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ARIEL HRS Tunes Derivation

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Abstract: This note clarifies three aspects: (1) What is the dominant source of the aberration that cannot be corrected by the multipole. This is the nonlinear coupling $(x|yy)$, and I could quantify how to minimize it with tuning for minimum vertical size, and a slit at the multipole. (2) The matching optics, because of the large vertical cubic aberration at large magnification, will give a large vertical halo and that his also is handled with the vertical slit at multipole. That slit restores high resolution with not much loss in transmission. (3) Unfortunately, the stable test ion source cannot easily explore the limit of the HRSs performance since its emittance is too small. A nominal surface source emittance is 3 times larger than the stable source and so is easily slit-selected down to the emittance needed to reach the ultimate combination of most acceptance for a 20000 resolution. The stable source cannot do this because the horizontal divergence at entry to the pure separator is limited by the matching optics and so cannot be expanded to fill the HRSs acceptance.

There is a git repo of the files I used. Further studies are encouraged by others wishing to extend the simulations to e.g. include misalignments, etc. but also to support commissioning.

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1 Introduction

High resolution separators have a reputation as being unstable and difficult to tune. The ARIEL HRS has been designed to overcome these difficult characteristics. To do this, it has two unique features. (1) The matching system into and out of the HRS acts as both a matcher and a dispersion-magnifier.[1] (2) The aberration correction is not performed using a conventional multipole. Instead, the correction element is a flat arrangement of electrodes that are programmed according to the correction function rather than a multipole-at-a-time approach.[2]

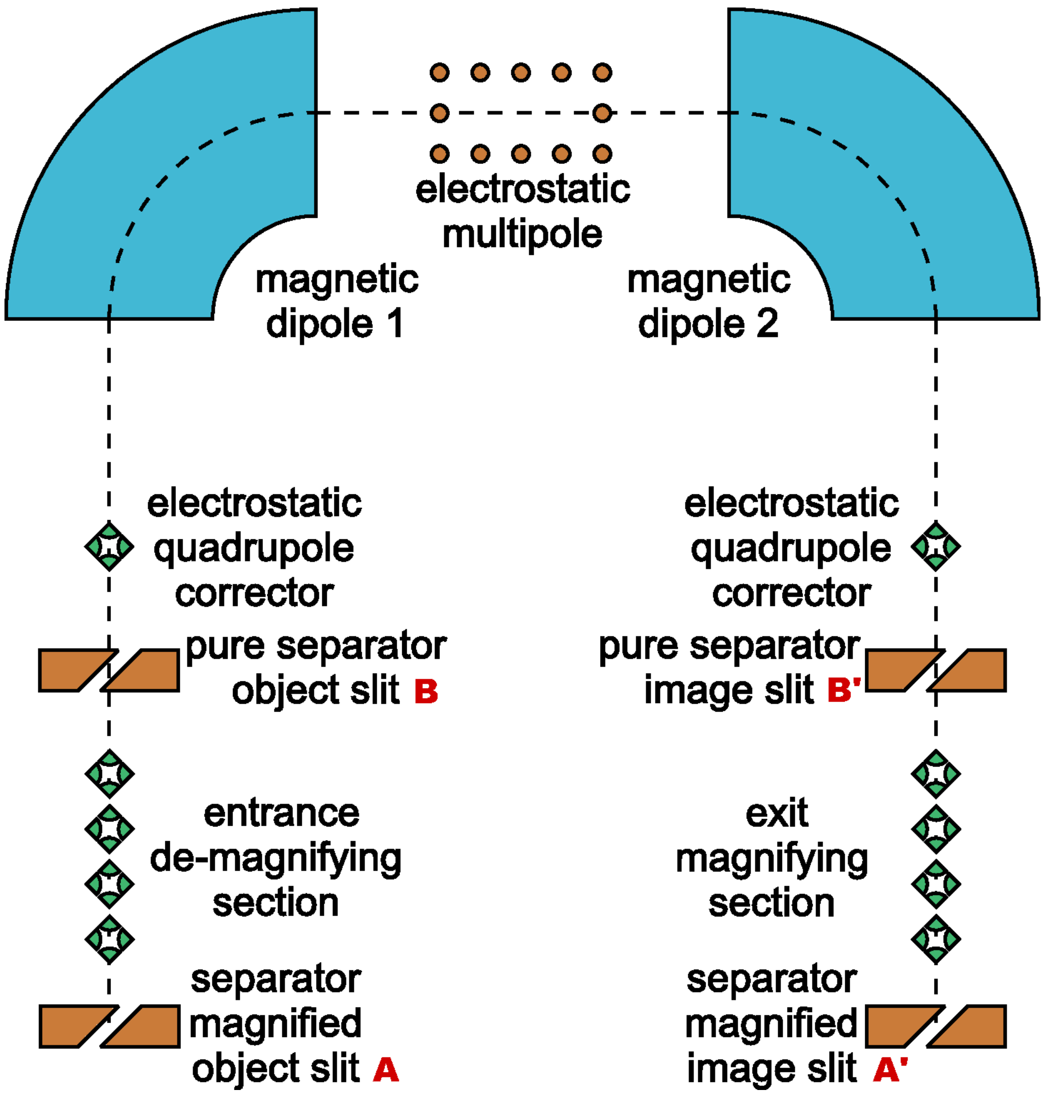

Figure 1: Schematic of ARIEL HRS showing the slits in red.

This note describes simulations used to characterize the performance of the HRS and its matching optics. All `COSY` maps used in simulations were to 7$^{\text{th}}$ order.

2 Pure Separator

The heart of the HRS is the two 90° dipoles, with the aberration corrector between them. See figure above. From entry (slit B) to exit (slit B'), the linear part of the horizontal transfer matrix is $-I$, the negative identity matrix. This is achieved with entry and exit edge angles in the dipoles. Further, the second order aberrations are cancelled with a curvature of the dipole edges (a conventional approach used in many other existing designs). The remaining aberration is corrected by the electrostatic corrector.

The vertical transfer matrix is also a $-I$, but only approximately so. The horizontal transfer matrix to the midpoint between the dipoles is precisely a $\pi/2$ phase advance. More than that, its matrix is symmetric $R_{11} = R_{22} = 0$, $R_{12} = d, R_{21} = -1/d$; this is assured by the fact of exact symmetry about the point halfway through the dipole. This has the handy feature that the emittance figure at the image (or mass-selection) point directly determines the electrode voltages needed to cancel the aberration observed. One simply interchanges x and x', scaling with a factor $d = 1.600\,\text{mm/mrad}$, to find the deflection function $x'(x)$ across the width of the beam at the multipole. The multipole is then programmed to give a negative of this function to correct it. See Fig. 2.

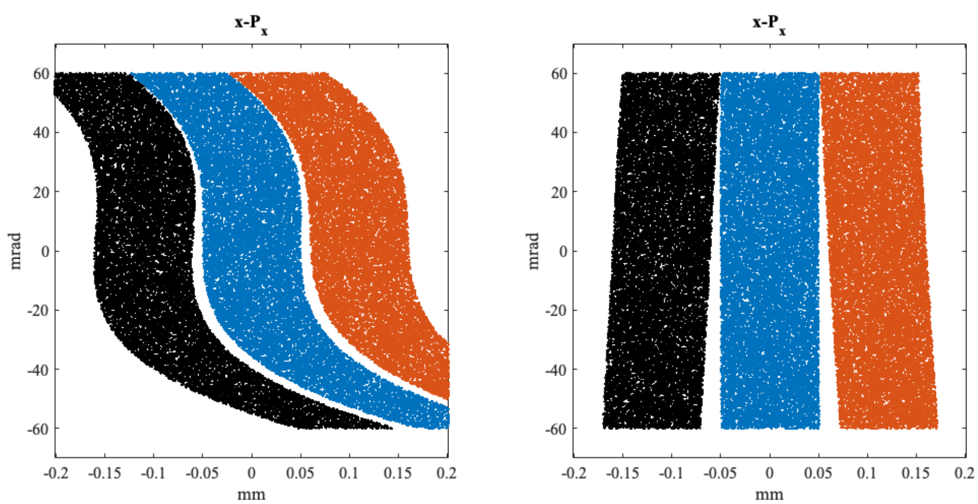

Figure 2: Corrected (right) and uncorrected (left) x-x' phase spaces are shown. The correction is up to decapole (fourth power in force), and the `COSY` maps used are to seventh order. For clarity, this is for a zero vertical emittance, and a horizontal beam that is $-50\,\mu\text{m} \leq x \leq 50\,\mu\text{m}$, $-60\,\text{mrad} \leq x' \leq 60\,\text{mrad}$. This would be an emittance of $3\,\mu\text{m}$ if elliptical, but we are using a uniformly filled rectangle of area $12\,\mu\text{m}$ rather than $3\pi\,\mu\text{m}$. The three masses are $\delta m/m = -1/21600, 0, 1/21600$.

2.1 Nonlinear coupling aberration

The multipole-corrected case with non-zero vertical emittance is shown in Fig. 3. This is for a uniformly filled rectangular phase space that would fit around an elliptical emittance of 6 μm (24 μm in actual area). The curvature on the right arises

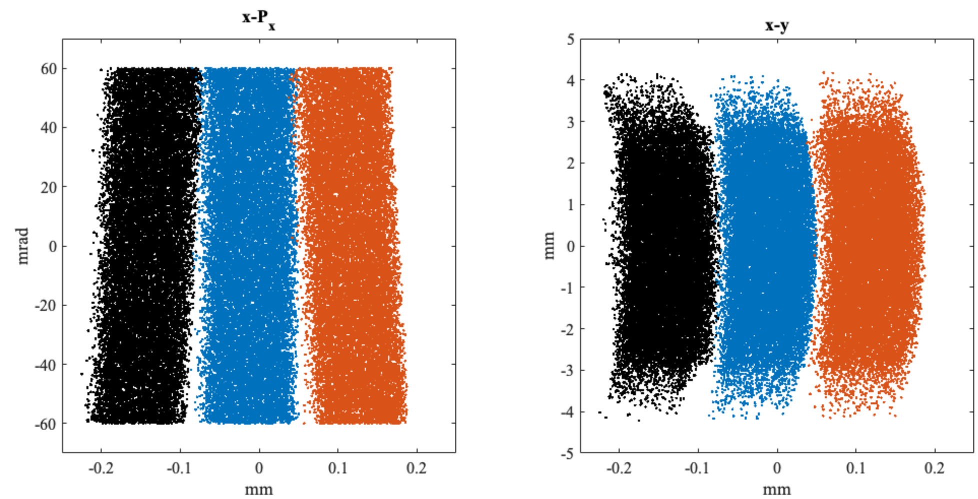

Figure 3: Corrected x-x' phase space on the left, real x-y space on right though x is expanded by factor of 20 compared with y. Note the parabolic distortion $x \sim y^2$ on right. To keep the masses separated for this fuzzier case, the mass separation has been increased to 1/18540.

from the dominant uncorrectable aberration that will most compromise the resolution, namely, the $(x|yy)$, meaning the shift in x due to y^2. This from the $P_x y^2$ term in the Hamiltonian for a sector magnet when it's expanded to third order[3]:

$$H(x, P_x, y, P_y) = \frac{1}{2}\left(h^2\, x^2 + P_x^2 + P_y^2 + hx(P_x^2 + P_y^2) + h'\, y^2 P_x\right) \tag{1}$$

Here, $h = h(s) = 1/\rho$ so h' is only non-zero at the dipole edges. From that term, we find $x' = \frac{\partial H}{\partial P_x} = h' y^2/2$. Since y is very close to constant over the short distance of the magnet edge, we easily integrate to find a shift in x:

$$\Delta x = \frac{y^2}{2\rho}. \tag{2}$$

Example: The smallest that a vertical emittance of 6 μm can be focused at the magnet edge is $y = 4.3$ mm, and $\rho = 1.2$ m. This results in a widening of the beam by 7.7 μm. The contributions of the four magnet edges add according to their betatron phases, and this results in a total growth of about 20 μm at the mass slit. Since the width of the slit to attain a resolution of 20000 is only 120 μm, this is already a significant degradation in performance. To maintain a high resolution requires the vertical aperture at the multipole midway between the dipoles to be no larger than 8 mm. As $y^2 \propto \epsilon_y$, this effect on resolution $\mathcal{R}$ can be styled as follows:

$$\mathcal{R} = \frac{24000}{1 + \frac{\epsilon_y}{36\,\mu\mathrm{m}}} \tag{3}$$

Including the horizontal limitation, where $\mathcal{R} = 24000$ for $\epsilon_x = 3\,\mu$m, we have finally:

$$\mathcal{R} = \frac{24000}{\frac{\epsilon_x}{3\,\mu\mathrm{m}} + \frac{\epsilon_y}{36\,\mu\mathrm{m}}} \tag{4}$$

2.1.1 Misalignment

The simulations in figures 2 and 3 are for a highly idealized uniform rectangular distribution, perfectly centred. In general, beams when not slit-selected would be closer to gaussian, and misaligned. Fig. 4 shows the simulations for cases where the beam is gaussian and misaligned vertically. This vertical misalignment of 8 mm

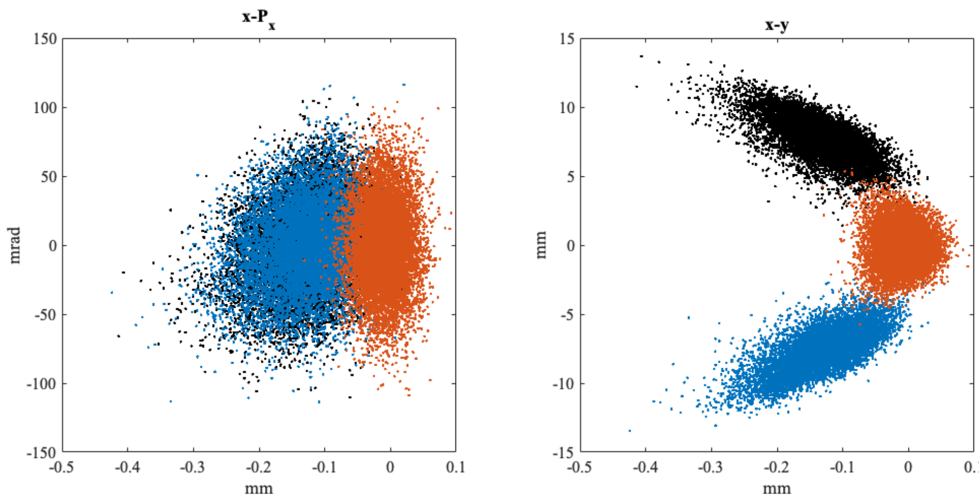

Figure 4: For 4rms emittances of $3\,\mu$m-by-$6\,\mu$m, three cases: misaligned by 8 mm (black) aligned (red), -8 mm (blue). (x-x' on the left, and x-y on the right.)

doubles the apparent horizontal emittance, thus reducing the resolution by a factor of 2. To limit the effect to 10% requires the misalignment through the dipoles to be less than 1 mm.

2.2 Chromaticity

Lastly, there is also a chromatic effect: the focal power of the pure separator focusing onto the mass selection slit is proportional to a particle's momentum. This is obvious from the above figures; the off-mass particles' distributions are tilted in x-x'-space. For all particles having same energy, this results in a small tilt in x-x' phase space for particles with mass deviation. This is evident in the figures. `COSY` map has this effect as the Δx (first element) in the 0100001 row, which is 3.6 m. (Or in other notation, $(x|x', dm/m)$.) The mass dispersion equation including chromaticity is then

$$\Delta x = [2.4\,\mathrm{m} + (3.6\,\mathrm{m})\,x'] \frac{\delta m}{m} = (1 + 1.5x')(2.4\,\mathrm{m}\,\frac{\delta m}{m}) \tag{5}$$

(The factor of 1.5 would be $\pi/2$ in a smooth approximation and can be understood as due to the slit-to-slit transport being a phase advance of π. In a smooth approximation, $x'' + k^2 x = 0$ would give $x = x_0 \sin ks$, and for the length of the system L, $kL = \pi$. So for a variance in k due to mass, we get $\Delta x = x_0 \Delta k L = \pi x_0 \Delta k/k = (\pi/2) x_0 (\delta m/m)$.)

Having used the angular full acceptance as $x' = 0.06$, this results in a total Δx diminished by 9% for $x' = 0.06$, augmented by the same amount for $x' = -0.06$. This is apparent in Fig. 2-right and Fig. 3-left. The resolution is thus reduced by this same amount, but proportional to the angular width. Thus the above equation for resolution becomes:

$$\mathcal{R} = \frac{22000}{\frac{\epsilon_x}{3\,\mu\mathrm{m}} + \frac{\epsilon_y}{36\,\mu\mathrm{m}}} \tag{6}$$

For reference, the slit-selected case of the more typical surface source emittance of 10 μm-by-10 μm and mass separation of 1/6000 is shown in Fig. 5. This shows that a typical case with high transmission should still have a resolution of ~ 6000.

3 Match/Magnifier

The match/magnifier section consists of only four quadrupoles. It performs the following functions: directly from the horizontal waist in the periodic transport (slit A), a second waist (slit B) is created that magnifies the divergence and demagnifies the beam size to the location of the pure separators object slit. This second waist is to be an image of the first by tuning the quadrupoles to make $R_{12} = 0$. The diagonal elements are then $1/M$ and M, M being the magnification factor. This factor can be tuned to be any value in the range from 3 to 10. See previous note on such tunes[4].

For the 4 quadrupoles, there are thus two conditions from horizontal: $R_{12} = 0$, and the desired value of M. The two remaining conditions can be used to create a

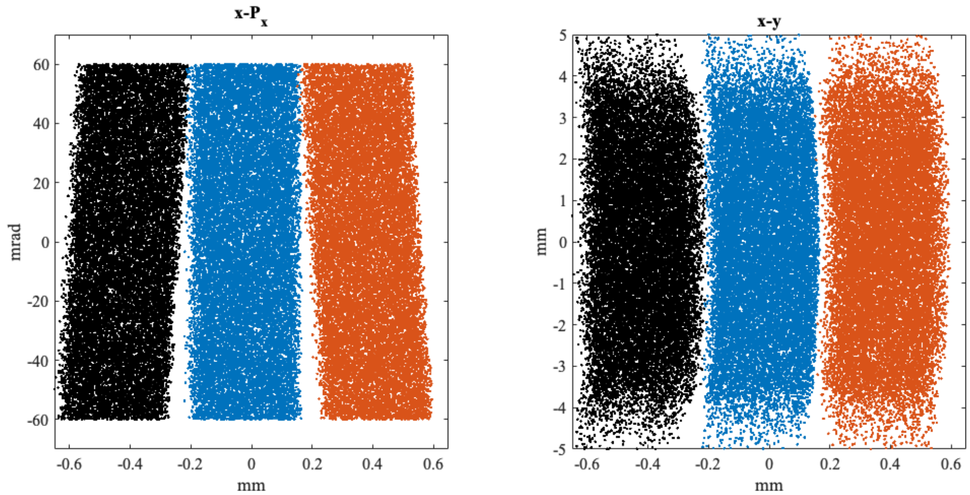

Figure 5: Case of a more typical emittance of 10 μm-by-10 μm and mass separation of 1/6000 x-x'. Phase space on the left, and x-y configuration space on the right.

vertical waist at the multipole. This waist is to be small as possible in order to minimize the second order nonlinear coupling aberration ($\Delta x \propto y^2$) due to finite vertical size in the dipoles, as mentioned in section 2.1. However, since the vertical dipole focusing is weak, the β_y-function cannot be reduced below about 2.2 m. It is this effect that limits the vertical acceptance of the HRS in high resolution mode. Since the pure separator does not change, there is only one solution where vertical size is minimized at the dipole edges. This is where there is a vertical waist at the multipole centre, with $\beta_y = 219$ cm.

There two distinct ways of using this section: one where the beam is selected using slit A, imaged at slit A', and the other using slit B, imaged at B'. The "pure separator", consisting of the section from slit B to slit B', has a mass dispersion of 2.4 m. This means masses whose relative difference is 1/20000 are separated by 0.12 mm at slit B'. This tiny separation can be resolved with slits B, B' of that size, but leads to a maintenance issue if used this way exclusively. The slits are necessarily delicate with micron-sized edges, and will erode after some use. In this mode, the slits are set to the desired resolution, and the quadrupoles only have the function of optimizing the transmission through the separator.

In the 'magnified' mode, the slits to use are A and A'. The magnification is as high as 9, meaning the dispersion at A' is 21.6 m. So slits would be set at no smaller than about 1 mm, and these can be thicker and more robust than the ones at B, B'. The disadvantage, though is that the match/magnifier 4-quad section brings in its own aberrations, and these limit the performance (resolution and acceptance) of the separator. In this mode, the quadrupoles are crucial to achieving the desired performance.

3.1 How Important is Matching?

A metric for performance is product of resolution and acceptance. It is accepted that the HRS is not capable of 20000 resolution at nominal emittances of $\sim 10\,\mu$m: as illustrated in Fig. 5 the resolution would then be only 6000. We have specified the 20000 resolution at $3\,\mu$m in x by $6\,\mu$m in y. But this fact that the highest resolution is not expected to have a transmission of more than about $18\%(= 3/10 \times 6/10)$ means the exact matching simply is not critical. We can match by using slits, and it is more flexible than using quads only. For example, it is not possible to match vertically (to get small enough vertical beam inside the dipoles) for any magnification above ~ 9. But if we slit-select down to $3\,\mu\text{m} \times 6\,\mu$m, matching is not a problem. Similar with x-direction. We cannot reach a magnification of > 9, but we don't have to. If we are throwing away about 80% of the beam anyway, we can slit-select in x down to the required beam size, whether we select at slit A (where we need 1 mm), or slit B (where we need ~ 0.1 mm).

To achieve maximum resolution-acceptance product, requires filling the separator's acceptance. As the beam is focused through the slit B, the divergence enlarges and this widens the beam at the multipole. The emittance scanner is then used to determine the multipole setting to straighten the emittance figure at the mass slit B'. Then if desired, slit A can be brought in and slits B,B' backed off, and slit A' be brought in to required size to select the mass.

3.2 Developing Match/Magnifier Tunes

This consists of 3 steps.

1. I start with the `TRANSOPTR` model, since fitting is easy. The fringe field integrals have been determined for the skimmer geometry used, and the Enge coefficients that give identical results in first order `COSY` have also been determined. This allows easy transfer of tunes between `TRANSOPTR` and `COSY`. For a given desired magnification M, a 4-quad tune is found using the fitting constraints mentioned above,
2. and then passed on to `COSY`. (The `TRANSOPTR` system code `sy.f` is written in a way that outputs the quad settings in `COSY` format.) The transfer map is found to 7^{th} order,
3. and then passed on to Matlab, where the simulation tools are used to analyze it.

The `TRANSOPTR` and `COSY` files are provided in the previous note [4], and are available in a git repo. The Matlab tools are described in a note[5], and available on another repo.

3.3 Example tune

In what follows, I use a nominal emittance beam of 10 μm-by-10 μm. I show how to obtain best resolution with slit selection.

As discovered in the previous note [4], the quadrupoles at a the magnification extreme cause severe cubic distortion. This is a well-known effect. For horizontal, this can be corrected with the multipole. In fact it turns out that pure HRS has an octupole component too, and of opposite sign, so it is partially compensated. But for vertical, the added amplitude causes $(x|yy)$ nonlinear coupling. This is illustrated in Fig. 6 for the tune with optimal vertical behaviour (tune 59), i.e., a vertical waist at multipole centre with smallest possible size.

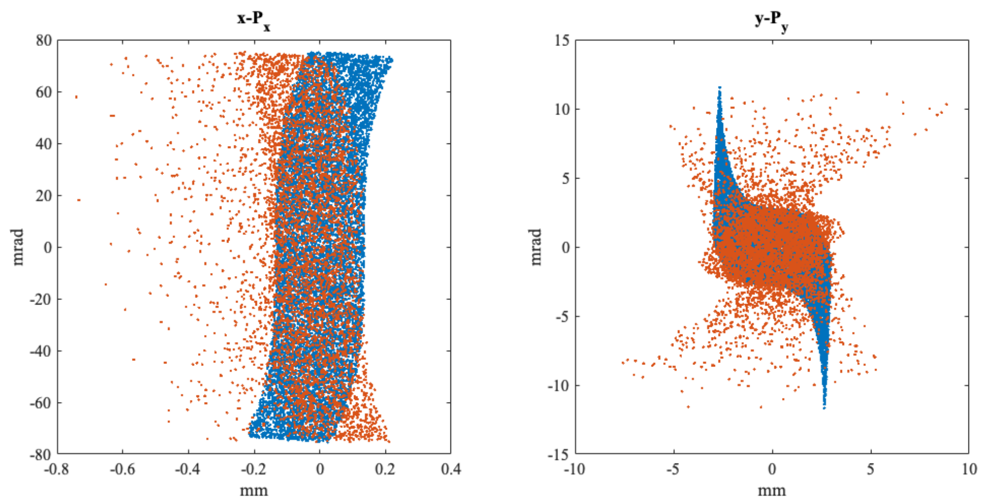

Figure 6: Tune 59 at slit B (blue) and at B' (red). Horizontal phase space on the left and vertical on the right.

But the 'fuzz' in vertical, which causes the smear in horizontal is easily handled by a vertical slit placed between the dipoles. (Let us call this slit 'C'.) That is because it's caused by the blue spikes appearing in Fig. 6 (right), and these are rotated 90° in phase space when at the multipole. The plots at this location are shown in Fig. 7. On the scale of this plot, the slit width of 12.5 mm cuts out all of the badly-behaved particles.

The comparison at the slit A' (final mass selection location if not using slits at B, B') with (red) and without this slit C is shown in Fig. 8.

The case of three masses separated at resolution of 7200 are shown in Fig. 9. Slit C has lost only 26% of the beam.

To obtain highest resolution, slit A must be used to also select horizontal phase

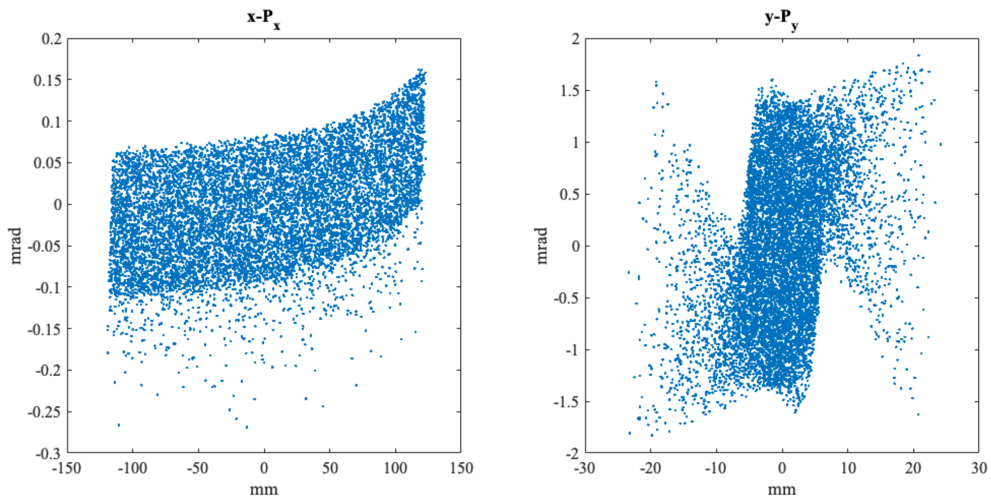

Figure 7: Tune 59 at the location of the multipole. Horizontal phase space on the left and vertical on the right.

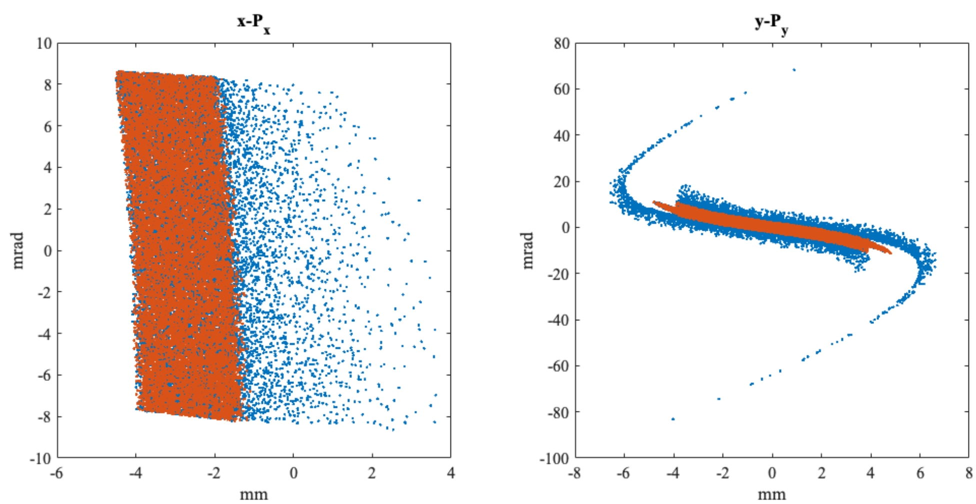

Figure 8: Tune 59 at the location of the final mass selection slit A'. Red is with slit C, and blue is without. Horizontal phase space on the left and vertical on the right.

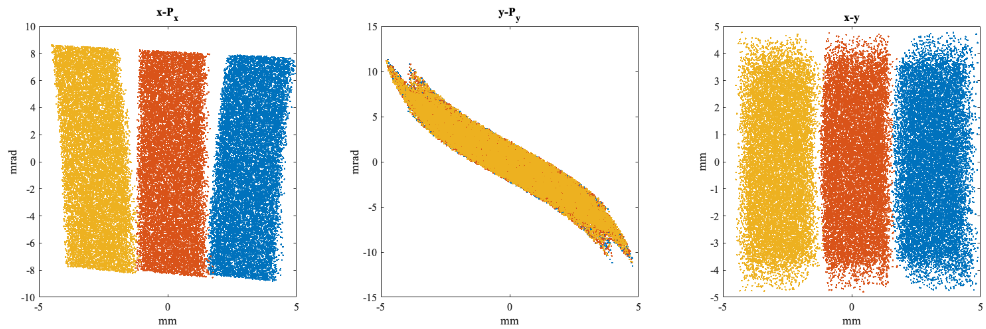

Figure 9: Tune 59 at the location of the final mass selection slit A'. 3 masses separated by $\delta m/m = 1/7200$. Left to right are x and y phase space, and xy space. Note that in this case, the xy aspect ratio is realistic reflecting the actual shape of the slit.

space out of the 10 μm emittance beam. By setting slit A width to 0.8 mm horizontally and 10mm vertically, slit C at 240 mm horizontally and 8 mm vertically, the transmission for the 10 μm-by-10 μm initial beam is 17.4%. This achieves a resolution of 20000, as shown in Fig. 10. The final 4rms emittances are 4.5 μm by 7.5 μm.

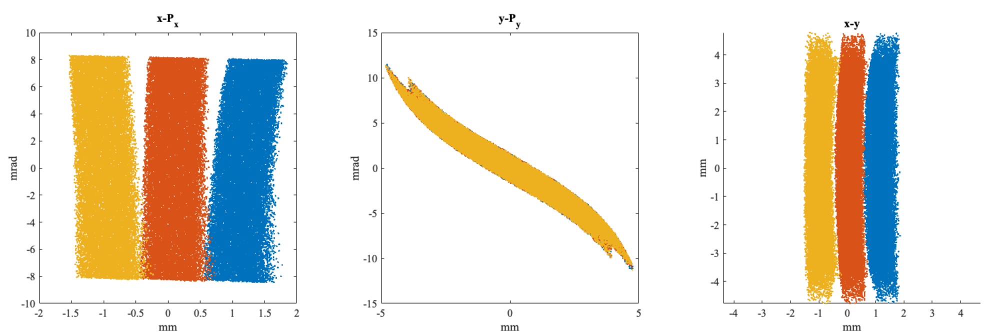

Figure 10: Tune 59 at the location of the final mass selection slit A'. 3 masses separated by $\delta m/m = 1/20000$. Left to right are x and y phase space, and xy space. Note that in this case, the xy aspect ratio is real.

4 Repository

Files to calculate `COSY` maps, `TRANSOPTR` quad fits, and scripts for plotting the above figures are all available in a git repo here (`https://gitlab.triumf.ca/beamphys/ariel-hrs-simulations`).

References

[1] J. Maloney, M. Marchetto, R. Baartman, ARIEL High Resolution Separator, Tech. Rep. TRI-DN-14-06, TRIUMF (2014).

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[4] R. Baartman, HRS match/magnifier section, Tech. Rep. TRI-BN-18-15, TRIUMF (2018).

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