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BL1A study using G4Beamline and TRANSOPTR: calibrate the beam parameters on the target

Lige Zhang

TRIUMF

Abstract: In this report, we studied the beam envelope in BL1A using G4Beamline and TRANSOPTR model. The initial protons' coordinates and momenta in G4Beamline are generated using the 6D σ matrix at the exit of the combination magnet given by TRANOPTR. The RMS scattering angle, energy loss and RMS energy loss straggling in TRANSOPTR are calculated using analytical expression based on a Gaussian approximation and compared with Monte Carlo simulation result. Finally, The comparison between the two BL1A models shows that the analytical expression is good enough for envelope computation.

4004 Wesbrook Mall, Vancouver, B.C. Canada V6T 2A3 · Tel: 604 222-1047 · Fax: 604 222-1074 · www.triumf.ca

1 Introduction

When the beam passes through the target, the interaction between the proton beam and the target would decrease the beam energy and increase the beam scattering angle. Energy loss is predominantly caused by ionization and excitation of atoms on the target, which could be calculated using Bethe-Bloch energy loss model. The growth of the beam scattering angle is mainly caused by Coulomb scattering, which could be calculated using Highland model based on a Gaussian approximation. In this report, the scattering, energy loss and energy loss straggling are studied using analytical method and built into the TRANSOPTR subroutine. The Monte Carlo simulation is conducted by G4Beamline.

2 Coulomb scattering

The 70-500 MeV proton beam with a pure Gaussian approximation leads to the Highland-Lynch-Dahl equation [1] to describe the scattering angle θ_{sc}

$$\theta_{\text{sc}} = \frac{13.6MeV}{\beta cp}\sqrt{\frac{x}{X_0}}\left[1 + 0.038\ln\left(\frac{x}{X_0}\right)\right] \tag{1}$$

Where X_0 is the radiation length, E is the beam energy, z is the travelling distance in the material of the beam. Under the condition of $10^{-3} < z/X_0 < 100$, this equation is accurate to 11% or better. With small energy loss in the target, E could be treated as a constant in Eq.(1). The radiation length of different target material is looked up in particle data group website[2]

$$X_0 = \begin{cases} 35.28cm\,(\text{beryllium}) \\ 19.32cm(\text{graphite}) \end{cases} \tag{2}$$

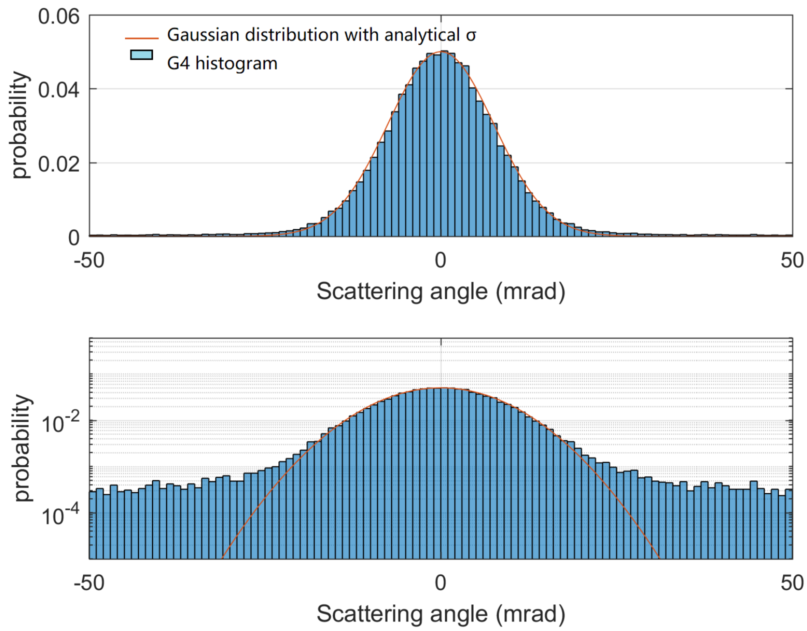

Figure 1: Angle distributions for 480 MeV protons through a 10 cm Beryllium target. The G4 result agrees with Gaussian using the Highland RMS scattering angle. However, it's obvious on the log scale that the Gaussian result peels away at 2σ.

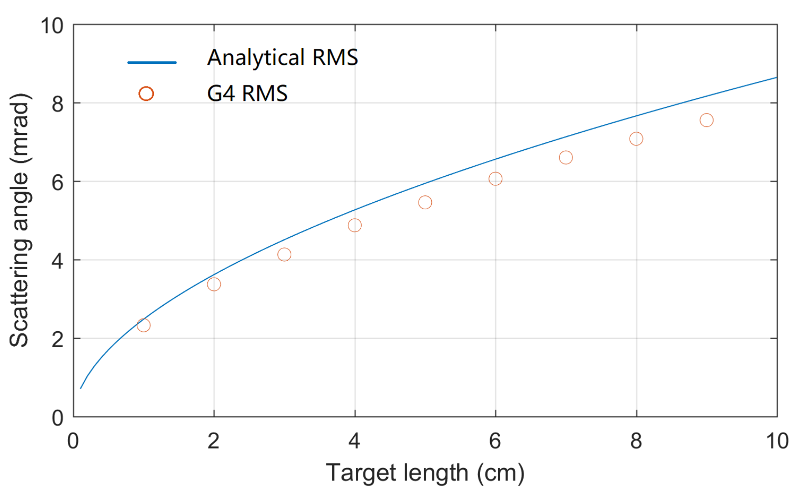

Figure 2: Highland RMS scattering angle vs. target length. The G4 RMS scattering angle is calculated under a 2σ cutting (95%). The difference between the Highland expressing and G4 simulation result is less than 5%, which is good enough for the envelope calculation in TRANSOPTR.

3 Energy loss and energy loss straggling

Energy loss of the proton beam could be given using Bethe-Bloch equation

$$-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_e c^2\beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right] \tag{3}$$

Eq.(3) describes a statistic energy loss result of a proton with energy E, travelling a distance x into a target of electron number density n and mean excitation potential I. Where ε_0 is the vacuum permittivity, e and m_e are the electron charge and rest mass respectively, and β is the ratio of particle speed to the speed of light c. The nozzle contains several materials, the total energy loss can be accumulated segment by segment with different materials using Eq.(6). The linear solution for Bethe-Block equation for a 480 MeV proton beam in the Be target is written as

$$dE = 4.3MeV * x \tag{4}$$

where the unit of the target length x is cm.

For relatively thick target such that the number of collisions is large, the energy loss distribution is shown to be Gaussian in form. For non-relativistic heavy particles the spread σ_0 of the Gaussian distribution is calculated by

$$\sigma_0^2 = 4\pi N_A r_e^2 \left(m_e c^2\right)^2 \rho Z x \tag{5}$$

where ρ is the density, Z is the atomic number of target, m_e is the electron mass.

4 Monte Carlo simulation

G4 beamline is a software based on a general purpose MC package geant4 for calculations of particle transport in a beamline and interactions with materials. In this paper, we use the model built by F.W. Jones[3], which track the beam from the exit of the combination magnet to TNF target. To compare the G4 result with TRANSOPTR, we have recalibrate all the quads setting. The initial protons are generated using the 6-D Gaussian distribution function. The

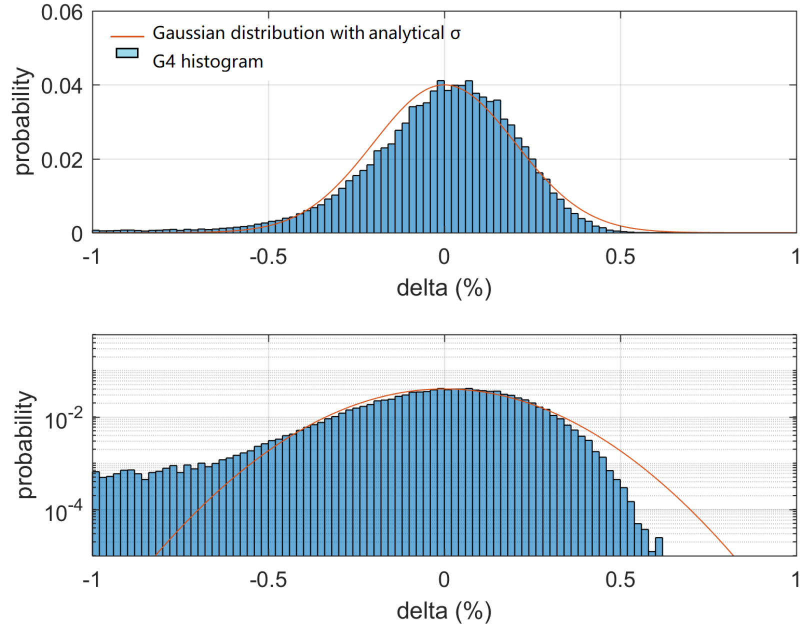

Figure 3: The energy loss straggling distribution for 480 MeV protons through a 10 cm Beryllium target. The energy loss straggling is the Landau distribution. which should be replaced by a RMS value to track the envelope in TRANSOPTR, we uses the Gaussian approximation in eq.5 to estimate the RMS straggling. For our case, it includes 95% of the particles despite skewing the G4 straggling distribution.

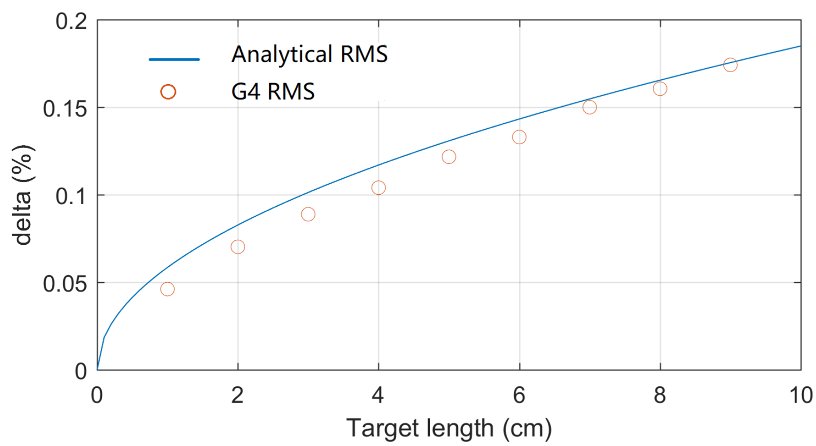

Figure 4: RMS energy loss straggling vs. target length. The G4 RMS energy loss straggling is calculated under a 2σ cutting (95%). With the target length increasing, the distribution trends to be Gaussian, thus the error between G4 and analytical result decreases.

6-D sigma matrix used to defined the beam ellipsoid is given by TRANSOPTR as shown in table 1.

Table 1: Sigma Matrix at the exit of the combination magnet (Normalized Form)

	Diagonal	**Off-Diagonals**				
x(cm)	0.484318					
theta(mrad)	0.791294	0.277967				
y(cm)	0.462270	0.00000	0.00000			
phi(mrad)	0.640501	0.00000	0.00000	0.926468		
l(cm)	64.9967	-0.00425	0.00251	0.00000	0.00000	
delta(%)	0.09319	0.285271	0.953712	0.00000	0.00000	-0.01483

Figure 5 shows the comparison of the envelopes between TRANSOPTR and G4Beamline. 10^5 particles are used in G4Beamline. The RMS scattering angle, energy loss and energy loss straggling calculated in TRANSOPTR are shown in table 2.

Table 2: Analytical RMS Scattering angle, energy loss and energy loss straggling results

Beam energy	**Target length**	**Energy loss**	$\Delta p/p$	**Scattering angle**
480 MeV	12 mm	5.16 MeV	0.063	2.74 mrad
475 MeV	12 mm	5.16 MeV	0.064	2.76 mrad
475 MeV	50 mm	21.5 MeV	0.13	5.96 mrad
475 MeV	120 mm	51.6 MeV	0.20	9.59 mrad

References

[1] G. R. Lynch, O. I. Dahl, Approximations to multiple Coulomb scattering, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 58 (1991) 6–10.

[2] https://pdg.lbl.gov/2020/AtomicNuclearProperties/index.html, Tech. rep.

[3] F. Jones, BL1A Simulation Model in G4Beamline, Tech. Rep. TRI-BN-19-03, TRIUMF (2019).

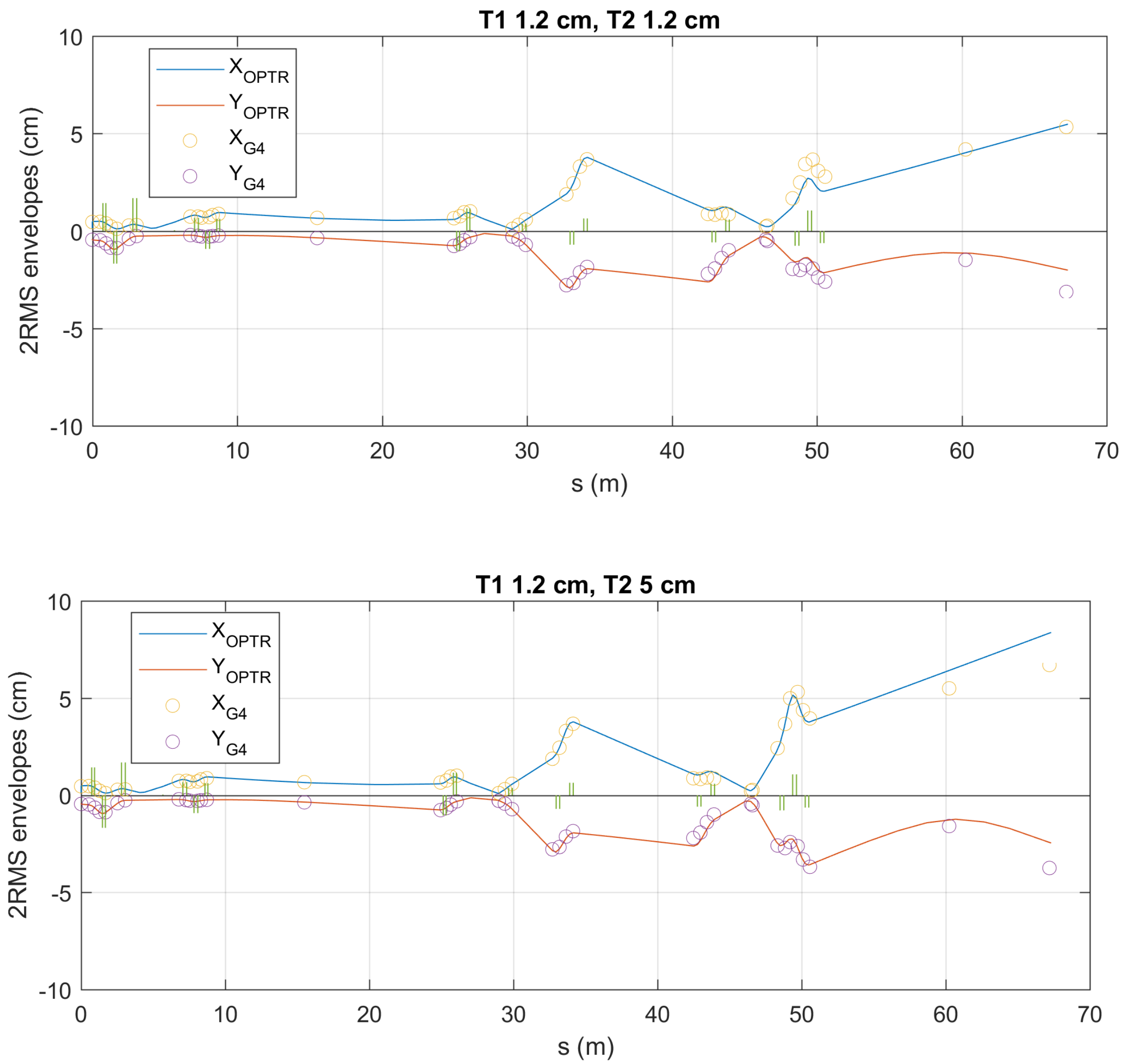

Figure 5: Beam envelopes with different T2 target length. The RMS scattering angle and energy loss straggling used in TRANSOPTR are all calculated using the analytical expression. The G4 RMS beam size is calculated under a 95% cutting between T1 to TNF. The envelope from TRANSOPTR and G4 agree very well from the combination magnet to T1 target. The difference from T1 to T2 is resulted from the none Gaussian beam profile given by G4 after scattering on the target. The scattered shoulder in the beam profile become higher after T2, resulting the difference become larger after T2. .