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Electrostatic Steerer Lensing Effect

R. Baartman, T. Planche

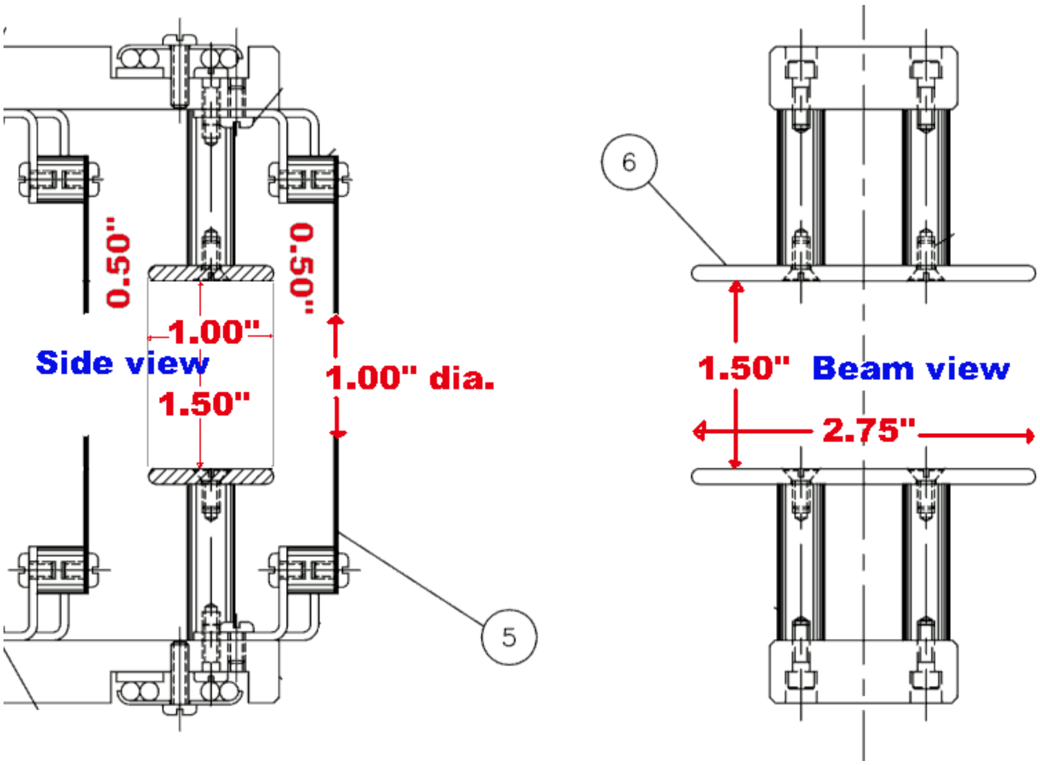
TRIUMF

Abstract: We show the lensing effect of electrostatic steerer, and calculate the case of the standard ISAC steerer which has plates of length 1 inch. Emittance measurement confirm the calculated focal length.

4004 Wesbrook Mall, Vancouver, B.C. Canada V6T 2A3 Tel: 604 222-1047 Fax: 604 222-1074
www.triumf.ca

1 Steerer Geometry

The standard ISAC LEBT steerer is shown in the figure.



This is a fragment of drawing number ILE0053C. The plates are 1.00 inch in the beam direction, 2.75 inches transversely. Their separation is 1.50 inches. The plates are sandwiched between two skimmers that have 1.00 inch circular apertures. The distance from plate edge to skimmer is 0.50 inch on either side.

The geometry has been calculated on a grid. There are two cases: (1) the steering case where one plate is at ground and the other at potential of 1 (it's scalable so units don't matter), (2) the non-steer case where both plates are at 1. The potential maps for the two cases are shown in the figure below.

2 COSY calculations

2.1 No steering

The potential of the bottom plot (powered but neutral-steering) in Fig. 1 was used in `COSY`-∞ with the routine `MELP`[1], that needs only the potentials on the symmetry plane. The integral is taken through the steering assembly to

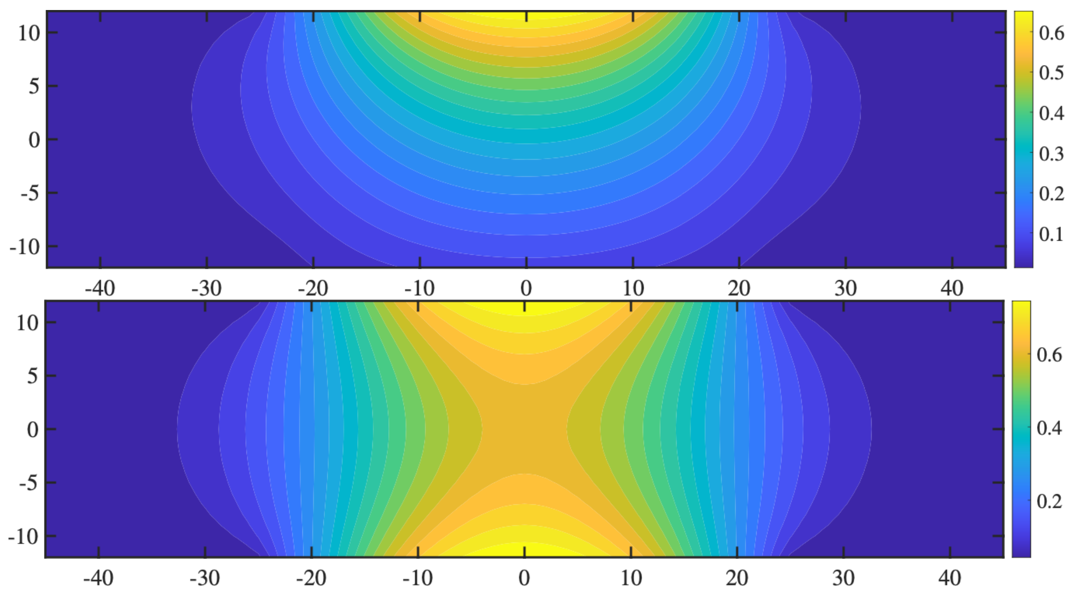


Figure 1: These are potential maps on the horizontal plane for plates that steer horizontally. Plots cover a length of 90 mm in the beam direction (horizontal axis). The skimmers are at +25.4 and −25.4 along this axis. The steering plates have a potential of 1. Units don't matter; refer to the colour bar at right. Plates are at +19 mm and −19 mm of the vertical scale, though the plot only covers ±12 mm. In the top plot, only one plate has voltage of 1, the other grounded. In the bottom plot, both plates are at 1.

distances sufficiently far from the skimmers at either end, and then a drift of half the total distance is removed before and after the integration through the field, so that the output matrix effectively is a 'zero-insertion' element.

For the case of $V_s = 1\,\text{kV}$ on the two plates, and a beam of kinetic energy per charge of $V_0 = 100\,\text{kV}$, the first order transfer map is shown below. (This is a standard format COSY map: The first 4 columns are (x, x', y, y'), the fifth column is related to longitudinal and not used here. The right-most column represents exponents of the input particle coordinates, resp. $(x, x', y, y', \beta c\delta t, \delta K/K)$, where K is kinetic energy. Again, the fifth and sixth digits are not used. See [2] for more info. For the case shown, the 4-by-4 matrix is in fact the transpose of the usual 'transfer matrix'.)

0.9999954	-0.2204609	0.000000	0.000000	0.000000	100000
0.4166467E-04	0.9999954	0.000000	0.000000	0.000000	010000
0.000000	0.000000	0.9999966	0.2198527	0.000000	001000
0.000000	0.000000	-0.3135005E-04	0.9999966	0.000000	000100

It represents a case where there is no steering intended, yet the map clearly is one of a thin lens of focal strength -0.22 dioptres in x and $+0.22$ dioptres in y: a thin lens quadrupole.

This map is scalable on energy and electric potential: In the general case, the focal strengths are given by

$$\Delta x'|_{\text{quad}} = -\frac{x}{f} = -\frac{22}{\text{m}}\frac{V_s}{V_0}\,x = -\frac{0.56}{\text{inch}}\frac{V_s}{V_0}\,x, \qquad (1)$$

and y' is same but opposite sign. (In fact it is also scalable geometrically: if the same geometry is scaled by a factor of s larger, replace the metre by $s\,\text{m}$.)

The `OPERA` data used is too coarse for any calculation above second order, and the second order effect is zero in this case by symmetry.

2.2 Steer case

The potential of the top plot in Fig. 1 was used in `COSY-`∞ to check the steering strength and investigate aberrations. With 100 keV per charge and one side at 1 kV and the other at zero, the map to second order is found to be:

0.9999933	-0.1104298	0.000000	0.000000	0.000000	100000
0.2078168E-04	1.000004	0.000000	0.000000	0.000000	010000
0.000000	0.000000	1.000005	0.1094657	0.000000	001000
0.000000	0.000000	-0.1565849E-04	0.9999937	0.000000	000100
-0.2938188E-03	-2.092660	0.000000	0.000000	0.000000	200000
0.000000	0.000000	0.6417674E-03	4.091154	0.000000	101000
-0.1450939E-04	2.045567	0.000000	0.000000	0.000000	002000

(Note that the rows with negligible entries have been erased for clarity.)

The quadrupole effect is still there, but precisely half the strength of the previous case. It is easy to see, by a superposition argument, therefore that the quadrupole effect of the steerer depends only upon the sum of the potentials of the two plates (commonly, the steering plate and the 'common' plate). Thus, the formula eq. 1 still applies, but with $V_s = (V_1 + V_2)/2$, the average of the two plates' potentials.

The second order is accurately a pure sextupolar effect: scales as

$$\Delta x'|_{\rm sex} = -\frac{205}{\rm m^2}\frac{V_s}{V_0}(x^2 - y^2), \text{ and } \Delta y'|_{\rm sex} = \frac{205}{\rm m^2}\frac{V_s}{V_0}(2\,xy). \quad (2)$$

This has a negligible effect: for particles at the aperture limit will have the focus effect of the steerer changed by only $\sim 10\%$.

2.3 Bipolar

Another interesting case is where the plates are at opposite polarity (+1 kV and −1 kV, again for a 100 keV per charge particle):

0.9999779	-0.1954165E-02	0.000000	0.000000	0.000000	100000
0.9890816E-07	1.000022	0.000000	0.000000	0.000000	010000
0.000000	0.000000	1.000022	-0.8509528E-03	0.000000	001000
0.000000	0.000000	0.1351127E-06	0.9999782	0.000000	000100
-0.4000149E-04	-4.078218	0.000000	0.000000	0.000000	200000
0.000000	0.000000	0.4339047E-04	8.152922	0.000000	101000
0.1626594E-04	4.076640	0.000000	0.000000	0.000000	002000

As predicted by the superposition principle above, the focus effect disappears, but the sextupole effect is doubled as expected since the deflection also is doubled. To eliminate the higher order effects, the apertures should be shaped as slits of same orientation as the plates[3],[4]. In such a case, only the intrinsic (non-pure-multipole) aberrations would remain.

2.4 Zero order effect

Lastly, the steer effect can be quantified. `COSY` with `MELP` outputs the trajectory of the particles. From the deflection in the above case (plates at zero and 1 kV, beam of 100 keV per charge) is found to be 4.14 mrad. The expected deflection is written as

$$\Delta x' = \frac{L_{\rm eff}}{2\,s}\frac{V_s}{V_0}, \quad (3)$$

where s is the plate separation. This gives a value for the effective length of 1.24 inch or 31.5 mm.

3 Experimental Validation

Refer to Fig. 4 for locations of steerers and the emittance scanner.

Ideally, when all of the beamline and emittance scanner parameters are known, this is straightforward. However, this is not the case. According to drawings and direct measurement, the distance from the separator slit MCOL3B to the emittance scanner is $D = 0.500\,\text{m}$. This would mean that the emittance figure cast by a very narrow slit MCOL3B would show a straight line with slope of $1/D$ or 2 mrad/mm. But the slope found from this simple experiment turns out to be closer to 1.75 mrad/mm or $D = 0.57\,\text{m}$. See Fig. 2. The calculation of angle in the scanner is $\propto \frac{L_p}{g}\frac{V_p}{E}$ (symbols are plate length, plate separation, plate voltage, and beam energy resp.), and any of these 4 parameters could be wrong by a factor of 7/8. So I insert this as a 'fudge factor' for the remainder of the following analysis.

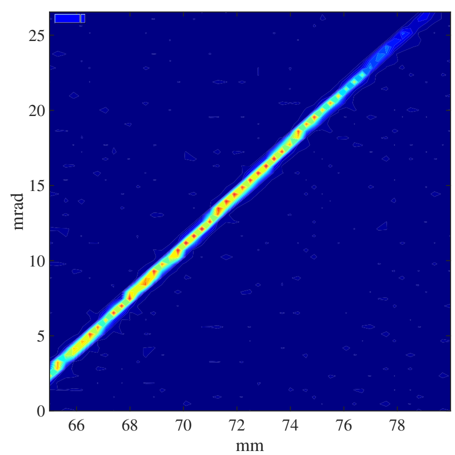


Figure 2: Emittance figure of a slit beam with all optics between MCOL3B and the scanner, set to zero.

The chosen steerer is OLIS:YCB6 as it is near and just upstream of the OLIS emittance scanner. As a first test, the two plates (YCB6 and CCB4, the 'common' connected to many steerers including YCB6) were set to (300V,300V), (600V,0V), (0V,600V). All three cases showed virtually identical emittance figures, confirming that the focus effect depends only upon the some of the voltages of the two plates.

Next, a tune was set up with steerer YCB6 and its common plate both at zero. This resulted in emittance figure of top left in Fig. 3. Then both the steer plates were set to 1 kV. The beam energy per charge was 8.16 kV, so this is a rather extreme case. The resulting emittance is shown as the top right in Fig. 3. Note that as the steerer is vertical, the emittance scan, being horizontal, is expected to show a defocus effect and it does; the figure is sheared in a counterclockwise direction.

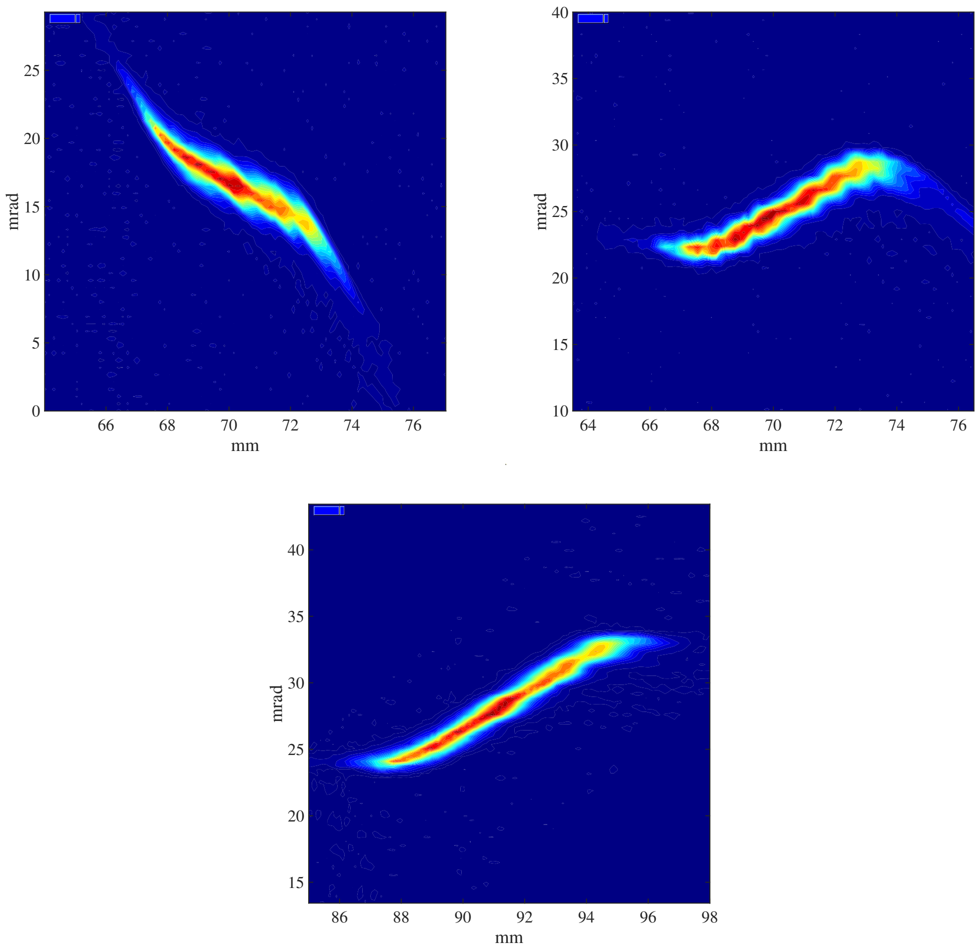


Figure 3: Emittance figures of beam at the OLIS emittance scanner. On the top left is the case with steerer YCB6 and its common plate at zero volts, while on the top right they are both at 1 kV. At bottom is the top left figure, transformed using the theoretically derived lens effect of the steerer YCB6 at 1 kV on both plates. All three plots have the same scale.

To make a comparison with the expected focus effect, the case shown at the top left was transformed by backtracking one drift distance of L, applying the defocus effect of the steerer, and then drifting forward by distance L, i.e., according to the following transfer matrix.

$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & -L \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 + \frac{L}{f} & -\frac{L^2}{f} \\ \frac{1}{f} & 1 - \frac{l}{f} \end{pmatrix} \tag{4}$$

For our case the distance L is not well-known, but the drawing ILE2200 from 2008 shows $\sim 4.5\,$inch, so that is the best guess. For $V_s = 1\,$kV, $V_0 = 8.16\,$kV, we get from eq. 1, $\frac{1}{f} = 2.70/$m, and thus

$$M = \begin{pmatrix} 1.308 & -0.035\,\mathrm{m} \\ 2.70/\mathrm{m} & 0.692 \end{pmatrix} \tag{5}$$

The result is shown in the lower part of Fig. 3. It is seen to agree fairly well with the measured figure when the plates were powered shown in the top right of the figure. An interesting difference is that the measured figure shows more third order aberration ('S'-shape). This is likely due to an octupole effect of the steerer.

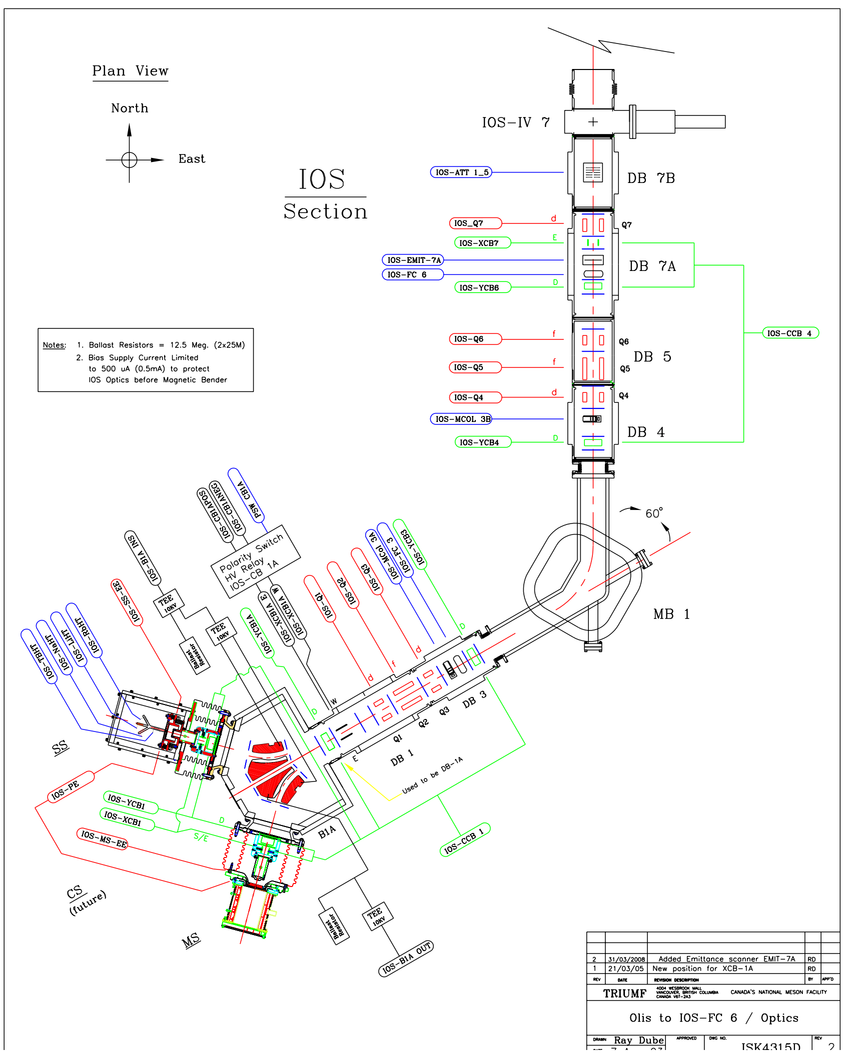


Figure 4: OLIS layout. Note location of steerer YCB6 and the emittance scanner.

References

[1] J. Maloney, New Electrostatic Element Procedures for COSY Infinity, Tech. Rep. TRI-BN-14-03, TRIUMF (2014).

[2] M. Berz, COSY INFINITY Version 8.1 — User's Guide and Reference Manual, Department of Physics and Astronomy MSUHEP-20704, Department of Physics and Astronomy, Michigan State University (2002). URL `http://cosy.pa.msu.edu/cosymanu/index.html`

[3] I. Bylinskii, R. Baartman, K. Jayamanna, T. Planche, Y.-N. Rao, Recent Improvements in Beam Delivery with the TRIUMF's 500 MeV Cyclotron, in: 21st Int. Conf. on Cyclotrons and Their Applications (Cyclotrons' 16), Zurich, Switzerland, September 11-16, 2016, JACOW, Geneva, Switzerland, 2017, pp. 133–136.

[4] J. Maloney, R. Baartman, T. Planche, S. Saminathan, Electrostatic potential map modelling with cosy infinity, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 376 (2016) 171–174.