**TRIUMF Beam Physics Note**
TRI-BN-24-23
September 2, 2024

# Optimal Excitation and Parameter Extraction for a Steering Quadrupole

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**Abstract:** This note discusses how to power the steering quadrupole to achieve the desired steering and focusing using the optimal coil current configuration. Additionally, the dipole and quadrupole components of the steering quadrupole 4Q8.5 are calculated using the FEA model and compared with the measurement data.

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# 1 Introduction

In beamline 1U, a well-centered beam is required to pass through the collimator and onto the target. However, there is insufficient space to accommodate additional steering magnets in the beamline. To provide steering capability, a magnetic quadrupole is equipped with four additional independent trim power supplies, one for each pole coil. This steering quadrupole can provide horizontal and vertical steering along with the beam focusing power.[1, 2].

# 2 Model

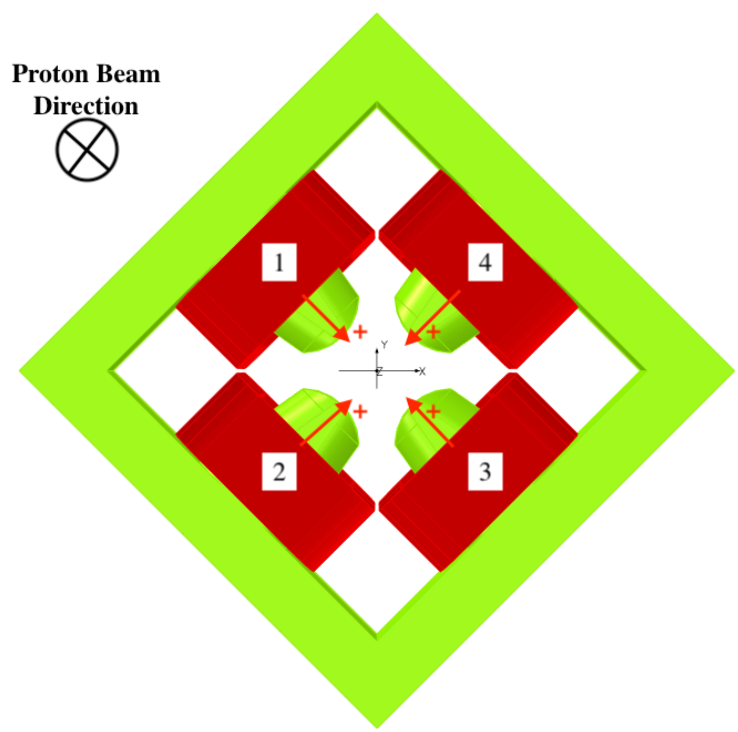


Figure 1: Opera model of the 4Q8.5 quadrupole. The positive current direction in each coil produces an inward magnetic field. The beam direction is into the plane of the paper. Therefore, to generate a focusing quadrupole field, the current signs for coils 1 through 4 should be -1, 1, -1, 1.

The magnet has 4 individual coils. By define a 4D vector to represent the excitation of the coils, The quadrupole, horizontal dipole and vertical dipole components could be written as

$$
\begin{aligned}
\vec{Q} &= [-1, 1, -1, 1] \\
\vec{H} &= [1, -1, -1, 1] \\
\vec{V} &= [-1, -1, 1, 1]
\end{aligned} \tag{1}
$$

These three vectors are orthogonal to each other, which can be easily verified by taking the dot product of any two of them. A fourth vector that is perpendicular to these three vectors is calculated using a 4D cross product method, which is a generalization of the cross product method in 3D for computing a vector that is perpendicular to two linearly independent vectors. Specifically, define $\vec{I_1} = (1, 0, 0, 0)$, $\vec{I_2} = (0, 1, 0, 0)$, $\vec{I_3} = (0, 0, 1, 0)$, and $\vec{I_4} = (0, 0, 0, 1)$. The fourth orthogonal vector is

$$
\vec{M} = det \begin{pmatrix} \vec{I_1} & \vec{I_2} & \vec{I_3} & \vec{I_4} \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix} = 4[1, 1, 1, 1] \tag{2}
$$

The vector $\vec{M}$ represents the monopole component. Along with the quadrupole component $\vec{Q}$, horizontal steering $\vec{H}$, and vertical steering $\vec{V}$, these vectors form an orthogonal basis for the 4D parameter space, meaning each component can be adjusted independently without affecting the others. Similarly, $\vec{I_1}$, $\vec{I_2}$, $\vec{I_3}$, and $\vec{I_4}$, the currents of each coil, is another orthogonal basis for the 4D parameter space. The transfer matrix that relates the optical components to the coil currents, and vice versa, is given by:

$$
\begin{aligned}
\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} &= \begin{pmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} Q \\ H \\ V \\ M \end{pmatrix} \\
\begin{pmatrix} Q \\ H \\ V \\ M \end{pmatrix} &= \frac{1}{4} \begin{pmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}
\end{aligned} \tag{3}
$$

## 3 Power the quad by 5 power supplies

For 1UQ1, the asymmetrical quads, it has been powered by 5 power supplies. Using the notation in figure 1, they could be expressed as 5 4D vectors as below

$$
\begin{aligned}
\vec{i_0} &= i_0[-1, 1, -1, 1] \\
\vec{i_1} &= i_1[-1, 0, 0, 0] \\
\vec{i_2} &= i_2[0, 1, 0, 0] \\
\vec{i_3} &= i_3[0, 0, -1, 0] \\
\vec{i_4} &= i_4[0, 0, 0, 1]
\end{aligned} \tag{4}
$$

Obviously, the solution to produce the given vector $[Q, H, V, M]$ using the 5 power supplies is not unique. Since $\vec{i_0}$ is identical to $\vec{Q}$, we can find the unique solution of $[i_1, i_2, i_3, i_4]$ to produce $[Q-i_0, H, V, M]$. The transfer matrix between them calculated using eq.3 is given below

$$
\begin{aligned}
\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \\
&= \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} Q - i_0 \\ H \\ V \\ M \end{pmatrix}
\end{aligned} \tag{5}
$$

Typically, the monopole component is set to zero because it does not contribute to the desired steering or focusing, but instead introduces undesirable higher-order aberrations. Additionally, the current of the trim power supplies for this quadrupole must be positive, as it can only add current to the coil. These conditions result in:

$$
Q - i_0 - |H| - |V| \geq 0 \tag{6}
$$

Since the main power supply has a larger capacity than the trim power supplies, it is preferable to run $i_0$ at the maximum value within the calculated range. Therefore, the settings for the five power supplies are given by

$$
\begin{aligned}
i_0 &= Q - |H| - |V| \\
i_1 &= |H| + |V| - H + V \\
i_2 &= |H| + |V| - H - V \\
i_3 &= |H| + |V| + H - V \\
i_4 &= |H| + |V| + H + V
\end{aligned} \tag{7}
$$

From the above equation, under all the steering settings, at least one of the trim power should be 0. In the following paragraph, 3 examples are given.

**Example 1**:

Default setting Q=100 without steering, the currents of power supplies are $i_0$=100, $i_1 = i_2 = i_3 = i_4 = 0$. Now we need H=10 and V=0.

The new settings calculated by eq.7 are $i_0 = 90, i_1 = 0, i_2 = 0, i_3 = 20, i_4 = 20$. In the notation shown in figure 1, the current in the coils are $I_1 = -90, I_2 = 90, I_3 = -110, I_4 = 110$. They equals to $[-100, 100, -100, 100] + [10, -10, -10, 10]$, that's the ideal setting.

**Example 2**:

Same initial condition. Change the steering to H=-10 and V=0.

The new settings calculated by eq.7 are $i_0 = 90, i_1 = 20, i_2 = 20, i_3 = 0, i_4 = 0$. In the notation shown in figure 1, the current in the coils are $I_1 = -110, I_2 = 110, I_3 = -90, I_4 = 90$. They equals to $[-100, 100, -100, 100] + [-10, 10, 10, -10]$, that's the ideal setting.

**Example 3**:

Same initial condition. Change the steering to H=20 and V=10.

The new settings calculated by eq.7 are $i_0 = 70, i_1 = 20, i_2 = 0, i_3 = 40, i_4 = 60$. In the notation shown in figure 1, the current in the coils are $I_1 = -90, I_2 = 70, I_3 = -110, I_4 = 130$. They equals to $[-100, 100, -100, 100] + [20, -20, -20, 20] + [-10, -10, 10, 10]$, that's the ideal setting.

# 4 Calculate the field parameters

## 4.1 FEA simulation

Using the FEA model shown in Figure 1, I calculated the field produced by a single coil on one pole of the magnet. Due to the symmetry of the magnet, the fields produced by the other three coils are rotated by N*90 degrees with respect to the beam axis. Assuming the model is linear,for different coil excitations to produce either a quadrupole or dipole component, the total field is the sum of the fields from all four coils. By taking an FFT, the amplitude of the field harmonics along the circle on the axis is calculated. The radius of the circle is 2 cm, which is half of the magnet aperture radius. The harmonics are shown in the plot below. The magnet parameters are calculated from the field data, which are summarized in table 1.

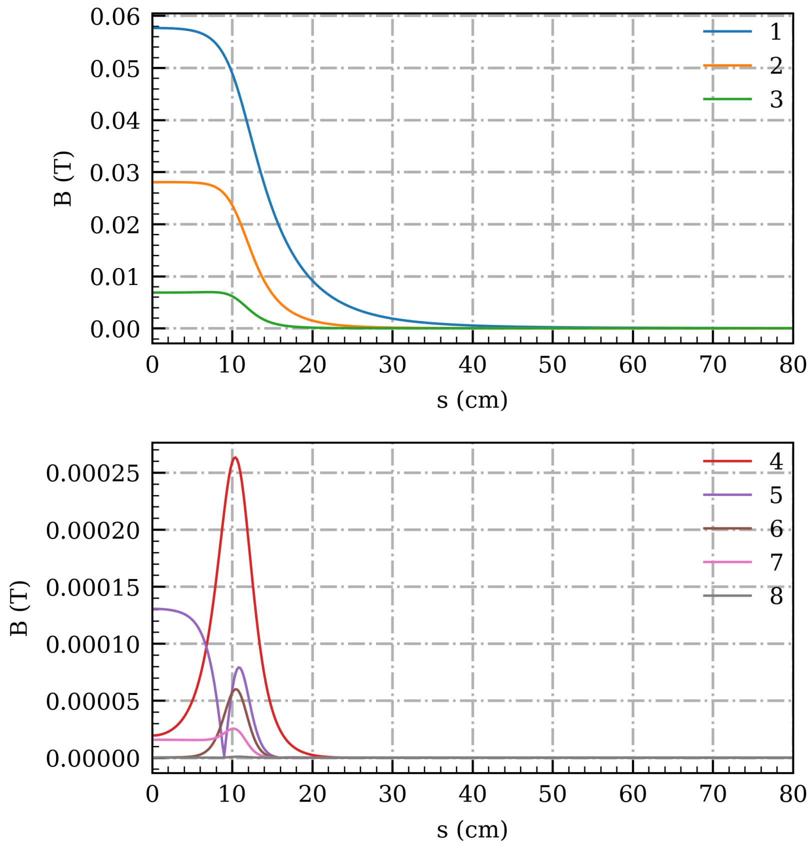


Figure 2: Field harmonics produced by a single coil at a 100 A current setting. 0 is at the center of the magnet, so the field on the left side, not shown in the plot, is symmetric with respect to the y-axis. The labels correspond to the harmonic numbers. The dominant field components are the dipole, quadrupole, and sextupole. The dipole field distribution has a softer edge, indicating that it has a longer effective lengths.

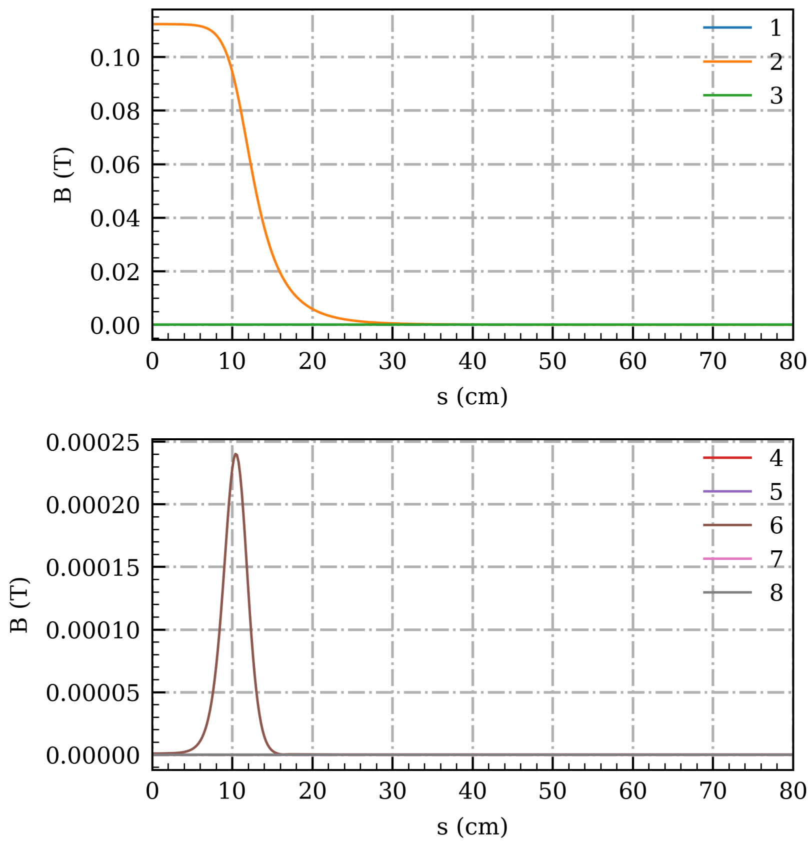


Figure 3: Filed harmonics produced by 4 coils at a 100 A current setting with the configuration given in eq.1 to produce a quadrupole component. Under the quadrupole configuration, only harmonics of the form 2+4*N exist, because the other harmonics are cancelled out by the symmetry.

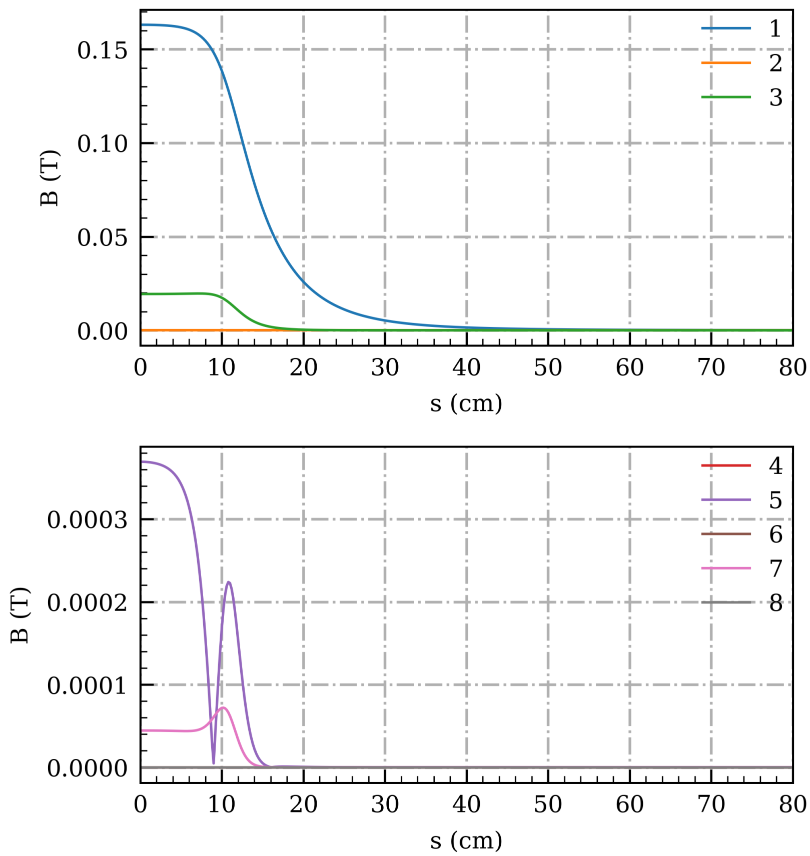


Figure 4: Filed harmonics produced by 4 coils at a 100 A current setting with the configuration given in eq.1 to produce a horizontal steering. Only harmonics of the form 1+2*N exist due to the symmetry. For the vertical steering , the field harmonics has the same amplitude with different angle. The angle difference is $N\pi/2$.

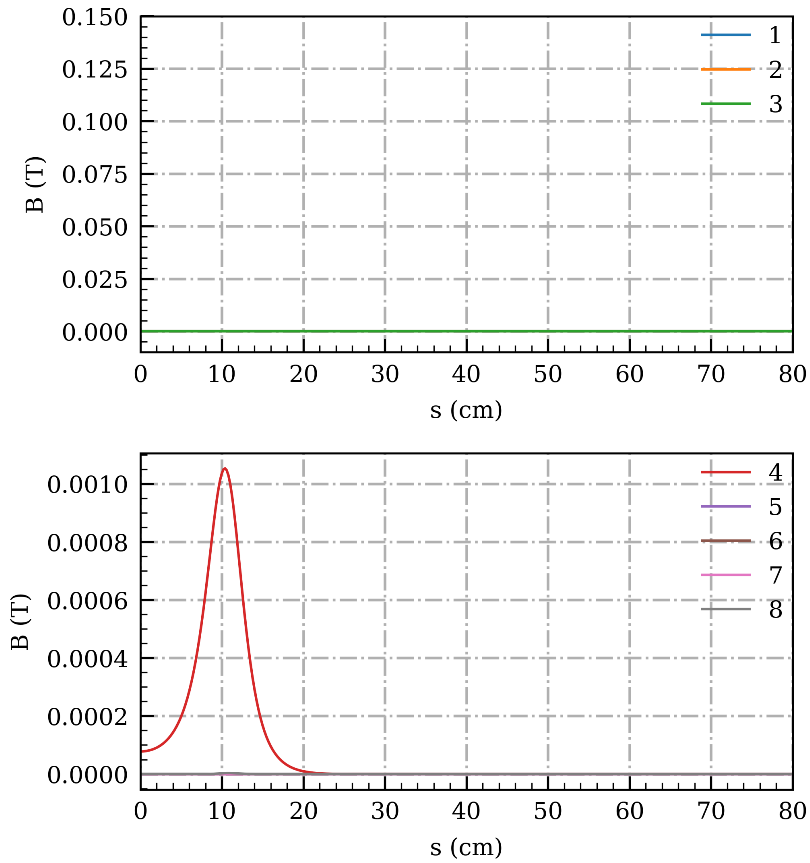


Figure 5: Filed harmonics produced by 4 coils at a 100 A current setting with the configuration given in eq.2 to produce a monopole. Only harmonics of the form 4*N exist due to the symmetry.

Table 1: Magnet parameters

| Component | B-I | Effective length |
|---|---|---|
| Quadruple | 5.6 Gs/(cm*A) | 26.4 cm |
| Dipole | 16.3 Gs/A | 30.4 cm |

## 4.2 Measured component

According to Doug Evan's field survey table, the effective length of the 4VQ8.5 is 26.18 cm, with a B-I slope of 5.8 Gs/(cm·A) and an integral field gradient of 151.84. The difference between the measured and calculated values is within 3%, which may be due to the nonlinearity of the iron core. In the absence of field survey data with steering coil excitation, we measured the steering power by scanning the steering and recording the beam positions at BPMs downstream of the magnet. The results are shown below.

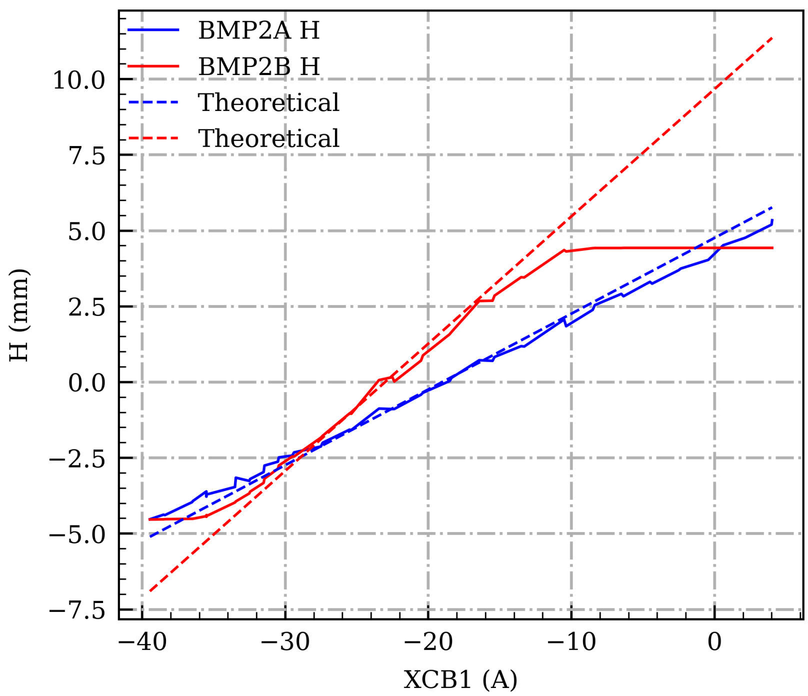


Figure 6: Horizontal steering measurement. The M12 elements of the transfer matrix from XCB1 to BPM2A and BPM2B are 0.1768 cm/mrad and 0.2997 cm/mrad, respectively. These values, along with the parameters in Table 1, provide the theoretical slope. The calculated slope agrees with the measured value.

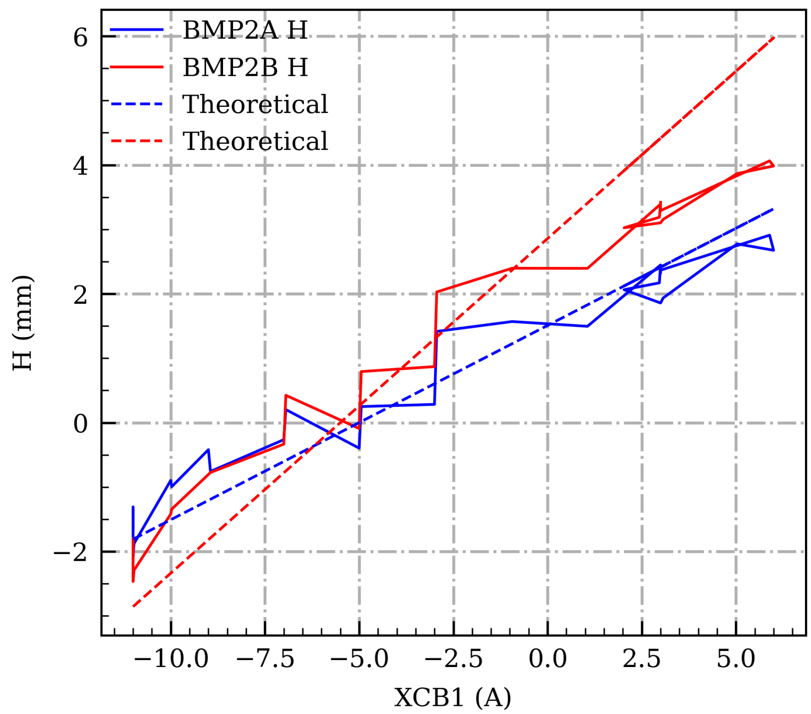


Figure 7: Vertical steering measurement. The M34 elements of the transfer matrix from YCB1 to BPM2A and BPM2B are 0.2159 cm/mrad and 0.3718 cm/mrad, respectively. These values, along with the parameters in Table 1, provide the theoretical slope. The calculated slope agrees with the measured value.

# 5 Conclusions

An optimal solution for powering the five power supplies of the asymmetric quadrupole is given, in which the trim supplies remain positive and the monopole component is zero, while maximizing the use of the main power supply rather than the trim supplies. The dipole and quadrupole components of the steering quads 4Q8.5 is calculated using FEA simulation, the calculated quadrupole component agrees with field survey result. And the dipole component was verified using the steering scan in the beamline 1U.

# 6 Acknoledgement

Grateful acknowledgment is extended to Yi-nong Rao for the insightful discussions and to Rick for the informative lectures on steering quadrupoles.

# References

[1] Y.-N. Rao, How to Power Quadrupole 1VQ5 Asymmetrically?, Tech. Rep. TRI-BN-13-03, TRIUMF (2013).

[2] G. W. Grime, A compact beam focusing and steering element using quadrupoles with independently excited poles, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 306 (2013) 12–16. doi:https://doi.org/10.1016/j.nimb.2012.10.041.
URL `https://www.sciencedirect.com/science/article/pii/S0168583X12006787`