments are, of course, subject to a POISSON verification of the suitability of such designs.

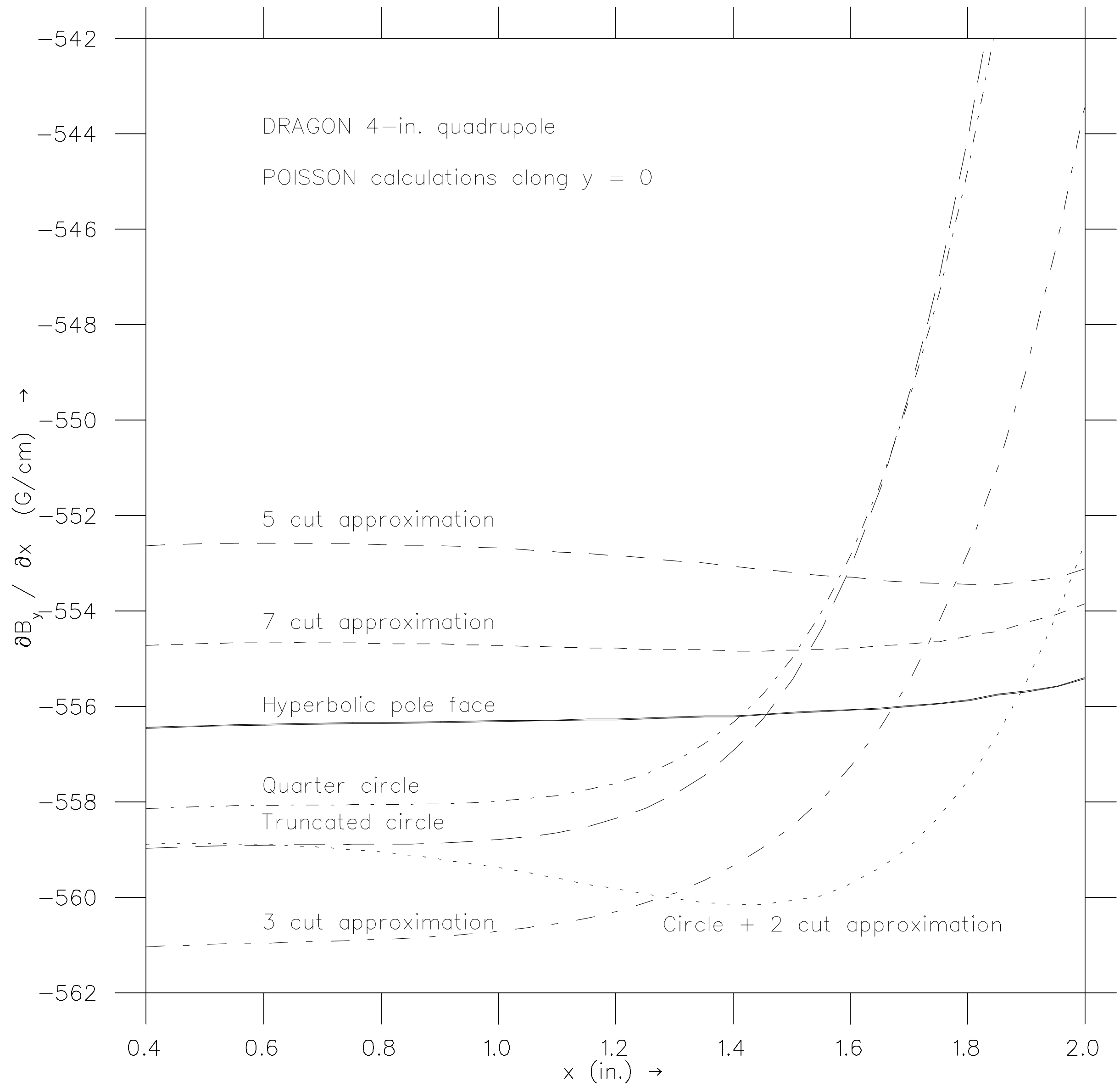

Fig. 1. Variation of the calculated field gradient along $y = 0$ as a function of x for the pole shapes considered for the 4-inch quadrupoles.

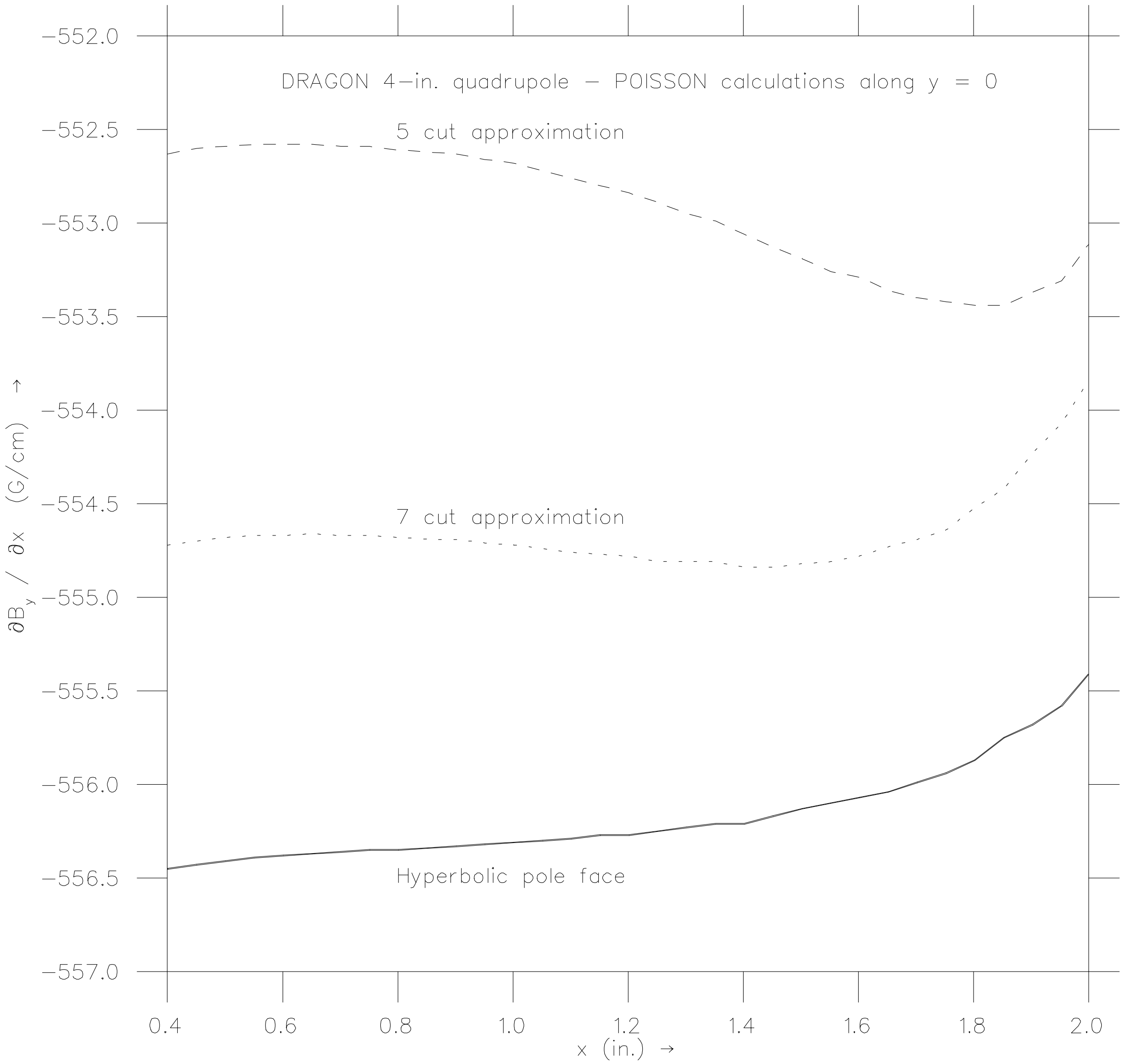

Fig. 2. Variation of the calculated field gradient along $y = 0$ as a function of x for the hyperbolic pole shape and the 7-cut and 5-cut approximations for the 4-inch quadrupoles.

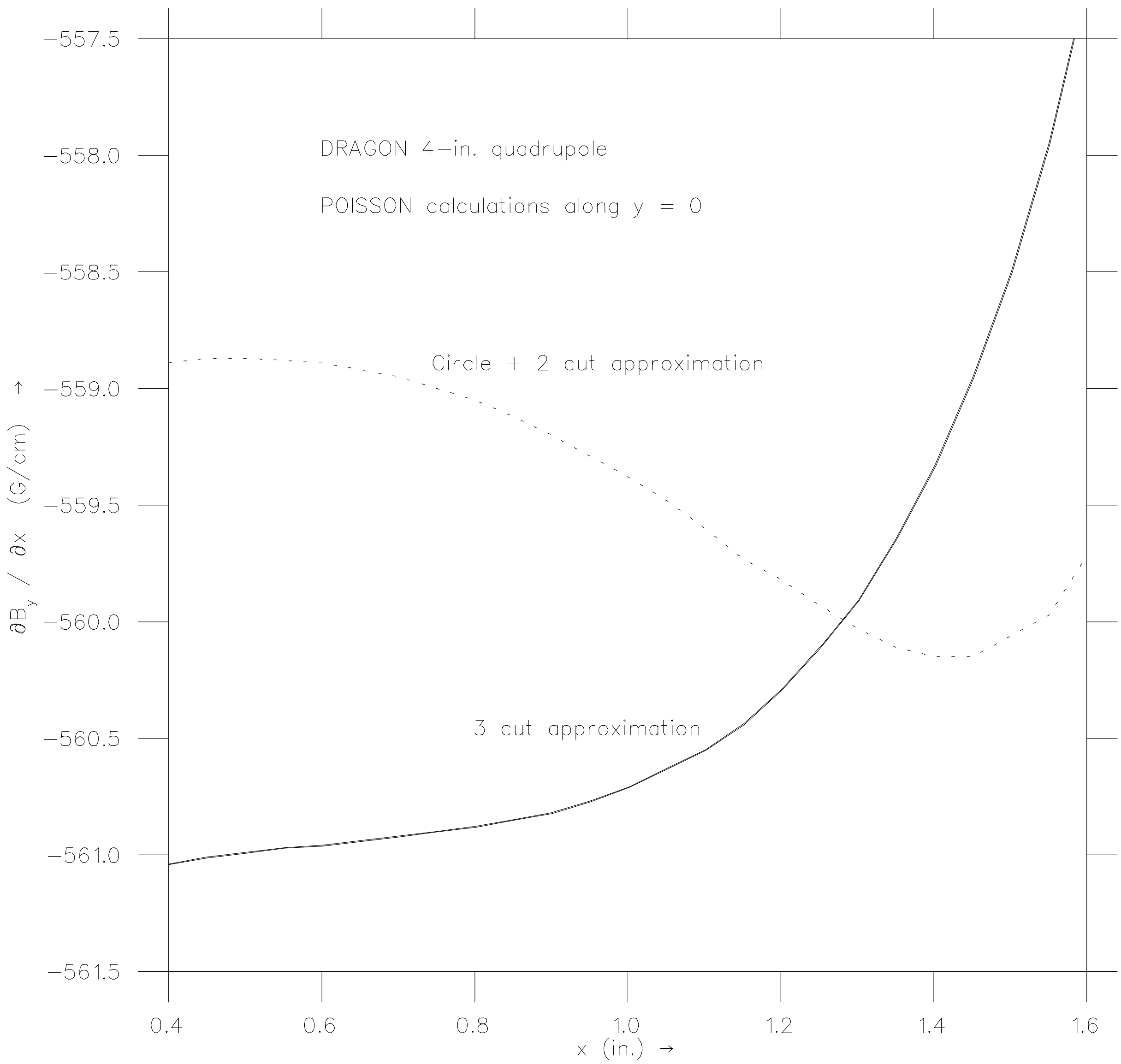

Fig. 3. Variation of the calculated field gradient along $y = 0$ as a function of x for the quarter circle and 3-cut approximations for the 4-inch quadrupoles.

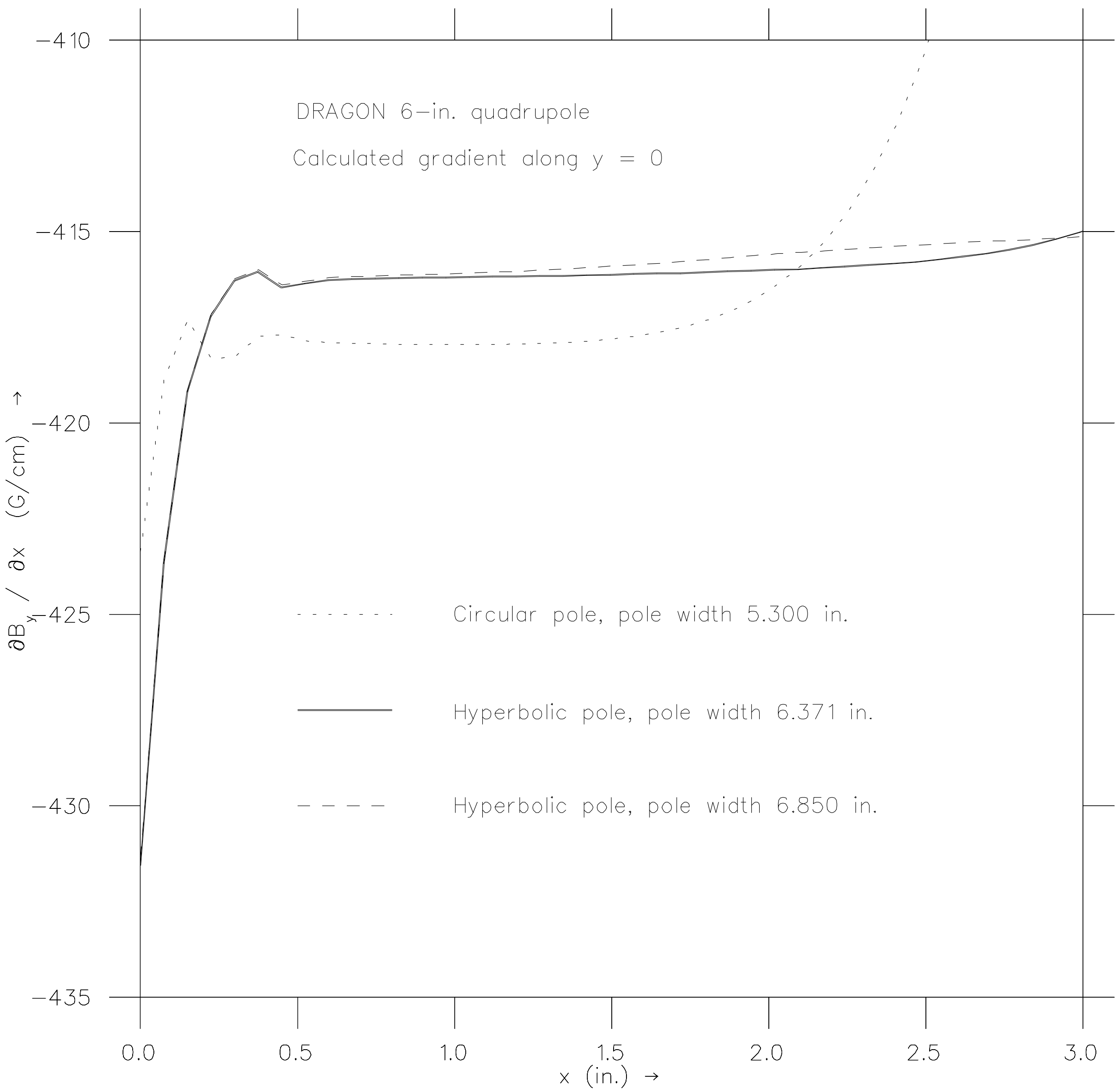

Fig. 4. Variation of the calculated field gradient along $y = 0$ as a function of x for the circular and hyperbolic pole faces for the 6-inch quadrupoles.

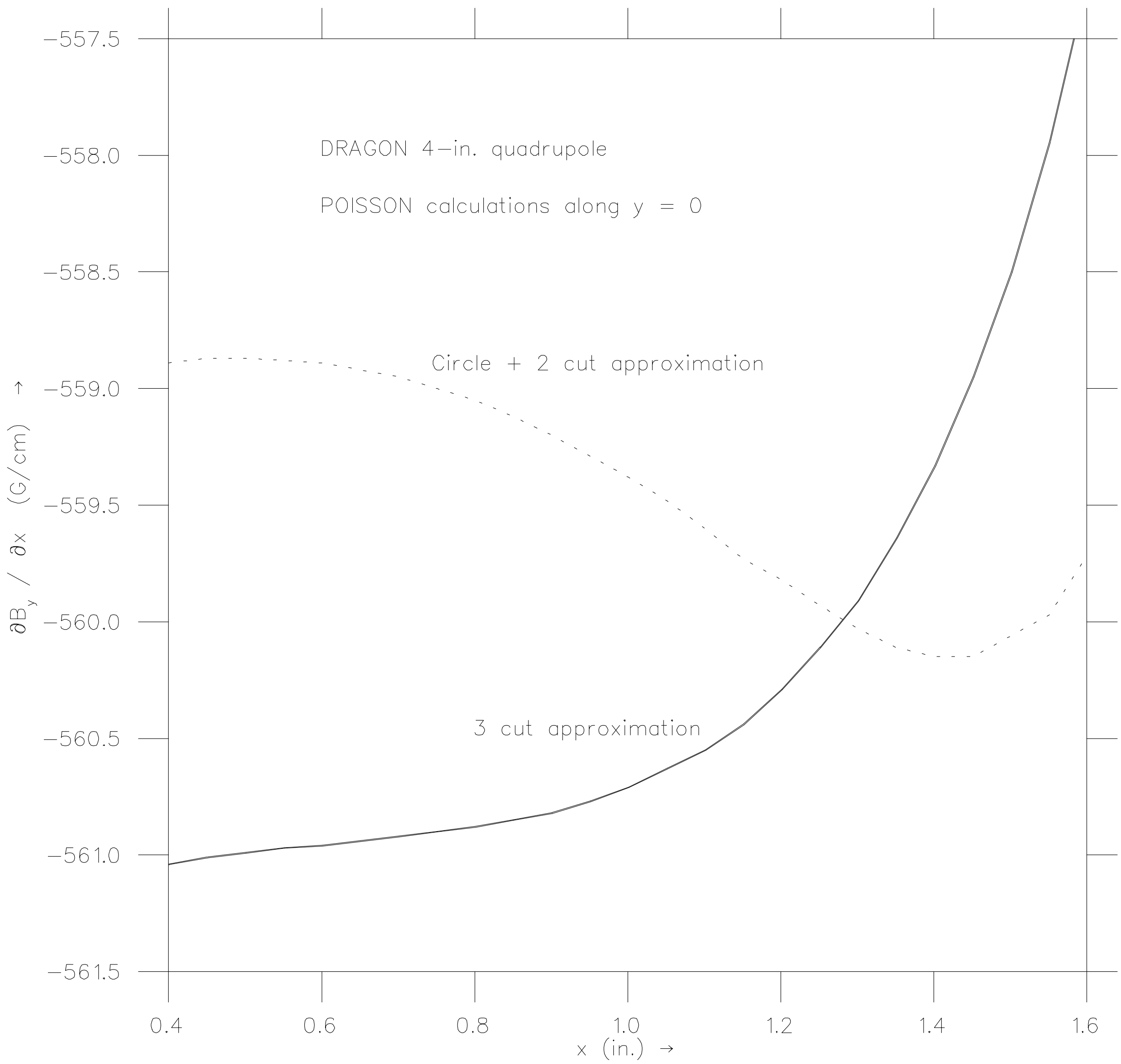

Fig. 5. Variation of the calculated field gradient along $y = 0$ as a function of x for the hyperbolic pole faces for the 6-inch quadrupoles.

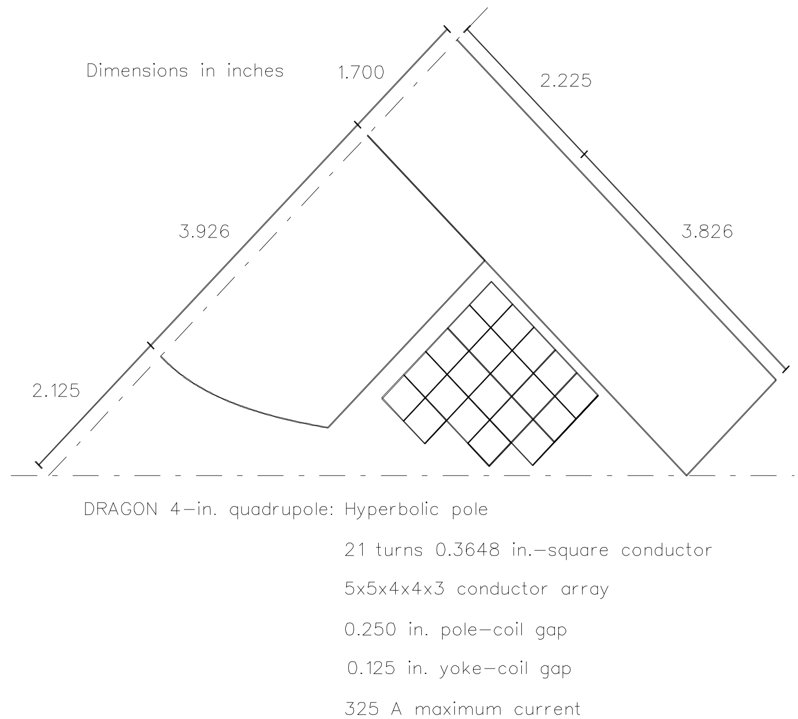

Fig. 6. Dimensions of an octant of a 4-in. quadrupole with hyperbolic pole; yoke thickness = 1.70 in.

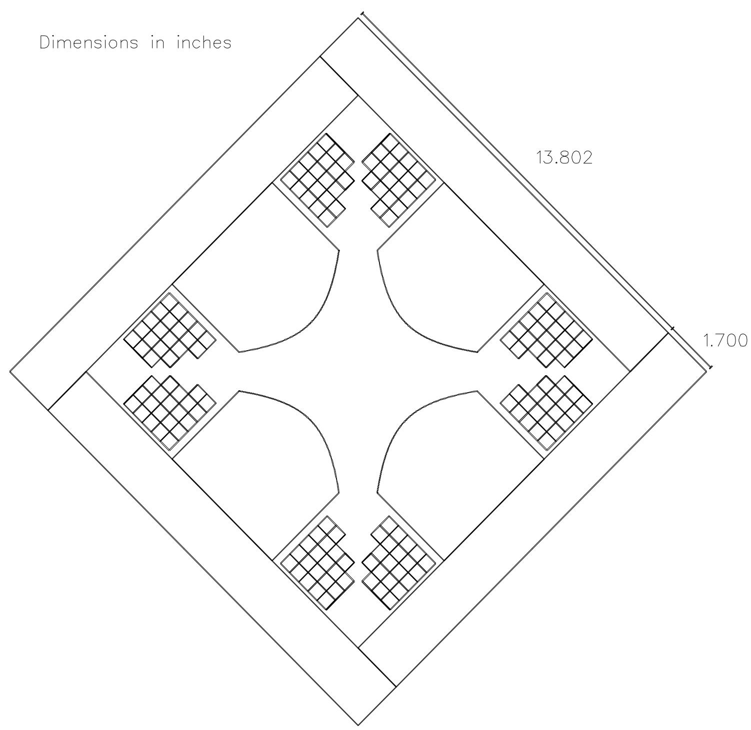

Fig. 7. Overall dimensions of such a quadrupole.

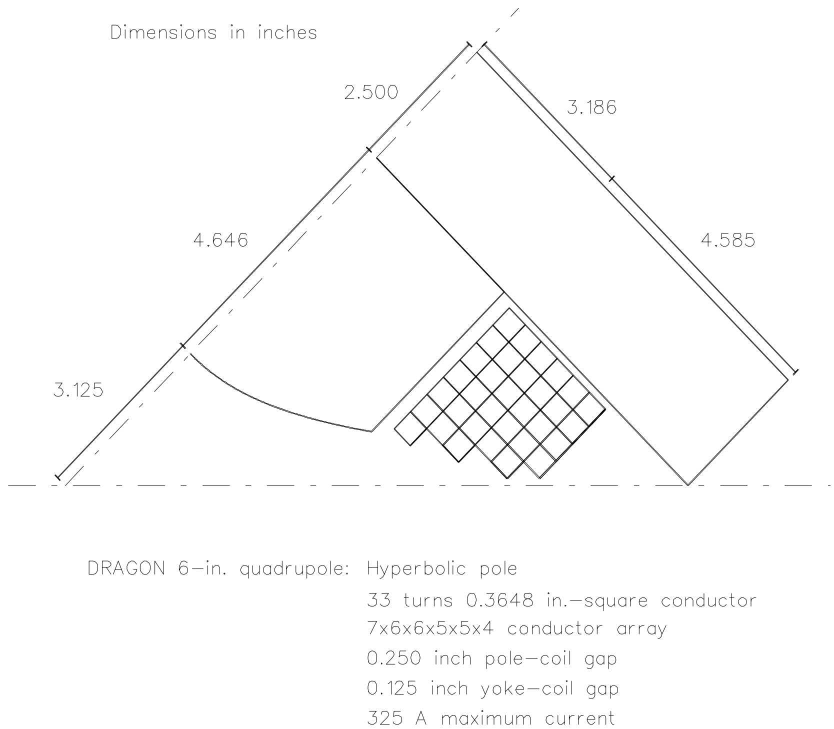

Fig. 8. Dimensions of an octant of a 6-in. quadrupole with hyperbolic pole; yoke thickness = 2.50 in.

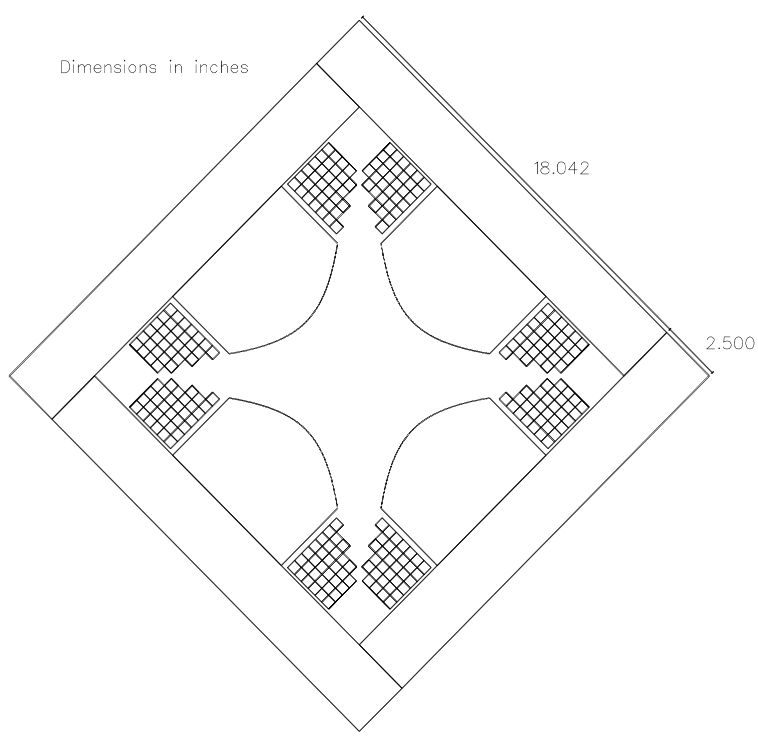

Fig. 9. Overall dimensions of such a quadrupole.

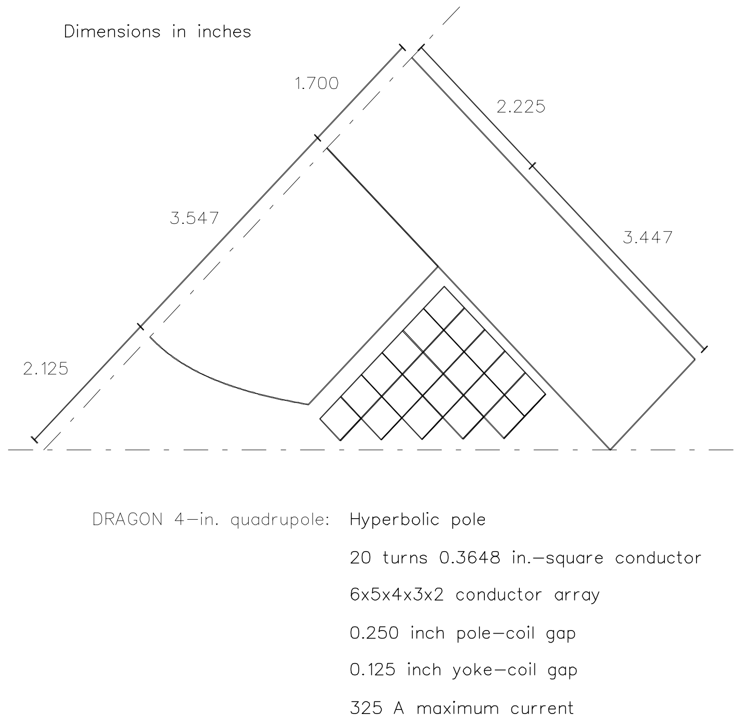

Fig. 10. Dimensions of an octant of a 4-in. quadrupole with hyperbolic pole; yoke thickness = 1.70 in.

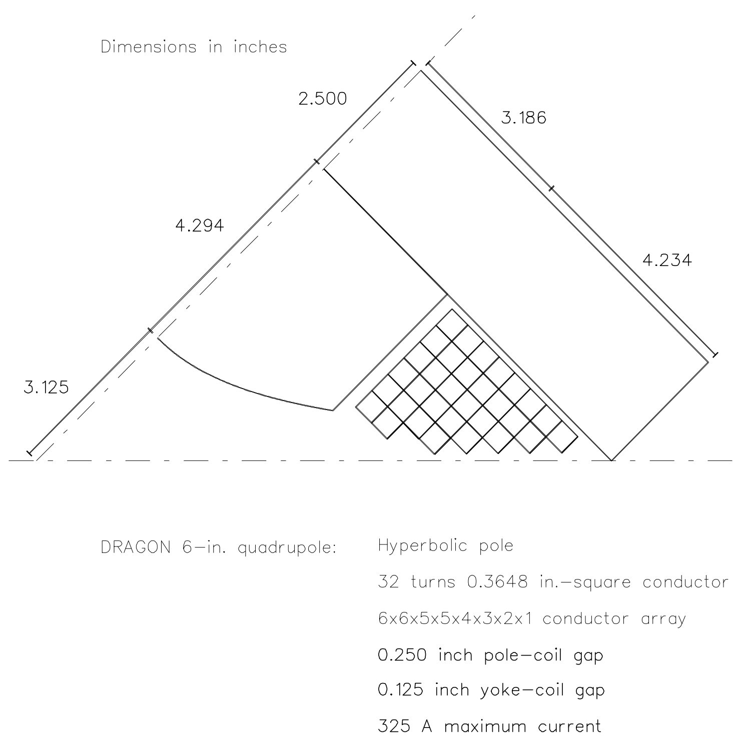

Fig. 11. Dimensions of an octant of a 6-in. quadrupole with hyperbolic pole; yoke thickness = 2.50 in.