1. Introduction

Optical design of the DRAGON facility [1] requires that the second quadrupole (Q2) be designed with an intrinsic sextupole component. This report presents the design of the pole faces for such a magnet.

2. Previous design work

Previous reports [2–4] have dealt with the designs for the 4-in. and 6-in. bore quadrupoles for the DRAGON facility. Each type of quadrupole was designed with a true hyperbolic pole profile. It is proposed that the quadrupole Q2 be identical in design to that of the other 6-in. quadrupoles with the exception that its pole profiles be modified to produce an intrinsic sextupole component of the desired magnitude.

As specified in ref [1], the required ratio of sextupole to quadrupole fields is

$$\frac{B_{sextupole}}{B_{quadrupole}} = \frac{0.100 \text{ kG}}{1.811 \text{ kG}} = 5.52\%$$

However, when the DRAGON optician [5] was questioned regarding the tolerances on this figure, his calculations indicated that a ratio closer to 6% would be more appropriate. Consequently, data presented in this report is for a sextupole-to-quadrupole ratio of that value.

$$\frac{B_{sextupole}}{B_{quadrupole}} = 6\%$$

3. The basis of the design of a quadrupole with an intrinsic sextupole component

Because the program RAYTRACE [6] was used in the optical design, we will use its notation in the following development. The multipole expansion of the potential used by RAYTRACE is of the form

$$\phi = \sum_{n=1}^{\infty} \frac{1}{n+1} \frac{B_n}{R^n} r^{n+1} \sin{(n+1)\theta}$$

with $n = 1$ the quadrupole term, $n = 2$ the sextupole term, etc. Here, R is a reference radius (the aperture of the element, for example) and r is an arbitrary radius. Thus, up to an octupole contribution we have

$$\begin{aligned} \phi &= \frac{1}{2}\frac{B_q}{R} r^2 \sin 2\theta + \frac{1}{3}\frac{B_s}{R^2} r^3 \sin 3\theta + \frac{1}{4}\frac{B_o}{R^3} r^4 \sin 4\theta \\ &= \frac{B_q R}{2}\left[\left(\frac{r}{R}\right)^2 \sin 2\theta + \frac{2}{3}\left(\frac{r}{R}\right)^3 \frac{B_s}{B_q} \sin 3\theta + \frac{2}{4}\left(\frac{r}{R}\right)^4 \frac{B_o}{B_q} \sin 4\theta\right] . \end{aligned}$$

We are interested only in the quadrupole and sextupole terms for the DRAGON quadrupole. In passing, it is noted that because the sextupole component grows with an additional power of r, and if the required sextupole component is *not* given at the radius of the aperture—that is, at $r \neq R$—the sextupole component must be normalized to the quadrupole component at the quadrupole aperture via

$$\left.\frac{B_s}{B_q}\right|_{r=R} = \frac{R}{r}\left[\frac{B_s}{B_q}\right]_{optical}$$

Our equation for quadrupole and sextupole components only becomes

$$\phi = \frac{B_q R}{2}\left[\left(\frac{r}{R}\right)^2 \sin 2\theta + \frac{2}{3}\left(\frac{r}{R}\right)^3 \frac{B_s}{B_q}\sin 3\theta\right]$$

which we normalize as

$$\begin{aligned}\phi' &= \frac{2\,\phi}{B_q R}\\ &= \left[\left(\frac{r}{R}\right)^2 \sin 2\theta + \frac{2}{3}\left(\frac{r}{R}\right)^3 \frac{B_s}{B_q}\sin 3\theta\right]\\ &= z^2 \sin 2\theta + \frac{2}{3}\frac{B_s}{B_q}\, z^3 \sin 3\theta\end{aligned}$$

with $z = r/R$. In our case, the quadrupole and sextupole components are given at the full aperture so that $r = R$. Using the sextupole-to-quadrupole ratio

$$\frac{B_s}{B_q} = 0.060\ ,$$

we determine the constant $\phi' = \phi'_{45}$ by evaluating the expression for ϕ' at $\theta = 45°$ and $z = 1$. Thus

$$\phi'_{45} = 1\sin[2(45)] + \frac{2}{3}\,(0.060)\,1^3 \sin[3(45)] = 1.028284$$

and our equation of the constant potential of the pole face is

$$z^2 \sin 2\theta + 0.040\, z^3 \sin 3\theta \;=\; 1.028284\ . \tag{1}$$

[Again, in passing, we note in particular that at $\theta = 60°$ this equation reduces to the quadratic

$$z^2 \sin[2(60)] = 1.028284$$

with the solution

$$z = \pm\sqrt{\frac{1.028284}{0.866025}} = \pm\, 1.089661\ .\ ]$$

We note that equation (1) is valid in the first quadrant with $0° < \theta < 90°$. In the second quadrant with $90° < \theta < 180°$ the equation becomes

$$z^2 \sin(2\theta) + 0.040\, z^3 \sin(3\theta) \;=\; -\ 1.028284\ . \tag{2}$$

Clearly, the solutions of equations (1) and (2) are different. Consequently, the pole shapes in the first and second quadrants will be different. In particular, the pole lengths—as measured along the centerline of the pole—will differ.

In anticipation of the results that follow, this is illustrated in figure 1(a) that shows the essential geometry of the upper right pole; that of the upper left pole is shown in figure 1(b). By symmetry, the lower right and left poles are identical to the upper right and left poles, respectively. Figure 2 shows the completed quadrupole.

4. Method and results of calculation

Equations (1) and (2) were solved using a bisection technique that, although crude, converged quickly. In the present context, bisection technique means choosing two values of z, z_{lower} and z_{upper}, such that the function

$$f(z,\theta) = \phi'(z,\theta) - \phi'_{45}$$

has the values

$$f(z_{lower}, \theta) < 0 \qquad \text{and} \qquad f(z_{upper}, \theta) > 0$$

An average value, $z_{avg} = (z_{upper} + z_{lower})/2$ is calculated and $f(z_{avg}, \theta)$ is obtained. If $f(z_{avg}, \theta) < 0$ then z_{lower} is set equal to z_{avg}; if $f(z_{avg}, \theta) > 0$ then z_{upper} is set equal to z_{avg}. The iteration is continued in this manner until $|f(z_{avg}, \theta)| \leq \epsilon$ with ϵ a predefined limit. For these calculations, ϵ was set to 10^{-6}. Although inefficient, this technique converged quickly.

In all cases, the pole widths of the quadrupoles were defined. Consequently, the first step was to determine the points of intersection with the pole. This then defined the angular range of θ. Calculation of intermediate points proceeded in a similar manner in steps of 0.5°. The program was written such that intermediate points were calculated at even and half angles. Thus, for example, the angular range in the first quadrant was found to be $9.024° \leq \theta \leq 79.084°$ and calculations for $\theta = 9.024°$, 9.5°, 10.0°, 10.5° ... 79.084° were made. In the second quadrant the angular range was found to be $98.415° \leq \theta \leq 169.576°$. Calculations were made for $\theta = 98.415°$, 98.5°, 99.0°, 99.5° ... 169.576°.

Independently, B. Milton[7)] wrote a program to solve this problem. His approach was basically that described above. However, instead of solving at constant angular intervals as described above, his program solved the problem for equal intervals along the $f(z, \theta)$ function. Thus his data served as a check of that of the author. The pole profiles generated by these two programs were identical insofar as they could be compared.

Because the method used by Milton generates a more uniform distribution of points along the pole face, we use data from his program here. Table 1 lists the coordinates that were obtained for the pole face in the first quadrant using a point separation of 0.100 in. along the pole face. The coordinates listed apply to a pole orientation with the line $y = x$ as the symmetry axis as in a pure quadrupole. These coordinates, when rotated 135° and shifted along the x-axis, were used in the generation of figure 1(a). Similar data for the pole face in the second quadrant is listed in table 2. These latter coordinates, when rotated 45° and shifted along the x-axis, were used in the generation of figure 1(b).

Table 3 lists the data required to generate figure 1(a); table 4 lists that required to generate figure 1(b).

Pole lengths given in figures 1(a) and 1(b) were calculated as follows. In ref[4)] the radial distance from the center of the quadrupole to the center of the pole at the outer yoke is given as

$$R(\text{pole center at outer yoke}) = R_{outer} = 3.125 \text{ in.} + 4.294 \text{ in.} = 7.419 \text{ in.}$$

Consequently, the coordinates of that point in the x-y coordinate system are

$$\begin{aligned} x_{outer} &= R_{outer} \cos 45 &= 5.24603 \text{ in.} \\ y_{outer} &= R_{outer} \sin 45 &= 5.24603 \text{ in.} \end{aligned}$$

Given a half-aperture of $R_0 = 3.125$ in., the coordinates of the center of the pole in the first quadrant are simply

$$\begin{aligned} x_{inner}^{quadrant1} &= R_0 \cos 45 &= 2.20971 \text{ in.} \\ y_{inner}^{quadrant1} &= R_0 \sin 45 &= 2.20971 \text{ in.} \end{aligned}$$

Consequently, the length of the poles in the first and fourth quadrants are

$$l_{pole1} = l_{pole4} = \sqrt{(5.24603 - 2.20971)^2 + (5.24603 - 2.20971)^2} = 4.29400 \text{ in.}$$

as is shown in figure 1(a). The coordinates of the point of intersection of the line $y = -x$ and $f(z, \theta)$ were found in the above calculations to be

$$x_{inner}^{quadrant2} = 2.27408 \text{ in.}$$
$$y_{inner}^{quadrant2} = 2.27408 \text{ in.}$$

Consequently, the lengths of the poles in the second and third quadrants are

$$l_{pole2} = l_{pole3} = \sqrt{(5.24603 - 2.27408)^2 + (5.24603 - 2.27408)^2} = 4.20297 \text{ in.}$$

as is shown in figure 1(b).

5. POISSON results

The pole coordinates generated by the above procedure were fed into the program POISSON[8)] to find the predicted harmonic contents of a quadrupole designed in this manner. Listed below are the contents of the control file that was used. (The default directory that is listed exists on the scratch disk at TRIUMF.)

```
$set def scr0:[stinson]
$dele plot.ps;*
$dele out*.lis;*
$dele tape*.dat;*
$run automesh
sex6pc.in
$run lattice
tape73
 *9 2.540000
 s
$run poisson
tty
0
 *6 0 *32 2 *42 1 *43 285 *44 1 *45 143
 *46 2 *110 10 100 3. 180. 3.125 0.
 s
-1
$run psfplot
0 0 0 s
s
go
1 0 20 s
s
go
-1 s
$rename plot.ps    sex6pc.ps
$rename outlat.lis sex6pc.lat
$rename outpoi.lis sex6pc.poi
$dele tape*.dat;*
```

The file containing the data for the run, `sex6pc.in`, is listed in table 5. This is the input data to the AUTOMESH routine of POISSON. Table 6 lists the predicted harmonic content, as calculated by POISSON, of the quadrupole. For this run the normalization and integration radii were both equal to the radius of the quadrupole aperture. Consequently, no normalization for radius is required. The computed harmonics at the bore radius are those listed in the table. From the table, we calculate

$$\frac{B_{dipole}}{B_{quadrupole}} = \frac{2.6601}{3,211.8} = 0.083\%$$
$$\frac{B_{sextupole}}{B_{quadrupole}} = \frac{183.95}{3,211.8} = 5.73\%$$

Thus, POISSON predicts a sextupole-to-quadrupole ratio of slightly less than the 6% that was the design goal.

6. Discussion

This note has presented the results of the calculations for the manufacture of a quadrupole for the DRAGON facility that has an inherent ratio of sextupole to quadrupole fields of approximately 6%. Data presented here is sufficient to pass on to the manufacturer.

For completeness we present in an appendix a listing of the Milton program that was used for the calculation of the sextupole-to-quadrupole ratio. We do this so that should a similar calculation be required in the future, a well-documented record of the technique used here is available. We also include in the appendix the coordinates for the hyperbolic pole faces for the 'standard' quadrupoles with 4-in. and 6 in. bores.

References

1. The Recoil Group, *DRAGON Recoil Separator Optics*, TRIUMF Report, January 18, 1999.
2. G. M. Stinson, *A conceptual design for the 4-inch diameter quadrupoles for the DRAGON facility*, TRIUMF Report TRI-DNA-98-4, August, 1998.
3. G. M. Stinson, *A conceptual design for the 6-inch diameter quadrupoles for the DRAGON facility*, TRIUMF Report TRI-DNA-98-5, August, 1998.
4. G. M. Stinson, *A further study of the design of quadrupoles of the DRAGON facility*, TRIUMF Report TRI-DNA-98-6, October, 1998.
5. D. Hunter, *Private communication*, TRIUMF, February, 1999.
6. S. Kowalski and H. A. Enge, *RAYTRACE*, Laboratory for Nuclear Science, M.I.T., July, 1987.
7. B. F. Milton, *Private communication*, TRIUMF, February, 1999.
8. M. T. Menzel and H. K. Stokes, *User's Guide for the POISSON/SUPERFISH Group of Codes*, Los Alamos National Laboratory Report LA-UR-87-115, January, 1987.

Table 1

Pole coordinates for quadrants one and four of the 6-inch quadrupole with an intrinsic sextupole component

Coordinates are relative to the 45° symmetry line with 0.100 inch point spacing

x (in.)	y (in.)	x (in.)	y (in.)
5.3562	0.8507	2.1652	2.2591
5.2595	0.8678	2.0998	2.3358
5.1520	0.8876	2.0363	2.4156
5.0486	0.9075	1.9745	2.4987
4.9490	0.9274	1.9144	2.5855
4.8532	0.9474	1.8557	2.6764
4.7488	0.9701	1.8013	2.7670
4.6487	0.9928	1.7480	2.8622
4.5524	1.0156	1.6985	2.9571
4.4492	1.0412	1.6526	3.0514
4.3505	1.0668	1.6075	3.1507
4.2460	1.0952	1.5657	3.2493
4.1462	1.1237	1.5271	3.3468
4.0419	1.1550	1.4891	3.4495
3.9423	1.1865	1.4541	3.5508
3.8392	1.2208	1.4220	3.6501
3.7409	1.2554	1.3904	3.7546
3.6398	1.2929	1.3615	3.8566
3.5434	1.3307	1.3354	3.9552
3.4450	1.3715	1.3096	4.0588
3.3449	1.4155	1.2864	4.1582
3.2498	1.4600	1.2635	4.2628
3.1535	1.5078	1.2432	4.3621
3.0565	1.5591	1.2231	4.4663
2.9642	1.6112	1.2055	4.5641
2.8714	1.6670	1.1882	4.6667
2.7829	1.7239	1.1712	4.7744
2.6940	1.7849	1.1565	4.8742
2.6091	1.8473	1.1421	4.9788
2.5278	1.9112	1.1298	5.0739
2.4498	1.9767	1.1178	5.1732
2.3748	2.0441	1.1062	5.2773
2.3025	2.1134	1.0948	5.3864
2.2327	2.1850	1.0855	5.4835
2.2097	2.2097	1.0766	5.5824

Table 2

Pole coordinates for quadrants two and three of the 6-inch quadrupole with an intrinsic sextupole component

Coordinates are relative to the 135° symmetry line with 0.100 inch point spacing

x (in.)	y (in.)	x (in.)	y (in.)
−0.7823	5.2879	−2.3182	2.2346
−0.8095	5.1824	−2.3948	2.1690
−0.8366	5.0803	−2.4741	2.1052
−0.8634	4.9816	−2.5564	2.0430
−0.8900	4.8861	−2.6419	1.9822
−0.9196	4.7830	−2.7310	1.9226
−0.9489	4.6837	−2.8241	1.8643
−0.9781	4.5878	−2.9166	1.8098
−1.0101	4.4858	−3.0083	1.7590
−1.0420	4.3876	−3.1044	1.7089
−1.0767	4.2843	−3.1995	1.6622
−1.1113	4.1852	−3.2991	1.6161
−1.1487	4.0819	−3.3976	1.5731
−1.1861	3.9829	−3.4945	1.5331
−1.2264	3.8806	−3.5961	1.4934
−1.2666	3.7826	−3.6957	1.4566
−1.3098	3.6821	−3.7926	1.4226
−1.3530	3.5859	−3.8941	1.3888
−1.3993	3.4878	−3.9924	1.3577
−1.4458	3.3939	−4.0951	1.3267
−1.4955	3.2985	−4.1939	1.2984
−1.5456	3.2072	−4.2970	1.2702
−1.5990	3.1147	−4.3953	1.2445
−1.6561	3.0213	−4.4979	1.2190
−1.7138	2.9320	−4.5946	1.1959
−1.7755	2.8420	−4.6952	1.1729
−1.8382	2.7559	−4.8001	1.1500
−1.9020	2.6731	−4.8978	1.1295
−1.9672	2.5935	−4.9994	1.1090
−2.0338	2.5169	−5.1051	1.0886
−2.1020	2.4429	−5.2020	1.0706
−2.1720	2.3713	−5.3026	1.0526
−2.2440	2.3019	−5.4071	1.0347
−2.2741	2.2741	−5.5010	1.0191
		−5.5215	1.0158

Table 3

Pole coordinates for quadrants one and four of the 6-inch quadrupole with an intrinsic sextupole component

Coordinates are those required to reproduce figure 1(a).

x (in.)	y (in.)	x (in.)	y (in.)
0.0000	0.0000	4.2906	−0.0664
0.0000	3.1859	4.2826	−0.1669
3.0301	3.1859	4.2711	−0.2683
3.0863	3.1054	4.2559	−0.3707
3.1484	3.0154	4.2371	−0.4746
3.2074	2.9282	4.2143	−0.5803
3.2637	2.8437	4.1887	−0.6829
3.3174	2.7618	4.1591	−0.7879
3.3751	2.6720	4.1270	−0.8900
3.4299	2.5851	4.0928	−0.9891
3.4818	2.5009	4.0544	−1.0912
3.5367	2.4099	4.0142	−1.1905
3.5884	2.3219	3.9726	−1.2867
3.6422	2.2280	3.9268	−1.3862
3.6926	2.1372	3.8800	−1.4826
3.7442	2.0413	3.8325	−1.5755
3.7924	1.9486	3.7809	−1.6718
3.8410	1.8514	3.7293	−1.7643
3.8861	1.7575	3.6780	−1.8525
3.9311	1.6595	3.6230	−1.9440
3.9725	1.5646	3.5691	−2.0307
4.0132	1.4661	3.5113	−2.1208
4.0528	1.3643	3.4555	−2.2054
4.0887	1.2656	3.3960	−2.2933
4.1230	1.1637	3.3392	−2.3749
4.1553	1.0588	3.2790	−2.4597
4.1837	0.9567	3.2148	−2.5479
4.2099	0.8516	3.1546	−2.6289
4.2322	0.7489	3.0909	−2.7130
4.2519	0.6428	3.0323	−2.7889
4.2678	0.5387	2.9705	−2.8676
4.2802	0.4360	2.9052	−2.9494
4.2890	0.3345	2.8361	−3.0346
4.2944	0.2339	2.7740	−3.1099
4.2964	0.1337	2.7104	−3.1861
4.2952	0.0337	0.0000	−3.1861
4.2940	0.0000	0.0000	0.0000

Table 4

Pole coordinates for quadrants two and three of the 6-inch quadrupole with an intrinsic sextupole component

Coordinates are those required to reproduce figure 1(b).

x (in.)	y (in.)	x (in.)	y (in.)
0.0000	0.0000	4.1997	−0.0591
0.0000	3.1860	4.1919	−0.1596
3.1267	3.1860	4.1809	−0.2608
3.1821	3.0921	4.1668	−0.3630
3.2352	3.0008	4.1493	−0.4665
3.2860	2.9120	4.1284	−0.5716
3.3347	2.8256	4.1038	−0.6787
3.3867	2.7319	4.0769	−0.7826
3.4362	2.6409	4.0480	−0.8834
3.4834	2.5524	4.0155	−0.9867
3.5328	2.4577	3.9813	−1.0870
3.5797	2.3657	3.9434	−1.1901
3.6282	2.2681	3.9042	−1.2902
3.6739	2.1735	3.8640	−1.3870
3.7204	2.0740	3.8202	−1.4869
3.7640	1.9776	3.7758	−1.5833
3.8079	1.8768	3.7313	−1.6759
3.8487	1.7791	3.6834	−1.7715
3.8892	1.6775	3.6360	−1.8630
3.9267	1.5789	3.5852	−1.9575
3.9633	1.4768	3.5354	−2.0474
3.9968	1.3775	3.4824	−2.1403
4.0292	1.2749	3.4310	−2.2279
4.0583	1.1750	3.3766	−2.3186
4.0859	1.0717	3.3245	−2.4032
4.1116	0.9654	3.2696	−2.4907
4.1339	0.8614	3.2117	−2.5811
4.1540	0.7542	3.1571	−2.6646
4.1706	0.6489	3.0998	−2.7509
4.1839	0.5453	3.0394	−2.8401
4.1941	0.4429	2.9836	−2.9213
4.2012	0.3416	2.9252	−3.0052
4.2053	0.2410	2.8640	−3.0917
4.2064	0.1409	2.8086	−3.1692
4.2046	0.0410	2.7965	−3.1860
4.2030	0.0000	0.0000	−3.1860
		0.0000	0.0000

Table 5

Input file to the AUTOMESH routine of POISSON

```
DRAGON Q2 with quadrupole + sextupole at 10,500 A-t, sex/quad=6%, 0.100 in. spacing, Milton
$reg    nreg=9,npoint=7,mat=1
xmin=-14.20000,xmax=14.20000,dx=0.10000
,ymin=0.000000,ymax=14.20000,dy=0.10000
$end
$po     x=−14.20000,y=0.00000 $end
$po     x=0.00000,y=0.00000 $end
$po     x=14.20000,y=0.00000 $end
$po     x=14.20000,y=14.2000 $end
$po     x=0.00000,y=14.2000 $end
$po     x=−14.20000,y=14.2000 $end
$po     x=−14.20000,y=0.00000 $end
$reg    npoint=69, mat=2 $end
$po     x=5.35621,y=0.85065 $end
$po     x=5.25951,y=0.86779 $end
$po     x=5.15198,y=0.88763 $end
 ⋮          ⋮    ,    ⋮                  ⟸ Remainder of data of table 1 in this format
$po     x=1.09482,y=5.38639 $end
$po     x=1.08545,y=5.48352 $end
$po     x=1.07661,y=5.58239 $end
$po     x=5.35621,y=0.85065 $end
$reg    npoint=9,mat=2 $end
$po     x=2.993621,y=7.498896 $end
$po     x=−1.767767,y=12.260284 $end
$po     x=0.000000,y=14.028051 $end
$po     x=14.028051,y=0.000000 $end
$po     x=10.492517,y=0.000000 $end
$po     x=7.498900,y=2.993618 $end
$po     x=5.35621,y=0.85065 $end
$po     x=1.07661,y=5.58237 $end
$po     x=2.993621,y=7.498896 $end
$reg    npoint=68, mat=2 $end
$po     x=−0.78230,y=5.28794 $end
$po     x=−0.80954,y=5.18239 $end
$po     x=−0.83657,y=5.08033 $end
 ⋮          ⋮    ,    ⋮                  ⟸ Remainder of data of table 2 in this format
$po     x=−5.30263,y=1.05264 $end
$po     x=−5.40705,y=1.03469 $end
$po     x=−5.51124,y=1.01748 $end
$po     x=−0.78230,y=5.28794 $end
$reg    npoint=9,mat=2 $end
$po     x=−7.498900,y=2.993618 $end
$po     x=−10.492517,y=0.000000 $end
$po     x=−14.028051,y=0.000000 $end
```

Table 5 (Continued)

```
$po   x=-1.767767,y=12.260284 $end
$po   x=0.000000,y=10.492517 $end
$po   x=-2.993621,y=7.498896 $end
$po   x=-0.78229,y=5.28794 $end
$po   x=-5.52155,y=1.01581 $end
$po   x=-7.498900,y=2.993618 $end
$reg  npoint=15,mat=1,cur=10500. $end
$po   x=5.826592,y=0.967756 $end
$po   x=6.399349,y=0.395000 $end
$po   x=6.692799,y=0.688449 $end
$po   x=7.265555,y=0.115693 $end
$po   x=7.559004,y=0.409142 $end
$po   x=7.845383,y=0.122764 $end
$po   x=8.138831,y=0.416213 $end
$po   x=8.425210,y=0.129835 $end
$po   x=8.718658,y=0.423284 $end
$po   x=9.005036,y=0.136906 $end
$po   x=9.298487,y=0.430355 $end
$po   x=9.584866,y=0.143977 $end
$po   x=9.878315,y=0.437427 $end
$po   x=7.587288,y=2.728452 $end
$po   x=5.826592,y=0.967756 $end
$reg  npoint=15,mat=1,cur=-10500. $end
$po   x=0.967756,y=5.826592 $end
$po   x=0.395000,y=6.399349 $end
$po   x=0.688449,y=6.692799 $end
$po   x=0.115693,y=7.265555 $end
$po   x=0.409142,y=7.559004 $end
$po   x=0.122764,y=7.845383 $end
$po   x=0.416213,y=8.138831 $end
$po   x=0.129835,y=8.425210 $end
$po   x=0.423284,y=8.718658 $end
$po   x=0.136906,y=9.005036 $end
$po   x=0.430355,y=9.298487 $end
$po   x=0.143977,y=9.584866 $end
$po   x=0.437427,y=9.878315 $end
$po   x=2.728452,y=7.587288 $end
$po   x=0.967756,y=5.826592 $end
$reg  npoint=15,mat=1,cur=-10500. $end
$po   x=-0.967756,y=5.826592 $end
$po   x=-0.395000,y=6.399349 $end
$po   x=-0.688449,y=6.692799 $end
$po   x=-0.115693,y=7.265555 $end
$po   x=-0.409142,y=7.559004 $end
$po   x=-0.122764,y=7.845383 $end
$po   x=-0.416213,y=8.138831 $end
```

Table 5 (Continued)

```
$po   x=-0.129835,y=8.425210 $end
$po   x=-0.423284,y=8.718658 $end
$po   x=-0.136906,y=9.005036 $end
$po   x=-0.430355,y=9.298487 $end
$po   x=-0.143977,y=9.584866 $end
$po   x=-0.437427,y=9.878315 $end
$po   x=-2.728452,y=7.587288 $end
$po   x=-0.967756,y=5.826592 $end
$reg  npoint=15,mat=1,cur=10500. $end
$po   x=-5.826592,y=0.967756 $end
$po   x=-6.399349,y=0.395000 $end
$po   x=-6.692799,y=0.688449 $end
$po   x=-7.265555,y=0.115693 $end
$po   x=-7.559004,y=0.409142 $end
$po   x=-7.845383,y=0.122764 $end
$po   x=-8.138831,y=0.416213 $end
$po   x=-8.425210,y=0.129835 $end
$po   x=-8.718658,y=0.423284 $end
$po   x=-9.005036,y=0.136906 $end
$po   x=-9.298487,y=0.430355 $end
$po   x=-9.584866,y=0.143977 $end
$po   x=-9.878315,y=0.437427 $end
$po   x=-7.587288,y=2.728452 $end
$po   x=-5.826592,y=0.967756 $end
```

Table 6

Harmonics of the DRAGON Q2 quadrupole as calculated by POISSON

TABLE FOR FIELD COEFFICIENTS

NORMALIZATION RADIUS =3.12500 in.

Harmonic Number	Field Value
1	2.6601E+00
2	3.2118E+03
3	1.8395E+02
4	−5.3840E−01
5	8.5730E+00
6	2.1786E−01
7	−5.3731E+00
8	−7.4542E−01
9	2.4252E+00
10	−3.6413E−01

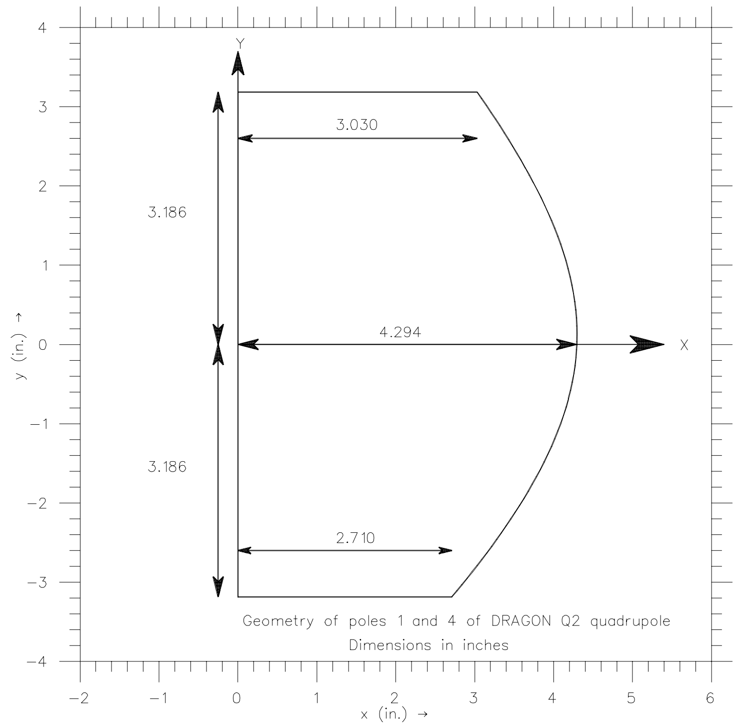

Fig. 1-a. Pole profile for first and fourth quadrants with inherent 6% sextupole component.

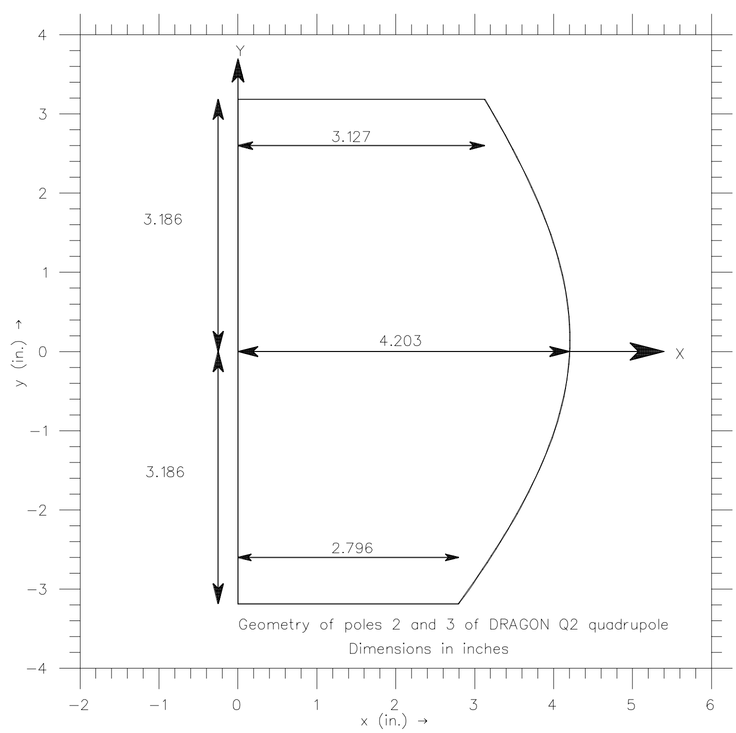

Fig. 1-b. Pole profile for second and third quadrants with inherent 6% sextupole component.

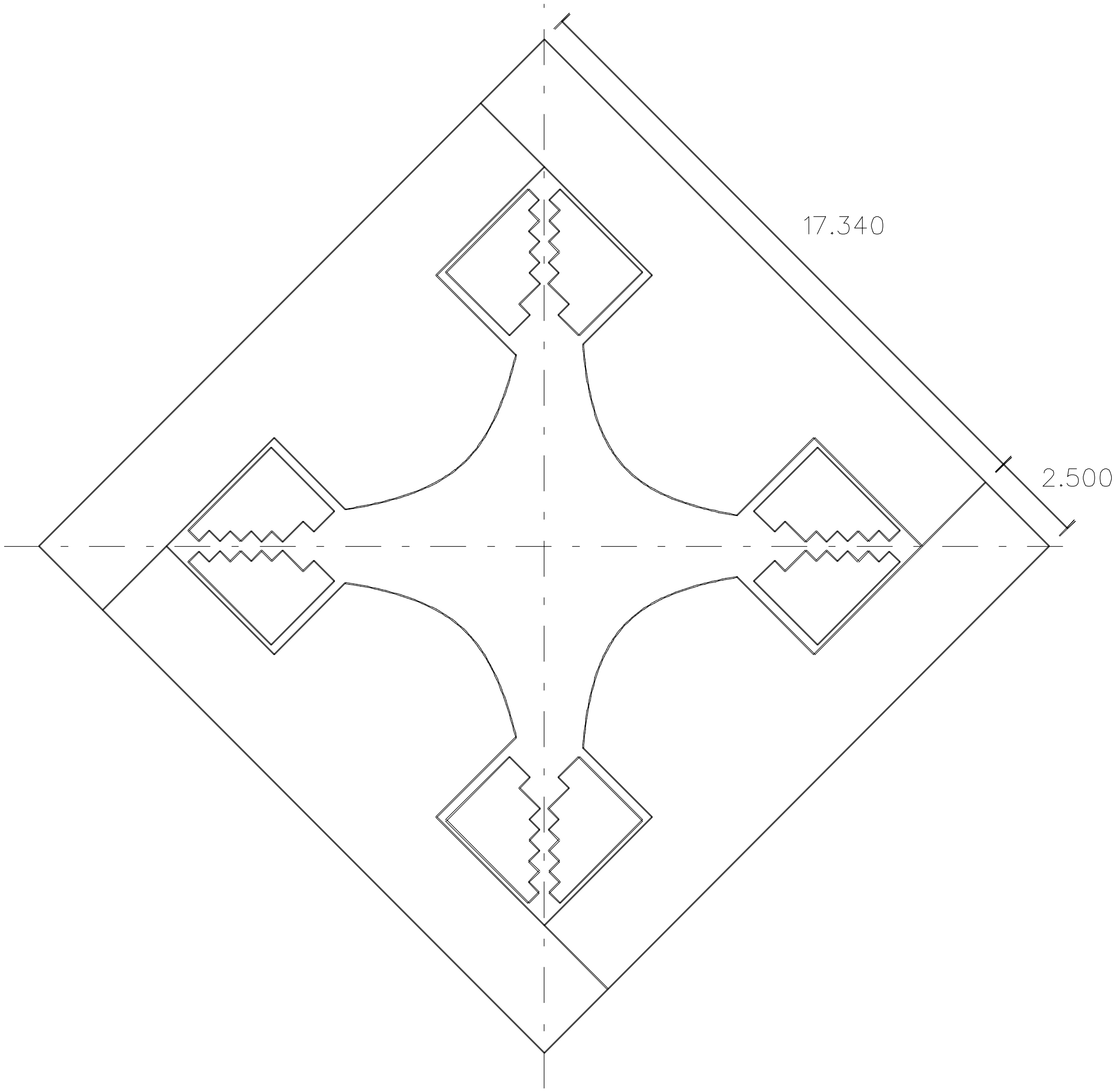

Fig. 2. The completed design for quadrupole Q2 of the DRAGON facility.

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Appendix

In this appendix we list the program of Milton that was used to generate the pole profile of a quadrupole with an inherent sextupole component. We also give data for the hyperbolic pole shapes of the 4-in. and 6-in. quadrupoles for the DRAGON facility.

A1 Program POLE

The program listed below will compile in f77 and the file output on unit 10 can be read into ACAD as a script file.

```
      Program Pole
      Implicit None
      Common/Poledata/Constant,Theta,Fraction
      Real Constant,Theta,Fraction

      Real Tcon
      Parameter (Tcon=57.29578)
      Integer Loopitermax
      Parameter (Loopitermax=5000)

      Real Rnorm,Dl
      Real Ratio                    ! Rad/Rnorm
      Real Rad                      ! Radius
      Real Xl,Xr
      Integer Ith,Ierr,Ipole
      Real  Xpos,Ypos
      Real  Xlast,Ylast,Dist
      Real  Theta1,Thpole
      Real  Offset,Off
      Logical  Near
      Real pminus
      External pminus
      Real Phi
      External Phi

      write(6,'(''Enter Normalization Radius (rnorm) '',$)')
      read(5,*)rnorm
      write(6,'(''Enter Ratio Bs/Bq  '',$)')
      read(5,*)fraction
      write(6,'(''Enter spacing between points '',$)')
      read(5,*)dl
      write(6,'(''Enter Pole 1/2 Width '',$)')
      read(5,*)offset

      theta=45.0
      CONSTANT=0.0
      constant=phi(1.0)

      write(6,'(''Constant='',g10.5)')constant
```

```
      write(6,'(''    Theta     Radius      X         Y       dl       ''
     + '' offset'')')

      theta1 = 45-offset/rnorm*tcon
      if(theta1 .lt. 0.0) theta1=9.0

      do ipole=1,2
         write(6,'('' Pole'',i2)')ipole
         theta=theta1
         thpole=45+(ipole-1)*90
! find theta with same offset from the centreline
         xl = 0.0
         xr = 100
         do ith = 1,100
            call bisect(phi,xl,xr,1.e-5,100,ratio,ierr)
            if(ierr .eq. -1) then
               write(6,'('' Illegal data in bisect '')')
               stop 'Error Stop'
            else if(ierr .eq. -2) then
               write(6,'('' Max Iterations in bisect '')')
               stop 'Error Stop'
            endif
            rad = ratio * rnorm
            off = rad*sind(thpole-theta) ! find the offset from the centreline
            if (abs(off-offset) .gt. 1.0e-4) then
               theta = theta + (off-offset)/rad * tcon/2 ! factor of 2 for damping
            else
               exit
            endif
         enddo
         if (ith .ge. 100) stop 'No Theta1 Found'
         write(10,'(''PLINE'')')

         xl = 0.0
         xr = 10
         theta1 = theta+90
         near = .false.
         do ith=1, LoopIterMax
            call bisect(phi,xl,xr,1.e-5,100,ratio,ierr)
            if(ierr .eq. -1) then
               write(6,'('' Illegal data in bisect '')')
               stop 'Error Stop'
            else if(ierr .eq. -2) then
               write(6,'('' Max Iterations in bisect '')')
               stop 'Error Stop'
            endif
            rad = ratio * rnorm
```

```
        xpos = rad*cosd(theta)
        ypos = rad*sind(theta)
        dist=sqrt((xpos-xlast)**2+(ypos-ylast)**2)
        off = rad*sind(theta-thpole)
        if((ith .ne. 1) .and. (dist-dl)/dl .gt. 0.1) then
           theta = theta - 0.05* dl/rad * tcon
        elseif (.not. near .and. (off-offset) .gt. 1.e-4) then
           theta = theta - (off-offset)/rad * tcon/2
           near=.true.
        elseif ( near .and. abs(off-offset) .gt. 1.e-4) then
           theta = theta - (off-offset)/rad * tcon/2
        else
           if(ith .eq. 1) dist = 0.0
           write(6,'(f10.3,6f10.5)')theta,rad,xpos,ypos,dist,off
           if(xpos .ge. 0.0) then
              write(10,'(f7.5,'','',f7.5)')xpos,ypos
           else
              write(10,'(f8.5,'','',f7.5)')xpos,ypos
           endif
           if (near) exit
           theta = theta + dl/rad * tcon
           xl=ratio-1.0
           xr=ratio+1.0
           xlast=xpos
           ylast=ypos
        endif
     enddo
     write(10,*)
     if (ith .ge. LoopIterMax) stop 'Did not complete'
     constant = -constant
  enddo

  end

!*****************************

  function phi(x)
  implicit none
  common/poledata/constant,theta,fraction
  real constant,theta,fraction
  real phi
  real x
  real th

  phi = x**2*sind(2*theta)+fraction*2/3*x**3*sind(3*theta)-constant

  return
  end
```

```
!****************************

      subroutine bisect(f,xl,xr,tol,max_iter,zero,error)
      implicit none

      real xl,xr                ! left and right starting values - must bracket root
      real tol                  ! tolerance in terms of interval size
      integer max_iter
      real zero                 ! this is the answer
      integer error             ! this is an error flag
      real f       ! function
      external f

! local variables
      real  x_left,x_mid,x_right,v_left,v_mid,v_right,delta
      integer  num_bisecs
      if(xl .lt. xr) then
         x_left = xl
         x_right = xr
      else
         x_left = xr
         x_right = xl
      endif

      v_left = f(x_left)
      v_right = f(x_right)

      If((v_left*v_right .ge. 0.0) .or. (tol .le. 0.0) .or.
     + (max_iter .lt. 1)) then
         error = -1
         return
      endif

      do num_bisecs = 0,max_iter
         delta=0.5*(x_right-x_left)
         x_mid = x_left+delta
!write(6,'(I3,4x,4f12.6)')num_bisecs,x_left,x_mid,x_right,v_mid
         if(delta .lt. tol) then
            error = 0
            zero = x_mid
            return
         endif
         v_mid = f(x_mid)
         if(v_left*v_mid .lt. 0.0) then
! a root lies in the left half of the interval
            x_right = x_mid
            v_right = v_mid
```

```
         else
! a root lies in the right half of the interval
            x_left = x_mid
            v_left = v_mid
         endif
      enddo

      error = -2
      zero = x_mid

      return
      end
```

A2 Pole coordinates for 4-inch quadrupoles

Table A1 lists the coordinates for the poles of the 4-in. quadrupoles in a coordinate system with its origin at the quadrupole center and the 45° line as the line of symmetry. Listed in table A2 are the coordinates of the pole in a coordinate system in which the midpoint of the pole base is the origin. The coordinates of table A2 were used to generate the pole profile shown in figure A1.

A3 Pole coordinates for 6-inch quadrupoles

Table A3 lists the coordinates for the poles of the 4-in. quadrupoles in a coordinate system with its origin at the quadrupole center and the 45° line as the line of symmetry. Listed in table A4 are the coordinates of the pole in a coordinate system in which the midpoint of the pole base is the origin. The coordinates of table A4 were used to generate the pole profile shown in figure A2.

Table A1

Pole coordinates for 4-inch quadrupole with hyperbolic pole face
Coordinates relative to the 45° symmetry line with 0.075 inch point spacing

x (in.)	y (in.)	x (in.)	y (in.)
3.7488	0.6023	1.5026	1.5026
3.6744	0.6145	1.4657	1.5404
3.6027	0.6267	1.4147	1.5960
3.5230	0.6409	1.3652	1.6538
3.4465	0.6551	1.3171	1.7142
3.3730	0.6694	1.2702	1.7776
3.2931	0.6856	1.2243	1.8441
3.2167	0.7019	1.1817	1.9107
3.1434	0.7183	1.1398	1.9809
3.0651	0.7366	1.1008	2.0512
2.9902	0.7551	1.0644	2.1213
2.9113	0.7755	1.0284	2.1954
2.8361	0.7961	0.9949	2.2694
2.7577	0.8187	0.9638	2.3427
2.6831	0.8415	0.9348	2.4152
2.6061	0.8664	0.9061	2.4917
2.5328	0.8914	0.8796	2.5670
2.4577	0.9187	0.8551	2.6404
2.3812	0.9482	0.8307	2.7179
2.3086	0.9780	0.8084	2.7931
2.2351	1.0102	0.7861	2.8722
2.1609	1.0448	0.7658	2.9484
2.0907	1.0800	0.7474	3.0207
2.0199	1.1178	0.7291	3.0966
1.9490	1.1584	0.7108	3.1763
1.8817	1.1999	0.6944	3.2513
1.8142	1.2445	0.6781	3.3298
1.7499	1.2903	0.6636	3.4024
1.6884	1.3373	0.6491	3.4783
1.6295	1.3856	0.6346	3.5577
1.5729	1.4355	0.6220	3.6298
1.5184	1.4870	0.6094	3.7050
		0.6023	3.7488

Table A2

Pole coordinates for 4-inch quadrupole with hyperbolic pole face
Coordinates are those required to reproduce figure A1.

x (in.)	y (in.)	x (in.)	y (in.)
0.0000	0.0000	3.5470	0.0000
0.0000	2.2250	3.5464	−0.0528
2.5953	2.2250	3.5431	−0.1282
2.6393	2.1637	3.5372	−0.2041
2.6813	2.1044	3.5285	−0.2808
2.7277	2.0380	3.5169	−0.3588
2.7717	1.9738	3.5023	−0.4382
2.8136	1.9118	3.4854	−0.5155
2.8586	1.8438	3.4653	−0.5947
2.9012	1.7782	3.4433	−0.6720
2.9414	1.7148	3.4194	−0.7473
2.9838	1.6464	3.3924	−0.8252
3.0237	1.5805	3.3638	−0.9012
3.0650	1.5102	3.3340	−0.9751
3.1036	1.4425	3.3032	−1.0468
3.1431	1.3711	3.2694	−1.1212
3.1797	1.3022	3.2349	−1.1932
3.2166	1.2302	3.2003	−1.2624
3.2507	1.1606	3.1627	−1.3345
3.2846	1.0882	3.1254	−1.4034
3.3178	1.0133	3.0852	−1.4751
3.3480	0.9409	3.0457	−1.5433
3.3773	0.8661	3.0075	−1.6074
3.4052	0.7892	2.9668	−1.6741
3.4300	0.7147	2.9234	−1.7434
3.4533	0.6379	2.8820	−1.8079
3.4747	0.5590	2.8380	−1.8750
3.4930	0.4821	2.7969	−1.9366
3.5092	0.4028	2.7535	−2.0005
3.5223	0.3250	2.7076	−2.0670
3.5325	0.2483	2.6655	−2.1268
3.5400	0.1725	2.6213	−2.1889
3.5448	0.0972	2.5953	−2.2250
3.5469	0.0222	0.0000	−2.2250
		0.0000	0.0000

Table A3

Pole coordinates for 6-inch quadrupole with hyperbolic pole face
Coordinates relative to the 45° symmetry line with 0.100 inch point spacing

x (in.)	y (in.)	x (in.)	y (in.)
5.4085	0.9028	2.1751	2.2448
5.3118	0.9192	2.1065	2.3180
5.2041	0.9383	2.0398	2.3938
5.1004	0.9573	1.9748	2.4726
5.0005	0.9765	1.9113	2.5547
4.9042	0.9956	1.8493	2.6404
4.7990	1.0174	1.7885	2.7301
4.6981	1.0393	1.7318	2.8195
4.6009	1.0613	1.6761	2.9132
4.4967	1.0859	1.6241	3.0066
4.3967	1.1106	1.5755	3.0993
4.2909	1.1379	1.5275	3.1967
4.1897	1.1654	1.4827	3.2932
4.0927	1.1931	1.4411	3.3883
3.9910	1.2234	1.3997	3.4884
3.8938	1.2540	1.3614	3.5867
3.7929	1.2873	1.3259	3.6827
3.6891	1.3236	1.2906	3.7834
3.5901	1.3601	1.2580	3.8812
3.4888	1.3996	1.2256	3.9839
3.3923	1.4394	1.1959	4.0828
3.2939	1.4824	1.1663	4.1865
3.1944	1.5286	1.1394	4.2856
3.0995	1.5754	1.1124	4.3894
3.0037	1.6256	1.0881	4.4874
2.9123	1.6766	1.0638	4.5899
2.8201	1.7314	1.0395	4.6971
2.7275	1.7902	1.0178	4.7972
2.6390	1.8503	0.9961	4.9018
2.5542	1.9117	0.9770	4.9979
2.4728	1.9746	0.9578	5.0978
2.3944	2.0392	0.9387	5.2020
2.3189	2.1057	0.9220	5.2958
2.2459	2.1741	0.9053	5.3933
2.2097	2.2097	0.9028	5.4084

Table A4

Pole coordinates for 6-inch quadrupole with hyperbolic pole face
Coordinates are those required to reproduce figure A2.

x (in.)	y (in.)	x (in.)	y (in.)
0.0000	0.0000	4.2936	−0.0493
0.0000	3.1860	4.2904	−0.1495
2.9562	3.1860	4.2840	−0.2503
3.0130	3.1060	4.2742	−0.3520
3.0757	3.0164	4.2611	−0.4550
3.1355	2.9296	4.2443	−0.5594
3.1926	2.8454	4.2239	−0.6658
3.2472	2.7637	4.2008	−0.7691
3.3061	2.6740	4.1739	−0.8748
3.3620	2.5871	4.1446	−0.9776
3.4152	2.5029	4.1134	−1.0775
3.4715	2.4118	4.0785	−1.1803
3.5248	2.3237	4.0419	−1.2802
3.5802	2.2295	4.0041	−1.3769
3.6323	2.1385	3.9626	−1.4769
3.6814	2.0503	3.9202	−1.5735
3.7318	1.9570	3.8774	−1.6665
3.7789	1.8667	3.8311	−1.7627
3.8267	1.7717	3.7850	−1.8549
3.8745	1.6727	3.7353	−1.9504
3.9187	1.5769	3.6864	−2.0413
3.9624	1.4773	3.6340	−2.1356
4.0025	1.3809	3.5830	−2.2248
4.0416	1.2810	3.5286	−2.3172
4.0794	1.1779	3.4765	−2.4037
4.1134	1.0777	3.4212	−2.4933
4.1456	0.9745	3.3626	−2.5863
4.1741	0.8738	3.3071	−2.6724
4.2006	0.7698	3.2486	−2.7617
4.2245	0.6628	3.1941	−2.8432
4.2446	0.5577	3.1370	−2.9274
4.2611	0.4543	3.0769	−3.0146
4.2742	0.3523	3.0223	−3.0928
4.2839	0.2512	2.9652	−3.1735
4.2904	0.1508	2.9563	−3.1859
4.2936	0.0507	0.0000	−3.1859
4.2940	0.0000	0.0000	0.0000

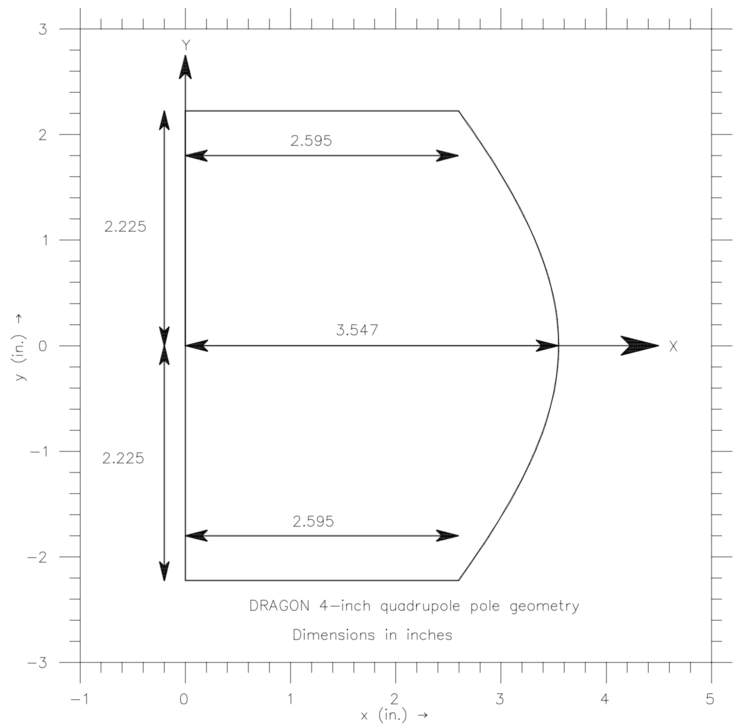

Fig. A1. Pole geometry for a 4-inch quadrupole with a hyperbolic pole.

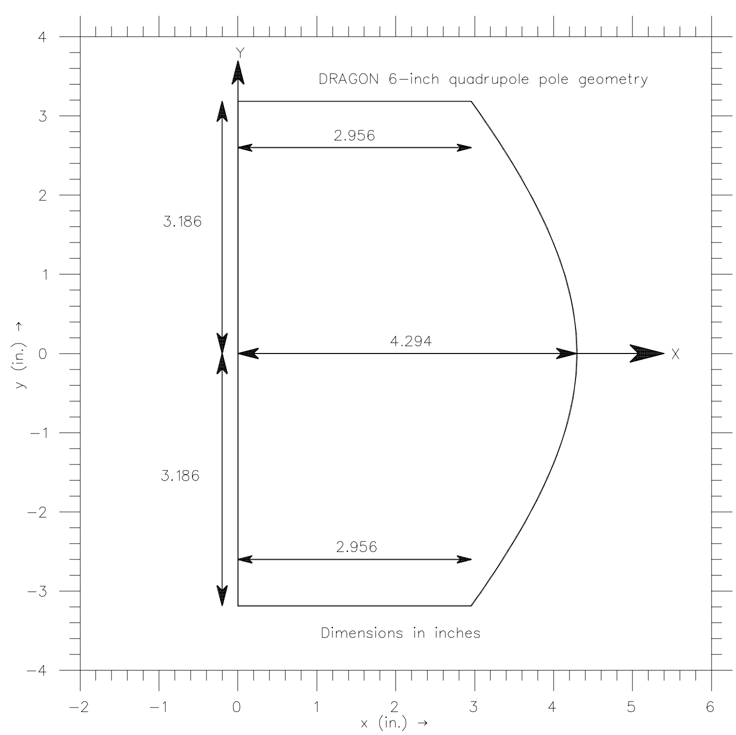

Fig. A2. Pole geometry for a 6-inch quadrupole with a hyperbolic pole.