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Dipole and Quadrupole Magnet's Installation Tolerances

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Abstract: In this note we give simple formulae to characterize the installation tolerances for dipole and quadrupole magnets in a beamline, and then apply to BL4N for calculations.

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1 Definitions

The installation tolerances for any magnet in a beam transport line cover 3 positional errors (Δx, Δy, Δs) and 3 angular errors (θ_x, θ_y, θ_s). Here the $x - y - s$ forms a right-handed curvilinear coordinate system as defined in `TRANSPORT`. It is a local frame, moving along the reference trajectory. Its origin is constantly riding on the reference trajectory, $+s$ axis is in tangential of the reference trajectory (no matter the beam particles are positively charged or negatively charged), and $+x$ axis is in the radial direction in the bending plane (for a straight section with no bending, it's regarded as a right bend with an infinitely large bend radius).

Usually, the magnet alignment errors are addressed along with orbit corrections in the x and y planes. Therefore, the transverse positional errors (Δx, Δy) and the angular errors (θ_x, θ_y, θ_s) are discussed more often than the longitudinal installation error Δs.

For a beamline lying in the horizontal plane, the 3 angular errors are called roll, yaw and pitch respectively. Specifically, the roll is a rotation about the s axis, the yaw is a rotation about the y axis, while the pitch is a rotation about the x axis.

Next we aim to derive simple formulae to characterize the tolerances, and then apply them to the BL4N for calculations.

2 Quadrupole Installation Tolerances

A quadrupole's positional and/or angular errors will cause an angular error to the **reference** trajectory locally, leading to a distortion to the **reference** trajectory downstream (called closed-orbit distortion in a synchrotron/storage ring). We can isolate these errors (based on the superposition principle of magnetic field) and discuss their affects separately.

2.1 Positional Error

Take the horizontal plane as an example. As is shown in the diagram Fig.1, there exists 2 frames: $x - o - x'$ denotes a frame sitting on the **reference** trajectory; $q - a - q'$ denotes a frame sitting on the axis of a magnet. Remember that the Courant-Snyder parameters $(\alpha(s), \beta(s), \gamma(s))$ are defined nowhere but on the reference trajectory. In the real life the beam ellipse is NOT necessarily centering on the reference trajectory, either due to an initial centering error at injection or due to the "closed-orbit distortion" somewhere in the beamline. It is important to minimize the COD.

The displacement (parallel shift) of a quadrupole in x will cause an error in x' to the reference trajectory, that is,

$$\Delta x' = \frac{x}{f} \tag{1}$$

where f denotes the focal length of the quad (in thin lens approximation). Note that here we ignore the sign of $\Delta x'$ and only look at its absolute value. We require that such an error be far smaller than the local beam divergence, i.e.

$$\Delta x' \ll \sqrt{\frac{\epsilon_x}{\beta_x}} \tag{2}$$

where β_x denotes the lattice function, ϵ_x is the beam emittance (conventionally prescribed as $4\epsilon_{rms}$ for the hadron machine, or ϵ_{rms} for the electron machine). It's worthy to point out that the local beam divergence $\sqrt{\epsilon_x/\beta_x}$ is NOT necessarily equal to the entire beam's divergence $\sqrt{\epsilon_x\gamma_x}$ unless the phase ellipse is upright (i.e. $\alpha_x = 0$).

We thus get

$$x \ll \sqrt{\frac{\epsilon_x}{\beta_x}}\, f = \sqrt{\frac{\epsilon_{xn}}{\beta\gamma\beta_x}}\, f \tag{3}$$

where ϵ_{xn} denotes the normalized beam emittance (which is constant). For the $\ll$ sign, we could use $\leq (0.05\times)$ if easily achieved, $\leq (0.1\times)$ if difficult. The equation is similar in the y plane.

When a beamline is set up to run at different energies without changing anything to the optics, the magnet excitation has to change as per relativistic factors $\beta\gamma$. In this case, the β_x and f are independent of beam energy. Thus, the tolerance requirement will become tighter at high energy than at low energy. This is understood as following.

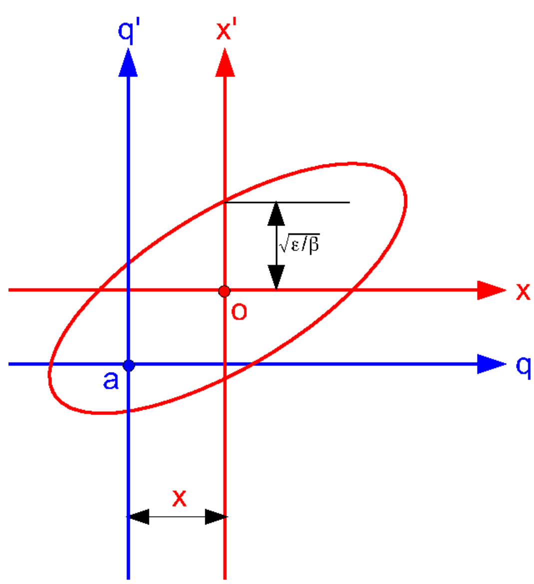

Figure 1: *Diagram showing displacement of a quadrupole (blue frame) relative to the reference trajectory (red frame).*

For a dipole, its bending radius has to remain unchanged and independent of beam energy,

$$\rho = \frac{p}{qB} = \frac{m_0\, c\, \beta\gamma}{qB} \ , \quad B = \frac{\mu_0 NI}{G} \ .$$

Thus, the excitation I has to scale with $\beta\gamma$. Here G denotes the dipole's pole gap (full).

For a quadrupole, its focal length has to remain unchanged and independent of beam energy,

$$\frac{1}{f} = k\, L_{eff} = \frac{g L_{eff}}{B\rho} \ , \quad g = \frac{B_{pole\ tip}}{r} = \frac{2\mu_0 NI}{r^2} \ , \quad B\rho = \frac{p}{q} = \frac{m_0 c\beta\gamma}{q} \ .$$

As well, the excitation I has to scale with $\beta\gamma$. Here g denotes the magnetic field gradient, r denotes the quadrupole's aperture radius.

2.2 Angular Errors

When a quadrupole is rotated by an angle around the $+s$ axis, it will cause beam coupling between transverse two planes. If the quad has a rotation angle θ_y around y axis (i.e. yaw), then the reference trajectory will deviate, in the horizontal plane, from the quad's axis by an amount of $(\tan\theta_y)\, L_{eff}/2 \simeq \theta_y\, L_{eff}/2$ at its entrance and exit with opposite sign. So, the kicks caused will be of opposite sign after passing through the 1st half and the 2nd half magnet, resulting in a canceled (at least partially canceled) effect to the angle. This does not mean that we can tolerate any large errors in the angle, instead, it suggests that we may impose a tight tolerance in the position to reduce the errors in the angle at the same time.

When the magnet has a rotation around x axis (i.e. pitch), the picture is similar to the yaw except that it is occurring in the vertical plane.

3 Dipole Installation Tolerances

For a dipole, the picture becomes somewhat complicated as the reference trajectory is curved, i.e., the $x - y - s$ coordinate frame is in a rotation passing through the magnet. Moreover, the magnet can be of rectangular shape or sector shape. To clarify, here we define the roll, yaw and pitch to be the angular errors of the magnet being rotated with respect to the $x - y - s$ frame sitting at the mid-point of reference trajectory inside the magnet (hard edge model).

For simplicity, we presume the beamline is lying in the horizontal plane and the beam is bending right (looking downstream). The magnet usually has a good field region which is specified in the x plane, while in the y plane the field is rather uniform. Thus the positional errors in x and y normally are not as concerned as the roll and pitch, because the latter two cause closed orbit distortion.

As is shown in the diagram Fig.2, a rolled dipole magnet generates a field component B_x in the bending plane, and this component exists at any location along the curved reference trajectory inside the magnet. This results in a vertical kick to the reference trajectory after exiting out of the magnet, represented as

$$\Delta y' = \frac{B_x L_{eff}}{B\rho} = \frac{B_y \tan\theta_s L_{eff}}{B\rho} \simeq \frac{B_y L_{eff} \theta_s}{B\rho} = \Theta\theta_s \tag{4}$$

where Θ denotes the nominal bend angle in the horizontal plane. Note that $\Delta y'$ is in $+y$ direction. Likewise, we require this angle be far smaller than the local beam divergence, i.e.

$$\Delta y' \ll \sqrt{\frac{\epsilon_y}{\beta_y}} \tag{5}$$

We thus get the roll tolerance

$$\theta_s \ll \frac{1}{\Theta}\sqrt{\frac{\epsilon_y}{\beta_y}}. \tag{6}$$

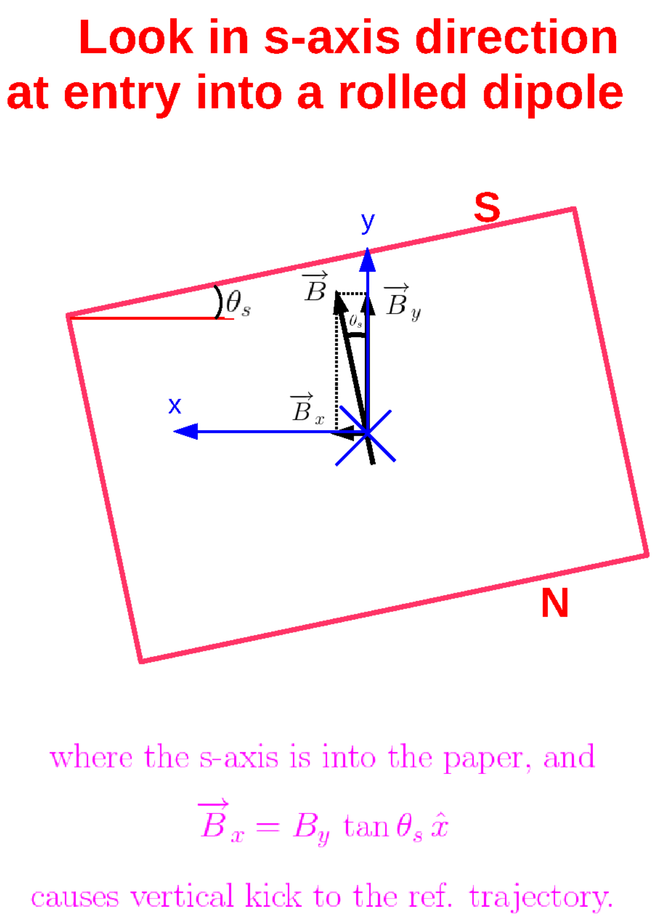

Figure 2: *Diagram showing a right bending dipole rolled around the s axis of a coordinate frame which sits at mid-point of reference trajectory inside the magnet.*

For a dipole tilted upward or downward, i.e. rotated around the x axis (where the $x-y-s$ frame is sitting at mid-point of reference trajectory inside the magnet), the situation becomes complicated. To simplify the picture, we assume that the beam is bending by an angle of 90° in the horizontal plane, and also assume that the magnet's pole face rotation angle is equal to zero at both ends. In this case, the magnet appears to be rolled at its exit; as a result, an overall kick caused to the reference trajectory in the vertical plane is represented as

$$\Delta y' = \tan\theta_x(1-\cos\Theta) \simeq \theta_x(1-\cos\Theta), \tag{7}$$

thus the pitch tolerance is

$$\theta_x \ll \frac{1}{1-\cos\Theta}\sqrt{\frac{\epsilon_y}{\beta_y}}, \tag{8}$$

where Θ denotes the nominal bend angle of the magnet. See diagrams Fig.(3) to Fig.(5) for details.

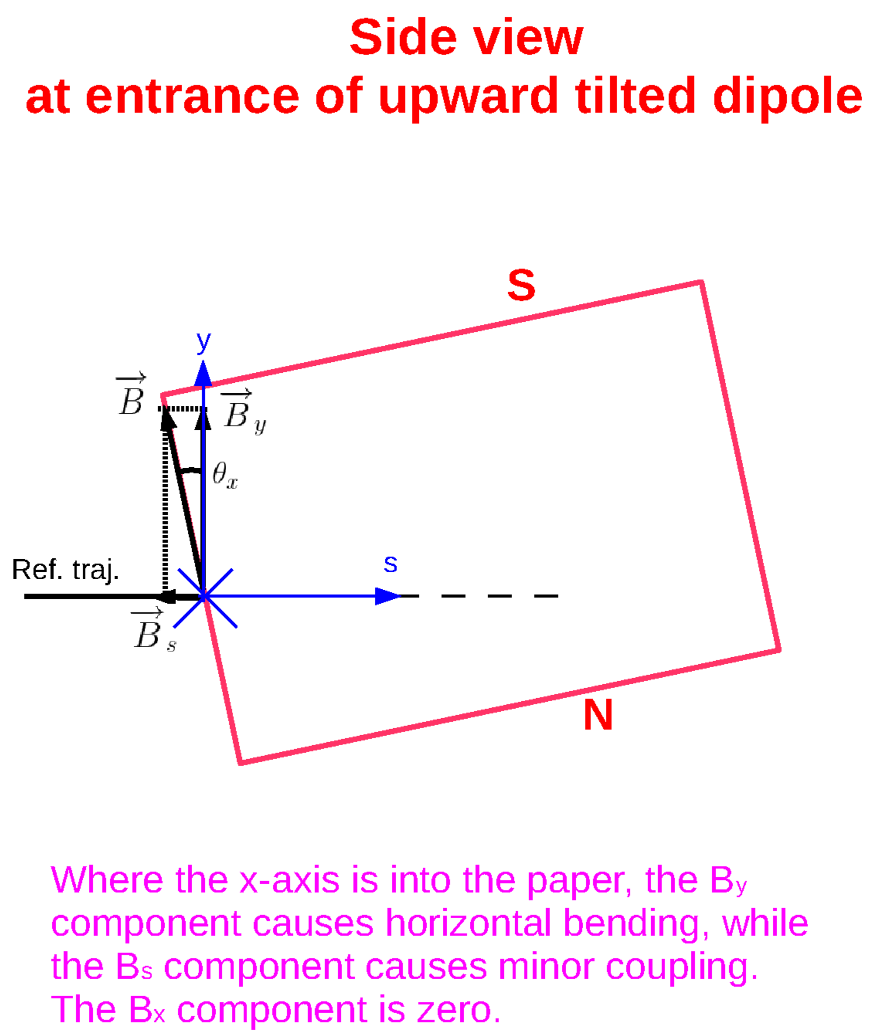

Figure 3: *Diagram showing a right bending dipole tilted upward. As a result of the tilt, a B_s component is generated causing minor coupling to the beam while the B_x component is zero at the magnet entrance.*

4 Application to BL4N

We applied the above formulae Eqs.(3),(6) and (8) to BL4N [1] to calculate the installation tolerances for quadrupole and dipole magnets. The results are listed in Tables 1 and 2 separately. It's seen that larger $\beta-$function value gives rise to more stringent tolerance; typically, the positional tolerance is $\sim \pm 150\,\mu$m for the quadrupoles; for the dipoles, the tolerance in roll is more stringent than that in pitch, and the former is less than $\pm 180\,$mrad.

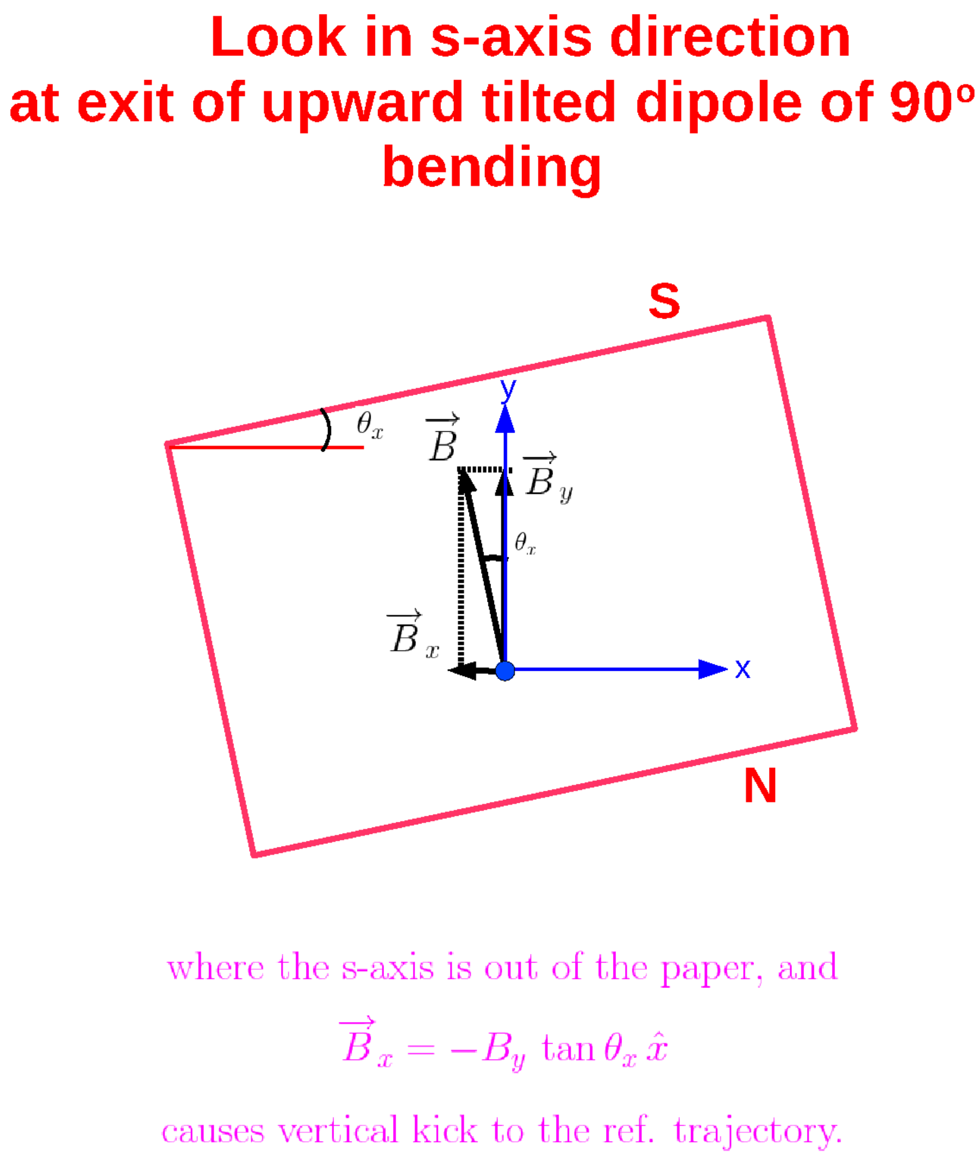

Figure 4: *Looking in s direction at exit of the upward tilted and right bending dipole. The bending angle is 90°; the magnet appears to be rolled around the s axis at the magnet exit, so that a field component B_x is generated, causing vertical kick.*

It's worthy to point out that the condition for the $\ll$ sign implies that there is no necessity to correct the orbit misalignment. But in reality the beamline will be equipped with steering magnets. The results shown here give us a fairly good sense of the installation tolerances that we could specify. These magnitudes, namely, $\pm 150\,\mu$m in position and $\pm 180\,$mrad in angle, are in line with the TRIUMF expert's experience in the beamline installation. Further, one could carry out sophisticated computations [2] about the orbit errors and propagation, considering that the installation errors are randomly populated in the beamline magnets.

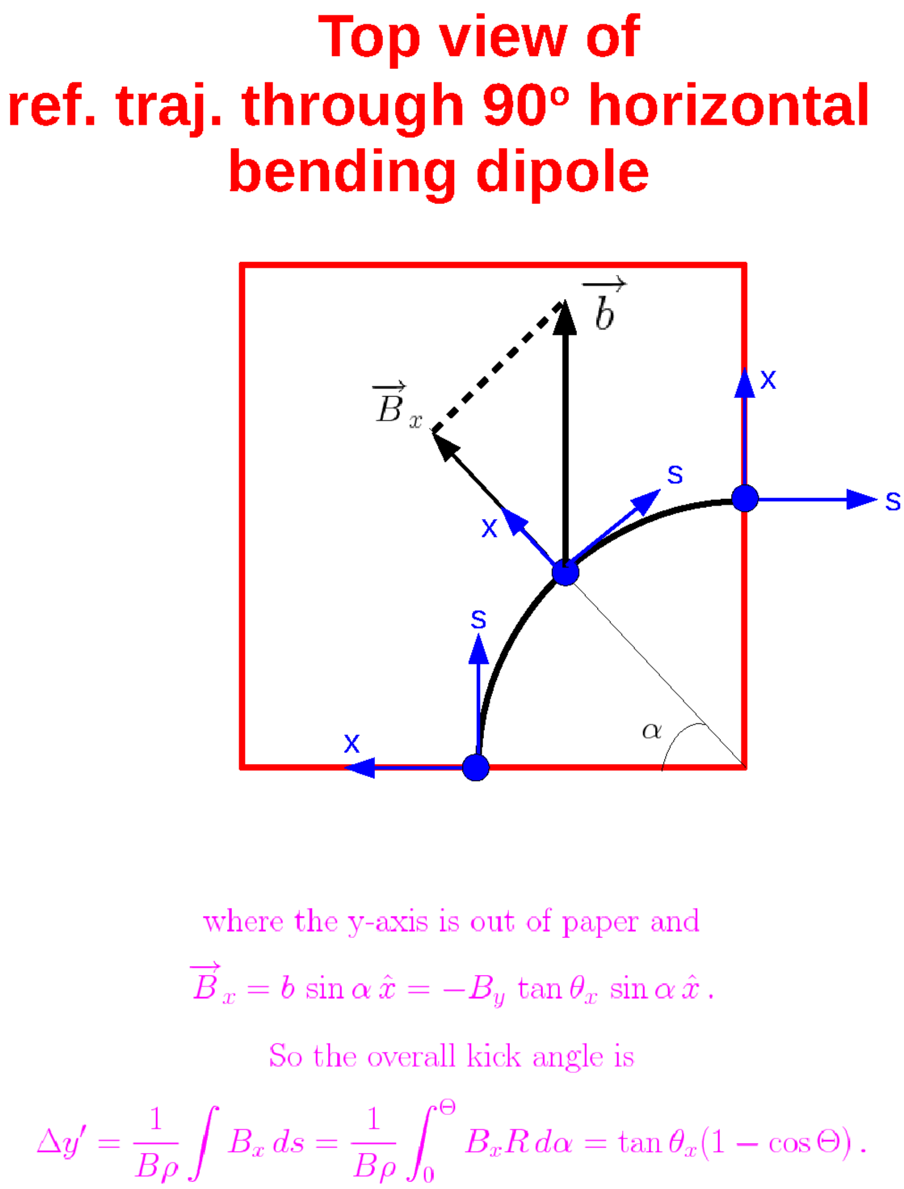

Figure 5: *Top view of reference trajectory passing through the* 90° *right bending dipole. For any bending angle* $0 < \Theta \leq 90^\circ$*, the overall vertical kick to the reference trajectory works out to be* $\Delta y' = \tan\theta_x(1-\cos\Theta) \simeq \theta_x(1-\cos\Theta)$*, where* θ_x *denotes the upward tilt of the magnet. Note that* $\Delta y'$ *is in* $+y$ *direction,*

Table 1: Installation tolerances in x and y calculated for BL4N quadrupole magnets. In the calculations, the beam emittance assumed was 0.84 and 1.42 πmm-mrad (4rms, unnormalized) in the x and y planes respectively; the $\beta_{x,y}$ values were taken in the mid-point of the magnets; and $\leq (0.2\times)$ was used for the $\ll$ sign.

Quad	gL_{eff} [T]	f [m]	β_x [cm]	β_y [cm]	x [mm]	y [mm]
4VQ1	-3.8787	-0.91	431.17	3576.17	0.081	0.036
4VQ2	5.9151	0.60	1193.18	424.02	0.032	0.069
4VQ3	8.2138	0.43	1111.48	956.52	0.024	0.033
4VQ4	-4.6530	-0.76	33.97	9113.86	0.240	0.019
4VQ5	3.8952	0.91	12023.00	220.52	0.015	0.146
4VQ6	-4.1376	-0.86	2183.36	2855.81	0.034	0.038
4NQ1	-1.0520	-3.37	2858.75	624.09	0.116	0.322
4NQ2	2.2024	1.61	7868.37	148.78	0.033	0.315
4NQ3	0.9128	3.88	5085.21	445.44	0.100	0.439
4NQ4	-1.8016	-1.97	2714.46	1086.08	0.069	0.142
4NQ5	-0.0338	-104.89	2901.83	1205.11	3.569	7.201
4NQ6	0.4823	7.35	2971.15	1241.87	0.247	0.497
4NQ7	-1.0456	-3.39	1208.49	1025.27	0.179	0.252
4NQ8	1.9899	1.78	1371.19	500.35	0.088	0.190
4NQ9	1.9899	1.78	1166.22	499.47	0.096	0.190
4NQ10	-1.0456	-3.39	1079.81	1023.35	0.189	0.253
4NQ11	1.7555	2.02	3132.91	1248.37	0.066	0.136
4NQ12	-1.3944	-2.54	1920.54	1796.03	0.106	0.143
4NQ13	1.6563	2.14	1117.24	859.33	0.117	0.174
4NQ14	-1.5031	-2.36	543.35	1207.83	0.185	0.162
4NQ15	3.1363	1.13	923.26	387.73	0.068	0.137
4NQ16	-2.7628	-1.28	422.63	854.67	0.114	0.105
4NQ17	2.8708	1.23	820.30	366.27	0.079	0.154
4NQ18	-2.8708	-1.23	334.98	854.14	0.124	0.101
4NQ19	2.8708	1.23	914.01	365.86	0.075	0.154
4NQ20	-2.8708	-1.23	409.25	853.30	0.112	0.101
4NQ21	-1.3690	-2.59	572.27	283.22	0.198	0.367
4NQ22	2.4851	1.43	945.14	181.35	0.085	0.252
4NQ23	-2.7961	-1.27	122.94	801.09	0.210	0.107
4NQ24	3.0992	1.14	234.98	396.04	0.137	0.137
4NQ25	3.0992	1.14	215.28	396.65	0.143	0.137
4NQ26	-2.7961	-1.27	107.66	802.81	0.224	0.107
4NQ27	2.4208	1.46	1097.17	167.57	0.081	0.270
4NQ28	-4.0310	-0.88	572.00	309.04	0.067	0.119
4NQ29	2.8868	1.23	4002.84	477.98	0.036	0.134
4NQ30	-2.9566	-1.20	1323.44	2039.68	0.060	0.063

Table 2: Installation tolerances in pitch and roll calculated for BL4N dipole magnets. In the calculations, the beam emittance assumed was 0.84 and 1.42 πmm-mrad (4rms, unnormalized) in the x and y planes respectively; the β_y value was taken in the mid-point of the magnets; and $\leq (0.2\times)$ was used for the $\ll$ sign.

Dipole	Bend. Angle [°]	β_y [cm]	θ_x [mrad]	θ_s [mrad]
4VMB4	24.8126	2003.64	0.577	0.123
4NMB6	45.0000	1502.57	0.210	0.078
4NMB10	45.0000	1499.66	0.210	0.078
4NMB22	34.0000	496.05	0.626	0.180
4NMB26	34.0000	497.72	0.625	0.180

References

[1] Y.-N. Rao, R. Baartman, *TRI-DN-13-13: Beam Line 4 North (BL4N) Optics Design*, Document-91008, Release 5, 2015-07-23.

[2] Y.-C. Chao, *BL4N Orbit Correction Configuration*, private communication, 2015-09-09.