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Design study for the TR100 spiral inflector

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Abstract: The spiral inflector is used to achieve the axial injection for the cyclotron with an external ion source. To study the injection for the TR100 cyclotron, we use the analytical expression of the reference orbit to design a conceptual model of the inflector. The fringe field is compensated for by carefully trimming the entry and exit of the inflector. Particle tracking result shows that the beam position and momentum at the exit of the inflector are consistent with the central region's initial ones. Beam envelope transport code TRANSOPTR and particle in cell (PIC) code OPAL are also used to study the envelopes of the injection beam. Both results show that the beam envelopes are expected to be well constrained in the inflector. The envelopes given by the 2 methods are different in the horizontal direction, and such difference is not negligible in a small electric radius (comparable with the gap size) inflector which remains to be studied further.

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1 Introduction

In order to study the design of the inflector for TR100, we have traced back to L. Root's [1] inflector design for the TRIUMF 500 MeV cyclotron and R. Baartman's [2] study of the inflector model. The spiral inflector steers the beam from the axial bore in the main magnet into the median plane of the cyclotron with 2 design goals, one is to center the beam into the central region with the proper initial coordinates and momentum, the other is to ensure a low beam loss inside the inflector.

Using the analytical model of the inflector in a flat magnetic field, we have given the conceptual design model of the inflector. The reference orbit at the exit of the inflector is centered to satisfy the injection coordinates and momentum by adjusting the 2 parameters, electric radius A and the tilt k'. The entry and exit of the inflector is also carefully trimmed to compensate for the fringe field of the inflector. Multiphysics simulation software COMSOL is used to study the electrical field in the inflector. The particle tracking module of COMSOL is also used to study the reference orbit in the inflector.

After designing the conceptual model of the inflector, we have studied the beam envelopes by using both TRANSOPTR [3] and OPAL [4].

2 Design study

2.1 Analytical approach for the reference orbit

In this report, we use the coordinate (α, β, γ) in the optical coordinate system, which moves along the reference orbit. The γ direction is the same with the velocity of the reference particle. The β direction is perpendicular to the γ direction and parallel to the median plane. At the same time, the cross product of the unit vector of the γ direction and β direction should have a positive projection on z-axis. The α direction is defined by the cross product of the unit vector of the γ direction and β direction.

The analytical expression of the reference orbit is calculated by Belmont and Pabot [5] based on the 4 assumptions below.

(1) Homogeneous magnetic field.

(2) Electrical field perpendicular to the reference trajectory.

(3) α component of the electric field is constant.

(4) β component of the electric field is proportional to the horizontal velocity.

I directly use Rick's expression [2], the reference orbit, using angle b which is $\pi/2$ at the exit of the inflector as the independent variable, is calculated as the following expression

$$\begin{aligned}
x_c(b) &= \frac{A}{2}\left[-\frac{2}{k^2-1}+\frac{\cos(k-1)b}{k-1}-\frac{\cos(k+1)b}{k+1}\right] \\
y_c(b) &= \frac{A}{2}\left[\frac{\sin(k+1)b}{k+1}-\frac{\sin(k-1)b}{k-1}\right] \\
z_c(b) &= -A\sin b
\end{aligned} \tag{1}$$

Where electric radius electric radius $A = 2E_0/E$, and parameter k is defined by

$$k = \frac{A}{\rho + k'} \tag{2}$$

The defination of the tilt parameter k' is the tangent of the angle between the magnetic field and the electric field at the inflector exit. ρ is the magnetic radius, that is the bending radius of the reference orbit in a uniform magnetic field.

To align the beam into the central region, the position and momentum of the reference orbit at the exit of the inflector should be consistent with the given value. The coordinates of the reference orbit at the exit of the inflector is given using Eq.1. The momentum at the exit is given by

$$\begin{aligned}
p_x &= \frac{p_0}{2}\left[\sin\frac{(k+1)\pi}{2}-\sin\frac{(k-1)\pi}{2}\right] \\
p_y &= \frac{p_0}{2}\left[\cos\frac{(k+1)\pi}{2}-\cos\frac{(k-1)\pi}{2}\right] \\
p_z &= 0
\end{aligned} \tag{3}$$

where p_0 is the total momentum of the particle.

2.2 Inflector conceptual design model

Since the inflector could rotate with respect to the z-axis, we only need two free value to center the beam. In this note, the 2 parameters, electric radius A

and title k', are adjustable to make sure that the beam is injected on the axis of the bore in the main magnet and the beam at the exit of the inflector could match the initial position on the median plane for proper beam centering. The electric radius A corresponds to the electrical field strength which is limited by the DC break down limitation in the vacuum.

The position (r, θ) and momentum (p_r, p_θ) of the reference orbit at the exit of inflector is optimized by the center region study code CYCLONE which ensures the beam could be injected into the main magnetic field with the minimum radial oscillation amplitude. The optimized design value for the spiral inflector design is given by solving the equation below

$$\begin{aligned} r^2 &= x_c(\pi/2)^2 + y_c(\pi/2)^2 \\ p_r &= \frac{y_c(\pi/2)p_x}{\sqrt{x_c(\pi/2)^2 + y_c(\pi/2)^2}} + \frac{x_c(\pi/2)p_y}{\sqrt{x_c(\pi/2)^2 + y_c(\pi/2)^2}} \end{aligned} \tag{4}$$

Substituting x_c, y_c, p_x, p_y into eq.4, we get 2 non-linear equation with 2 variables A and k', solving the equation using Trust-Region-Reflective method. The optimized value for the inflector is given in table 1.

Table 1: Design parameters

Parameter	Symbol	Value
Injection energy	E_0	34.7 (keV)
Magnetic field	B_0	2.0 (T)
Magnetic radius	ρ	1.9 (cm)
Voltage on the electrods	V_d	$\pm$12.6 (kV)
Electric radius	A	2.2 (cm)
Tilt	k'	-0.72
Gap between the electrods at the entry	d_{en}	0.80 (cm)
Gap between the electrods at the exit	d_{ex}	0.65 (cm)
Width of the electrods	l	1.6 (cm)

The electric potential function in the inflector is given[2] based on the assumption that the equipotential surface in the vicinity of the reference trajectory are ruled surface.

$$\begin{aligned} qV = &-\frac{m_0 v_0^2}{A}(\alpha + k'S\beta) \\ &- \frac{m_0 v_0^2}{2A^2}\left[\xi(\alpha + k'S\beta)^2 + 4TC\beta\gamma - 2kk'CS\alpha\gamma - \left(1 + kk'S^2\right)\gamma^2\right] \end{aligned} \tag{5}$$

where

$$\xi = \frac{1 + kk'S^2}{1 + k'^2 S^2} \tag{6}$$

$$C = \cos(b) \tag{7}$$

$$S = \sin(b) \tag{8}$$

$$T = \frac{k + k'}{2} \tag{9}$$

Because the analytical result is based on the assumption that the electrical field in the α direction is constant, which is not satisfied in the constant gap inflector. Thus, the gap should decrease along the reference orbit. The electrodes are on the two equipotential surface. Because of the infinite width of the equipotential surface, the larger width of the electrodes is preferred. The surface is build numerically using Eq. 1 and Eq .5. Figure 1 shows the conceptual design model of the inflector with the parameters given in table 1.

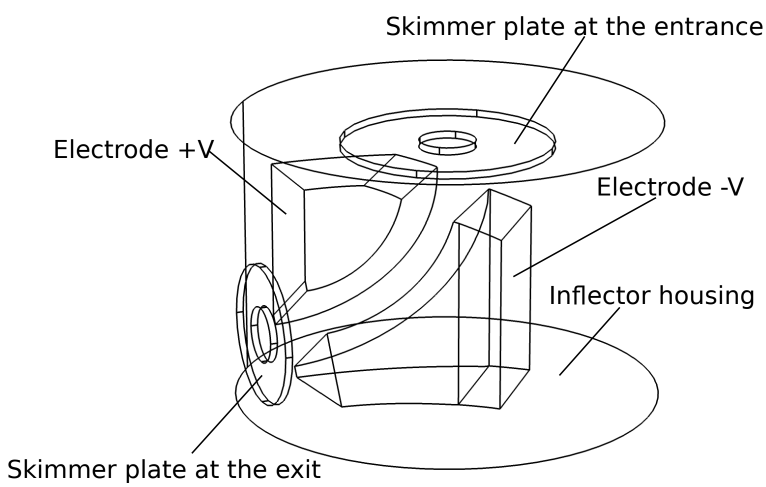

Figure 1: Model of the spiral inflector. The cylindrical house is set with the 0 potential. The upper electrode is set with the positive potential, while the lower one with negative potential.

The model electrical potential is solved using finite element analysis (FEA) method using COMSOL. The maximum mesh size in the inflector gap is 0.3 mm and in other domain is 1 mm. The total number of 3×10^6 elements is used in the model. Figure 2 shows the equipotential line on the entry surface plane.

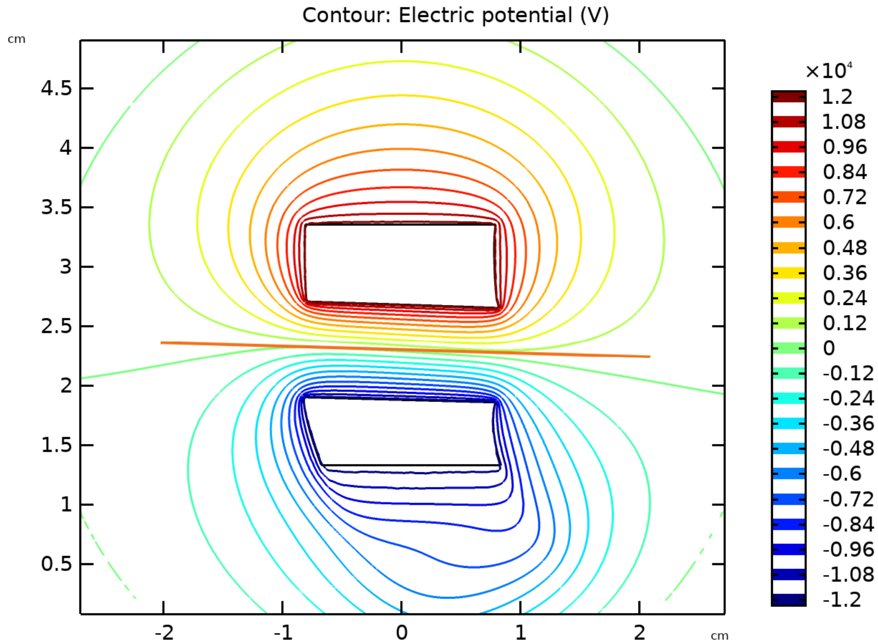

Figure 2: Equipotential at the entry of the inflector. A simple parallel capacitor model could be used to estimate the potential on the electrodes, when the electrical field strength is given by the analytical model.

2.3 Fringe field

At the entry of the inflector, the momentum of the beam is dominated by the p_z component, while the magnetic flux density on the main magnet bore axis only consist of the vertical componant B_z, thus the electro static deflection of the beam is only to be considered at the entry of the inflector. The effective length of the fringe field is given as

$$\eta^* = \gamma_{\mathrm{b}} - \int_{\gamma_a}^{\gamma_b} \frac{E_\alpha(0,\alpha,0)}{E_0} \mathrm{d}\gamma \tag{10}$$

The index a and b refers to the point $\gamma = \gamma_a$ in the field free region, and $\gamma = \gamma_b$ in the homogeneous region. To narrow the fringe field region, a skimmer plate is placed in front of the entry of the spiral inflector. The aperture is 0.8 cm diameter, which is the same with the gap at the entry, and distance between the skimmer plate and the inflector effective entry is 0.4 cm. The effective length of the fringe field must be determined by a numerical integration over the FEA model field distribution.

The electric field exported from COMSOL is under the cartesian coordinate system. After converting into the (α, β, γ) coordinate system, figure 3 shows the electric field along the reference orbit.

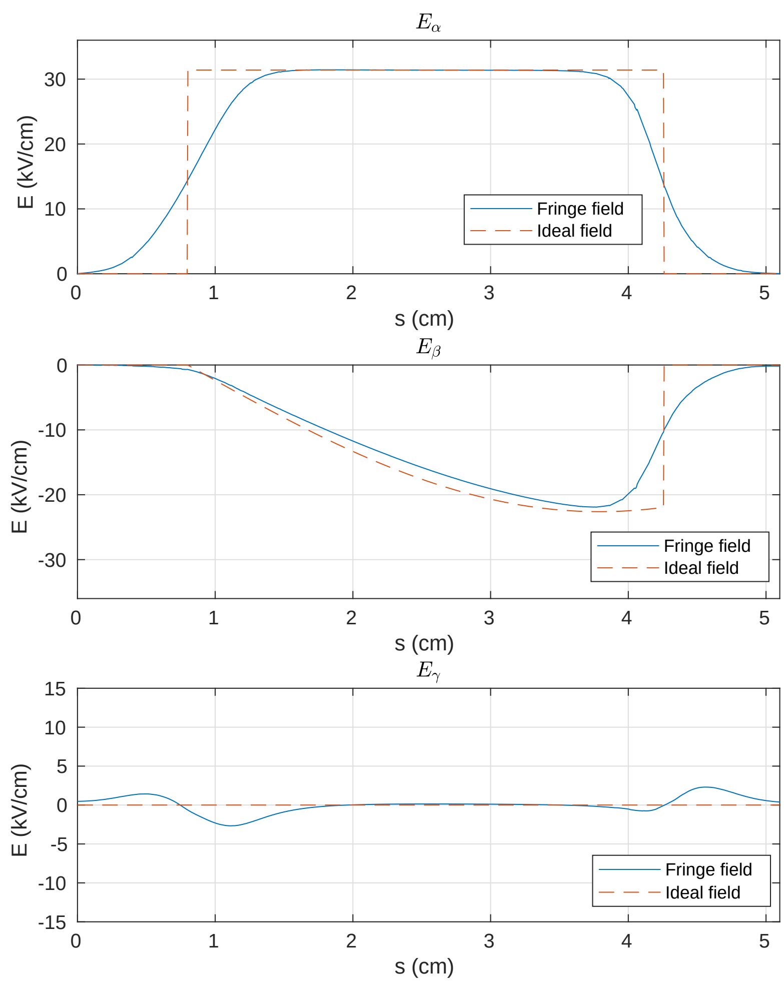

Figure 3: Electric field along the reference orbit. The electric field in the 3 directions agrees with the ideal field. The line integral for the E_β values is the same as the ideal field. The difference between the analytical result and the FEA result comes from the fringe field at the entry and exit and the finite width of the electrode.

2.4 Orbit centering

The reference orbit from the analytical result is shown in figure 4 and figure 5 in red color. The radius of the analytical reference orbit at the inflector exit is 2.17 cm and the momentum ratio p_θ/p_r is 2.06. Rotating the inflector by 121.66 degrees with respect to the z-axis., the reference orbit would be consistent with the TR100 central region injection beam position and momentum.

The initial position of the particle is at the entry of the inflector, the coordinate is (0,0,0.8). Figure 4 and 5 shows the result before and after trimming the entry and the exit respectively. The reference orbit is computed using COMSOL particle tracking module.

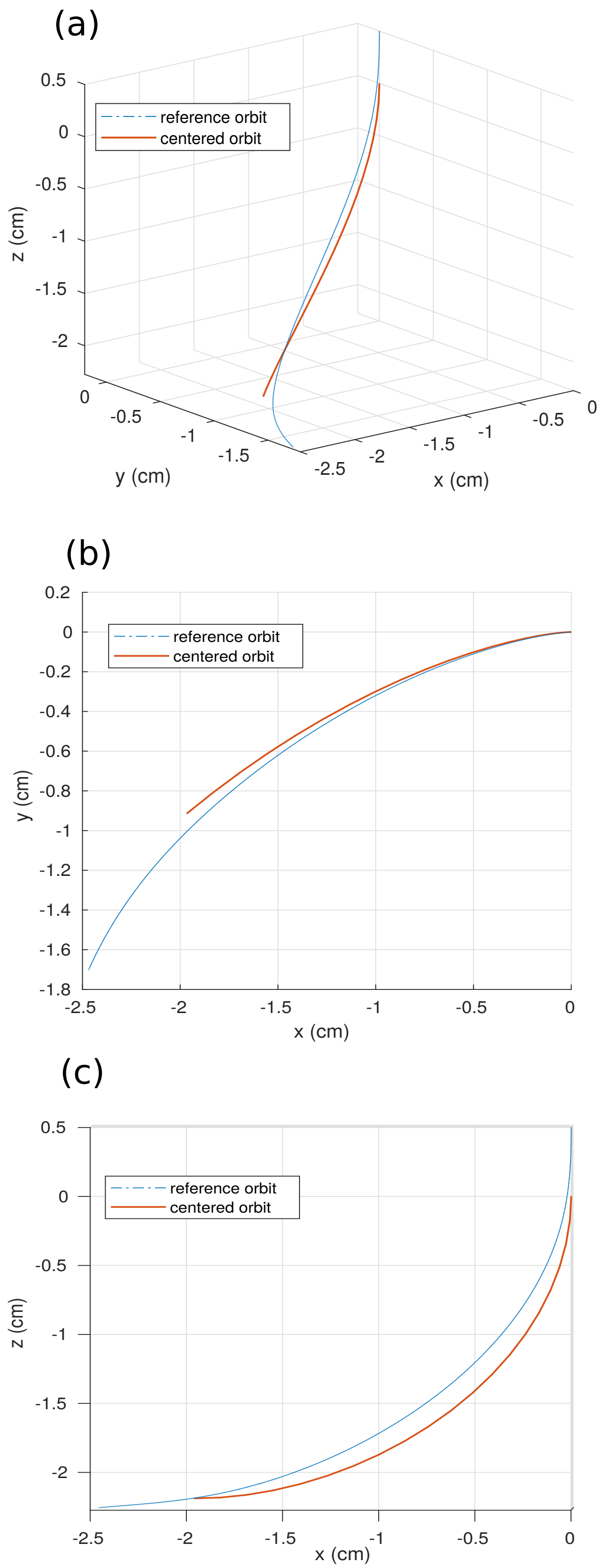

Figure 4: Single particle tracking result of the reference orbit without trimming the entry and exit of the inflector. The error between the centered orbit and the particle tracking result is about 0.15 cm. The model could not meet the requirement before trimming.

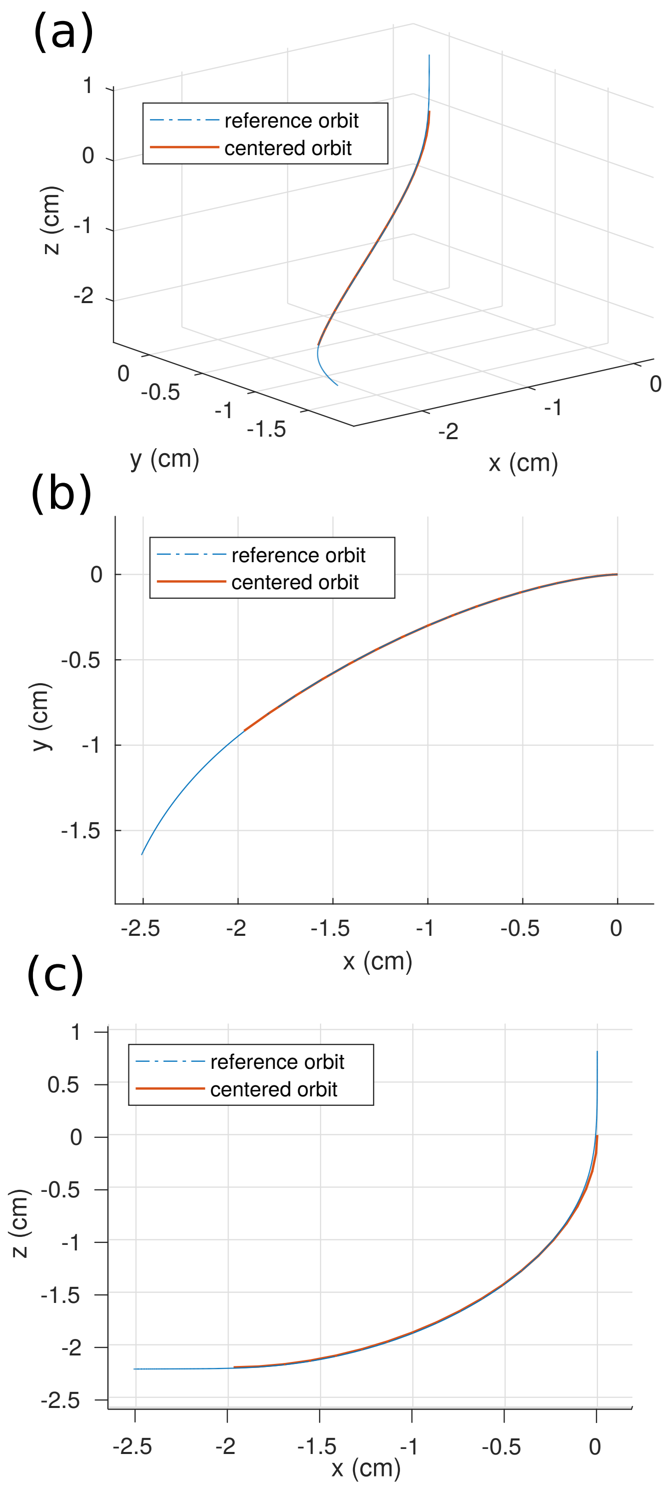

Figure 5: Single particle tracking result of the reference orbit after trimming the entry and exit of the inflector. The error between the centered orbit and the particle tracking result of the reference orbit is less than 30 μm. The reference orbit is well aligned with the central region injection orbit at the exit of the inflector.

2.5 Beam envelope study

Using Rick's Hamiltolian [2] for the spiral inflector, we can derive the infinitesimal transfer matrix F through the inflector. Linear spacing charge effect can also be easily included in the F matrix approach. Tracking the σ matrix through the inflector, the envelopes could be evaluated. The F matrix approach of the inflector is provided in the envelope code TRANSOPTR. We directly use the inflector module to study the beam envelopes in the inflector. In front of the inflector is a drift space with the length of 0.8 cm, and following the inflector is a bending magnet which is the main magnet with a flat magnetic field in the central region. The transfer matrix M given by TRANSOPTR is shown below.

$$M = \begin{pmatrix} 3.737\times10^{-1} & 3.361\times10^{-3} & -1.808\times10^{-1} & 2.930\times10^{-4} & 0 & 2.457\times10^{-4} \\ -3.410\times10^{2} & -4.474\times10^{-1} & -3.897\times10^{1} & -5.207\times10^{-2} & 0 & 5.400\times10^{-1} \\ 1.242\times10^{-1} & -1.858\times10^{-5} & -1.332 & -1.903\times10^{-4} & 0 & 4.121\times10^{-3} \\ 9.886\times10^{1} & 1.529\times10^{-1} & -4.460\times10^{2} & -7.991\times10^{-1} & 0 & 7.478\times10^{-1} \\ 2.893\times10^{-2} & -1.281\times10^{-3} & -7.544\times10^{-1} & -3.322\times10^{-3} & 1 & 5.649\times10^{-4} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

The canonical variables are not $\alpha, \alpha', \beta, \beta', \gamma, \gamma'$. The matrix below should be applied before integrating through the inflector.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2\rho} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{-1}{2\rho} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

where ρ is the magnetic radius.

When study the envelopes using OPAL, we track 1×10^5 particles. The electric field used in OPAL is extracted from COMSOL with the uniform grid size of 0.3 mm in each direction. The magnetic field is flat with the value of 2 T. The mesh size in the beam region, which used to evaluate the space charge field in the moving frame is less than 0.1 mm. The time step size is automatically controlled by OPAL.

To fix the difference between the x envelopes between the 2 codes, 2 thin lenses are added at the entry and the exit of the inflector respectively. A fit subroutine is used in to fit the focal power of the thin lens. Figure 7 shows

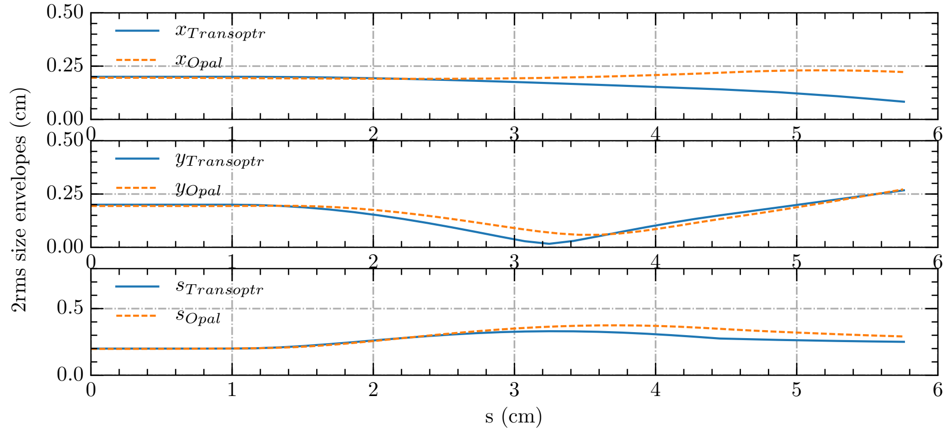

Figure 6: Beam envelopes. To compare the envelopes given by the 2 codes. We use a parallel beam with the 1σ size of 1 mm in all the 3 directions as the injection beam. The beam current is 0. The momentum spread is 0. The maximum beam 2 rms size in both directions is less than 0.2 mm in the inflector.

the result.

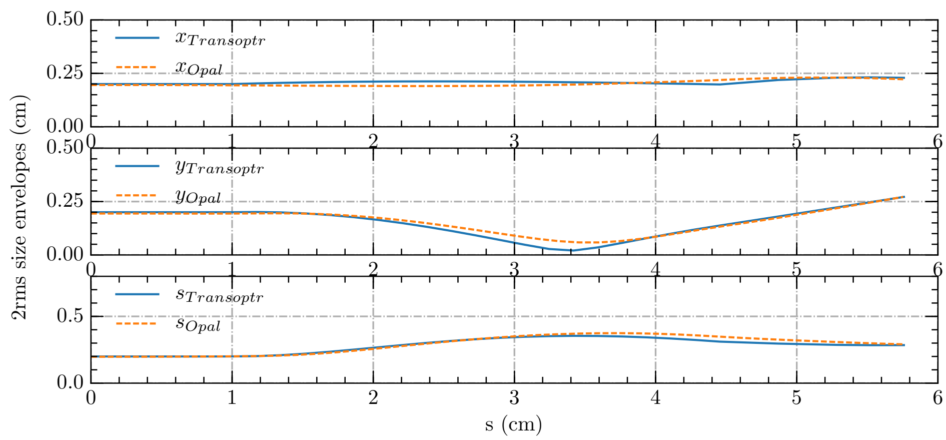

Figure 7: Beam envelopes after adding 2 thin lenses at the entry and the exit of the inflector. The reason remains to be studied.

To study the space charge effect in the inflector, we assumed a bunched beam with 2 mA average current. The bunch length is 1.5 mm for 30 degree phase acceptance of the central region. The normalized 4 rms emittance of 0.36 μm is used for the injection beam. Figure 8 shows the envelopes.

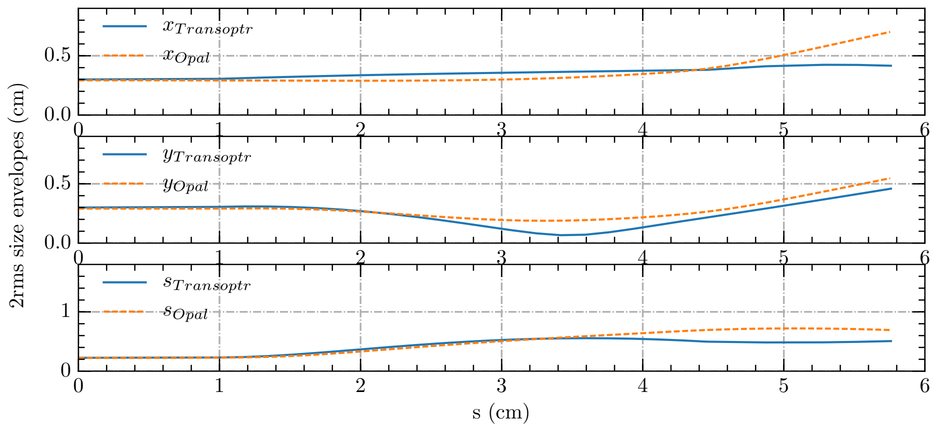

Figure 8: Beam envelopes with the beam current of 2 mA. The normalized 4 rms emittance is 0.36 um. The inflector exit located at s=4.25 cm. The 4 rms beam size is less than 6 mm in the inflector, which is smaller than the inflector aperture.

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