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Vertical Tune Measurement and Correction for the 500 MeV Cyclotron

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Abstract: The cyclotron vertical tune at the different radius is measured by scanning the trim coil radial field. Comparing with the historical tune calculated from the field survey data, a peak of the vertical tune at cyclotron radius of around 235 inches confirmed an extra $\nu_r - \nu_z = 1$ coupling resonance passage. To avoid the passage, the vertical tune at 235 inches is reduced using the trim coil axial field. After correcting the vertical tune, both the coherent and incoherent oscillations, induced by the crossing of the coupling resonance, are removed.

1 Introduction

TRIUMF 500 MeV cyclotron tune diagram, calculated from the magnetic field survey data, is shown below. The $\nu_r - \nu_z = 1$ resonance passage happens at around 166 MeV and 291 MeV. The exchange of both coherent and incoherent oscillation amplitude in the vertical and horizontal direction while passing the resonance will increase the beam loss in the cyclotron, as the radial aperture is much larger than the vertical one in the cyclotron magnet gap.

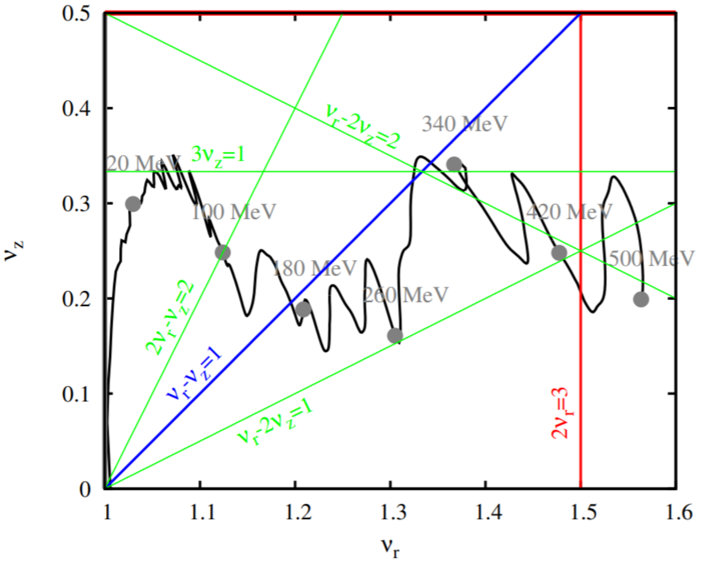


Figure 1: Tune diagram of the 500 MeV cyclotron calculated from historical field survey data. The blue line is the $\nu_r - \nu_z = 1$ resonance line.

The $\nu_r - \nu_z = 1$ coupling resonance is driven by the first harmonic of the radial gradient of the radial magnetic field, which is introduced by the asymmetry of the cyclotron median plane [1] [2]. For correction of the resonance, 3 harmonic coils are used to compensate for the first harmonic radial field imperfection of the main magnet at the radius of the coupling resonance passage. The resonance correction is verified by measuring the vertical coherent oscillation after detuning the beam radial centring. The detuning of the beam radial centring causes a radial coherent oscillation, which can be converted to the vertical direction during the coupling resonance passage. The measurement of the vertical center of gravity in figure 2 shows that the coherent oscillation in the vertical direction is removed after correction.[3] However, the reappearance of the oscillation at 235" indicates that there exists some unknown coupling resonance passage at this radius.

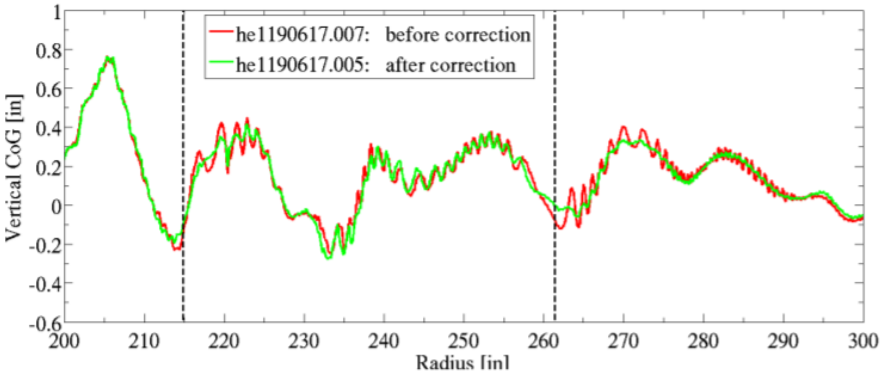


Figure 2: Vertical center of gravity vs. radius with and without correction. The vertical dash line shows the radial position of the $\nu_r - \nu_z = 1$ resonance passage.

Thus, measuring the vertical tune will help us to figure out the passage that is not shown by the calculated tune from the field survey data 50 years ago. During the commissioning of the cyclotron, the trim coil (TC) in the main magnet is used to measure the vertical tune [4]. However, the large error bar makes it hard to recognize the passage around the 235". In this report, we carefully calibrated the probe position and reduced the error by creating a flat-top in the radial field. To avoid the unwanted passage through the coupling resonance, the TC axial field is also used to correct the vertical tune.

2 Method

2.1 High energy probe

The existing diagnostics in the 500 MeV cyclotron are 4 radial scanning probes. 2 for the low energy region and 2 for the high energy region. The high energy probe [5], shown is figure 3 (a) is used to measure the vertical beam center of gravity and vertical beam size. It has 7 fingers which are Tantalum foils with a thickness of 0.005". The vertical beam center of gravity CoG_z and vertical beam RMS size σ_z are given by

$$\begin{aligned} CoG_z &= \frac{\sum_{i=1}^{7} F_i \cdot D_i}{\sum_{i=1}^{7} F_i} \\ \sigma_z &= \sqrt{\frac{\sum_{i=1}^{7} F_i \cdot (D_i - \mathrm{CoG_z})^2}{\sum_{i=1}^{7} F_i}} \end{aligned} \tag{1}$$

where F_i is the signal amplitude on the i-th finger, D_i is the distance between the i-th finger center and the median plane.

In the vertical dimension, the middle 5 fingers are 0.25” and the 2 outer fingers are 0.625”. The smaller size of the middle fingers makes the spatial resolution near the cyclotron median plane higher. Thus, if the beam deposited more on the outer fingers, the error of the measurement is larger. Figure 3 (b) compares the CoG calculated by the center 5 fingers and all the 7 fingers with vertically scanning the beam through the probe. The error in both of the results is small when the CoG is less than 0.3”. With the beam traveling far away from the middle finger, the error increases. Before the saturation of the measurement results, the 7 finger results always overestimate the CoG, while the 5 fingers results underestimate the CoG. Thus, we put an error bar on the measured CoG using the 5 finger result as the lower boundary and the 7 one as the upper boundary.

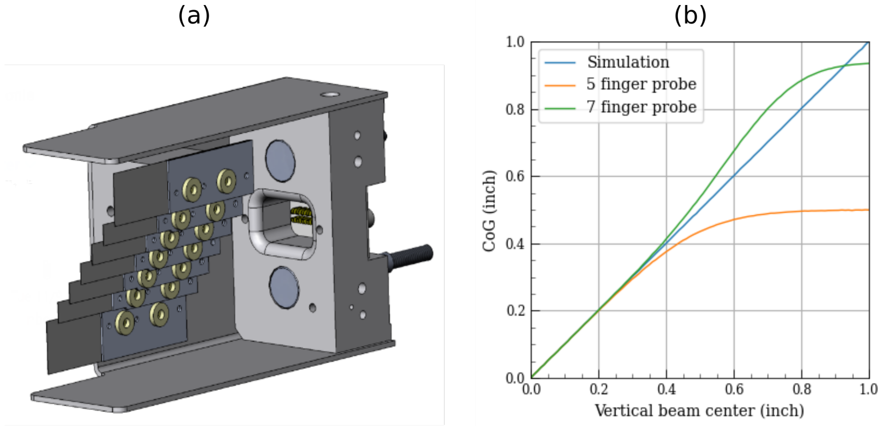


Figure 3: Measure the beam vertical center of gravity using the 7 finger HE2 probe. (a) Structure of the 7 finger probe head. (b) CoG results from 2 processing method.

2.2 Measure the vertical tune using trim coil radial field

The radial magnetic field in the median plane produces a vertical Lorentz force on the particles travelling in the azimuthal direction. Thus, the beam center orbit on the median plane is deflected vertically.[6] The low axial field in TRIUMF cyclotron makes the beam vertical position very sensitive to the radial field error, which has been carefully studied during the commissioning of the 500 MeV cyclotron.[7] The sum of $k-th$ component $B_{r,k}$ in the Fourier expansion of the median plane radial magnetic field drives the vertical motion equation as follows

$$\frac{d^2z}{d\theta^2}+\nu_z^2 z=\frac{r_0}{\bar{B}}\sum_{k=0}^{n}B_{r,k}\cos\left(k\theta+\phi_k\right) \tag{2}$$

In TRIUMF 500 MeV cyclotron, The trim coil pairs are symmetric with respect to the median plane. When the upper and lower coil carries the same current, the magnetic field is perpendicular to the median plane. Otherwise, the difference of the current in the upper and lower coil produces a radial magnetic field. Because the trim coil is axially symmetric, the introduced radial magnetic field $\Delta B_r(\theta)$ is constant at the same radius. However, the orbit scalloping makes the radial field seen by the particles is a periodic function with the base frequency of the sector number of 6. The closed orbit solution of Eq.3 including all the component of $B_{r,k}$ in the Fourier expansion is studied by J.L. Bolduc and G.H. Machenzie [8]. Considering the low vertical tune ν_z, the dominated driving term of the vertical motion is only the average radial field $\Delta\bar{B}_r$. Thus, the vertical displacement Δz as a function of r, $\Delta\bar{B}_r$ and ν_z is written as

$$\Delta z = \frac{qr\Delta B_r\sqrt{1-\left(\frac{\omega r}{c}\right)^2}}{m_o\nu_z^2\omega} \tag{3}$$

where m_0 is the particle's rest mass and w is the angular frequency, which are 2 constants. The detailed derivation could be found in L. Root's note [9]. Using Eq.3, the vertical tune could be calculated from the vertical displacement produced by scanning the TC radial field.

HE probe, travels along the radial direction on the cyclotron median plane, is used to measure the vertical CoG of the beam. In the cyclotron with the spiral sector, the scalloping of the SEO makes the radial position of the probe R_{probe} vs. the average SEO radius R_{av} is nonlinear. Figure 1 shows the difference between R_{probe} and R_{av} along the probe radius. To calculate the tune at different average radius, the R_{probe} should be calibrated against the nonlinear calibration curve.

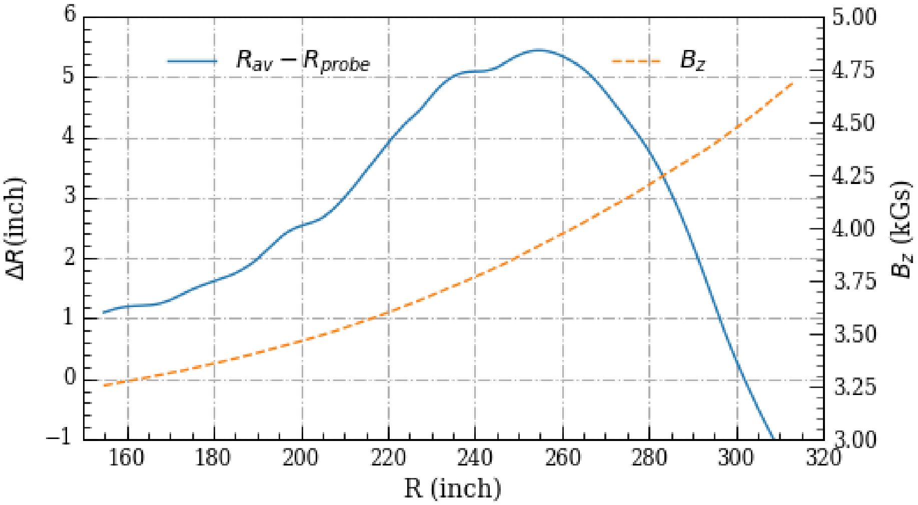


Figure 4: Calibrate the probe radius to the average radius of SEO.

Figure 5 shows the vertical tune measured by scanning 2 single TC radial field. The usable data with low error is only available near the field peaks. In the TC field overlap regions, the measured vertical tune from the 2 different TC scanning has some discrepancy. The discrepancy results from the radial position measurement error of the probe. To reduce the radial position error, we create a flat radial field by operating TC in pairs. The flat-top radial field is shown by the green line in figure 5(a).

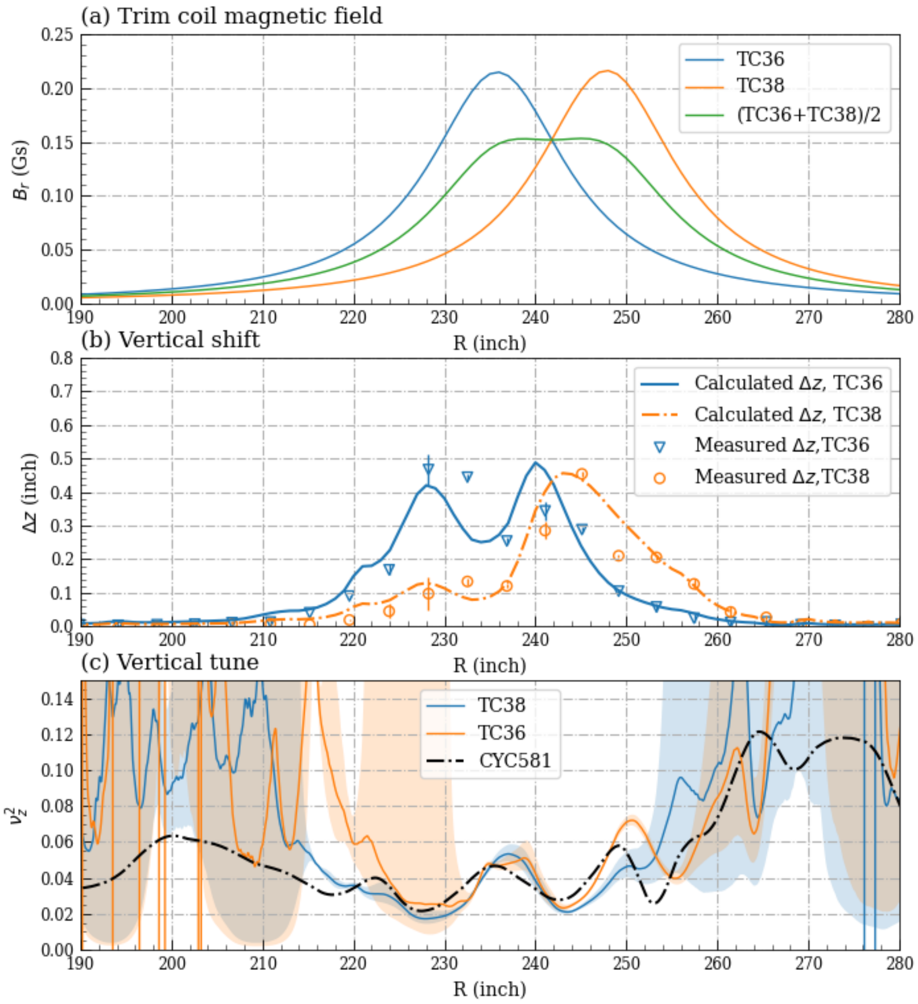


Figure 5: Measure vertical tune by scanning trim coil radial field. CYC581 is the tune calculated from the field survey data. The colored area shows the error, which is larger at lower field region.

2.3 Vertical tune correction

For a N sector cyclotron with spiral angle of ξ, an approximation of the radial and vertical tune is given as [10]

$$\begin{aligned}
\nu_r^2 &= 1 + k + \frac{3N^2}{\left(N^2-1\right)\left(N^2-4\right)} F\left(1+\tan^2\xi\right) \\
\nu_z^2 &= -k + \frac{N^2}{\left(N^2-1\right)} F\left(1+2\tan^2\xi\right)
\end{aligned} \tag{4}$$

where k is the field index. If we adjust the TC axial field, the change of the vertical tune is dominated by the change of the field index Δk. Because the tune contribution from the flutter difference resulted from the TC field adjusting is small for a 6 sector cyclotron. The tune difference contributed by a single TC axial field is shown in figure 6. The local bump created by a single TC could be used to correct the cyclotron vertical tune.

For isochronous orbits. The field index k satisfies

$$k = \frac{R}{B}\frac{\mathrm{d}B}{\mathrm{d}R} = \beta^2\gamma^2 \tag{5}$$

Where R is the average radius of the SEO, and B is the average axial field along the SEO. The adjusting of the field index causes the isochronous error. To maintain the isochronism after correcting the vertical tune, other TCs should be used to compensate for the total phase excursion.

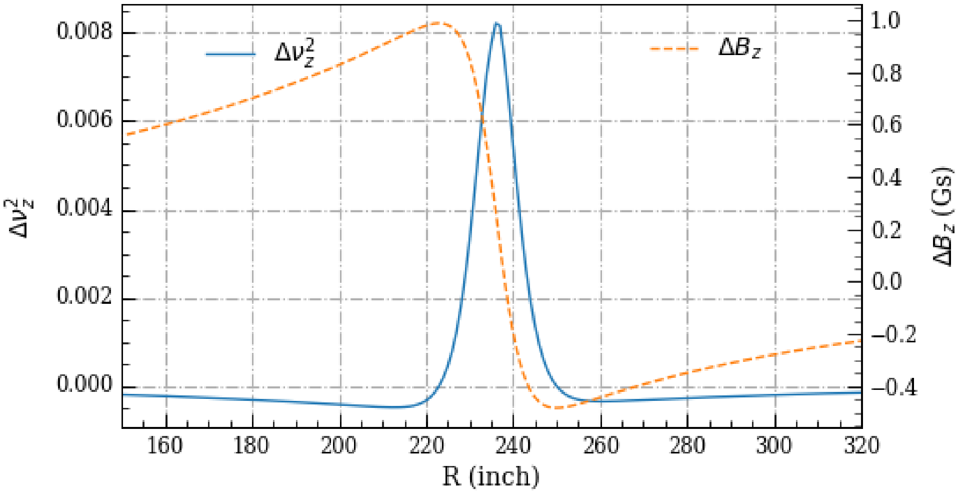


Figure 6: Field and vertical tune contributed by the trim coil axial field.

3 Result and Discussion

First, we've measured the vertical tune by scanning the radial field of 5 pairs of trim coil. Each pair of trim coil produces a flat-top covering a radial range of about 10 inches. The measured vertical tune is also compared with the calculated tune from the historical field survey data. Then we corrected the vertical tune bump around 235 inches which approached the linear coupling resonance. After correcting the vertical tune, the vertical coherent oscillation and vertical beam size is compared with the ones under the previous production trim coil settings.

3.1 Verticle tune

The vertical tune measured by scanning the 5 pairs of TCs is shown below. The scanning step is 5 At (ampere-turns). An increase of 20 At produces a beam vertical displacement of about half an inch, which is enough for the tune measurement, at the same time it could also keep the beam in the vertical position measurement range of the HE probe. The "CYC581" data in the plot is the tune calculated by CYCLOPE using the historical magnetic field survey data.

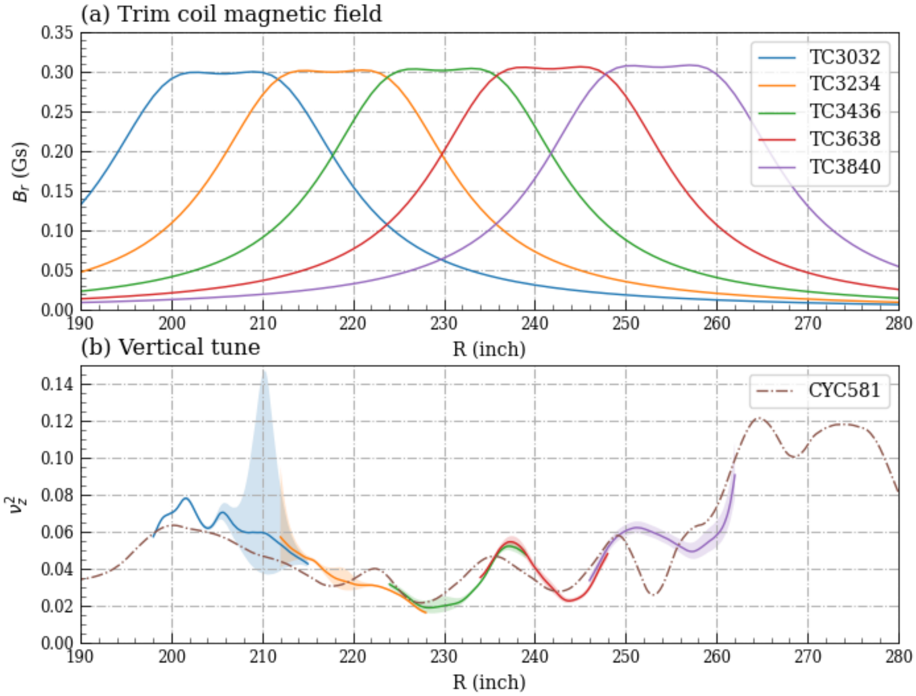


Figure 7: Measured vertical tune using different trim coil pairs. The measured vertical tune from different TC pairs agree well in the field overlap region, which means both the trim coil field and the HE probe are well calibrated with the average radius. The measured ν_r^2 reproduces several bumps in the CYC581 data. The difference between the measured vertical tune and the theoretical is below 0.02, which may be resulted from 2 reasons, one is the field survey data may be smoothed, the other is the trim coil setting is different from the one used in the field survey data.

The tune diagram is plotted in figure 8. The radial tune ν_r in the colored line is calculated by subtracting $\Delta\nu_z^2$, the difference between the square of CYC581 and measured vertical tune, from the CYC581 ν_r^2.

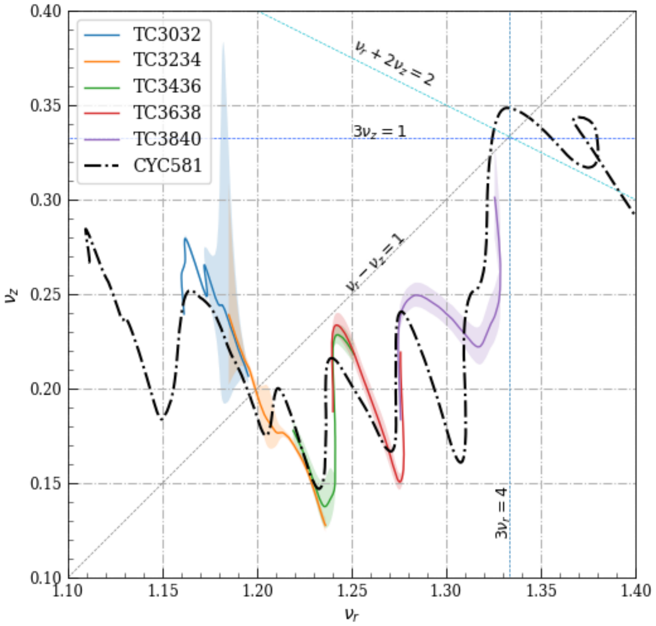


Figure 8: Tune diagram. Comparing with the CYC581 data, the higher bump shown by the red line approaches the coupling resonance line, which makes the tune very sensitive to the adjusting of the TC coil axial field at this radius.

3.2 Vertical tune correction

By increasing the axial field of TC36, we corrected the vertical tune bump around 235” which approaches the linear coupling resonance. Figure 9 shows the tune diagram before and after the correction. After the correction, TC35 and TC37 radial field is scanned to measure the vertical tune around 235”.

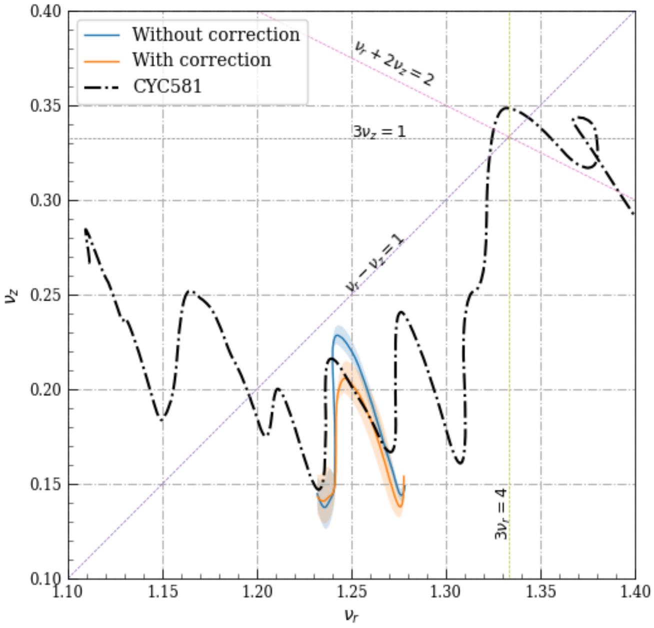


Figure 9: Tune diagram with and without correction. After the correction, the working point is far away from the coupling resonance line.

The isochronism is broken after adjusting the TC36 axial field, blue lines in figure 10 show the axial field profile and the contributed $\Delta sin(\phi)$ from a single TC. The axial field of a single TC extends to 0 radius, which could make the phase excursion increase continuously from the injection. To maintain the isochronism, we using 3 TCs as a triplet. The triplet axial field creates a local peak at a narrow range of radius, the current setting of each coil is optimized to minimize the tails on both sides of its $\Delta sin(\phi)$ peak. The orange lines in figure 9 show the optimal triplet axial field and the contributed $\Delta sin(\phi)$. The triplet setting is 63.8 100 33.3 At for TC33 36 41 respectively.

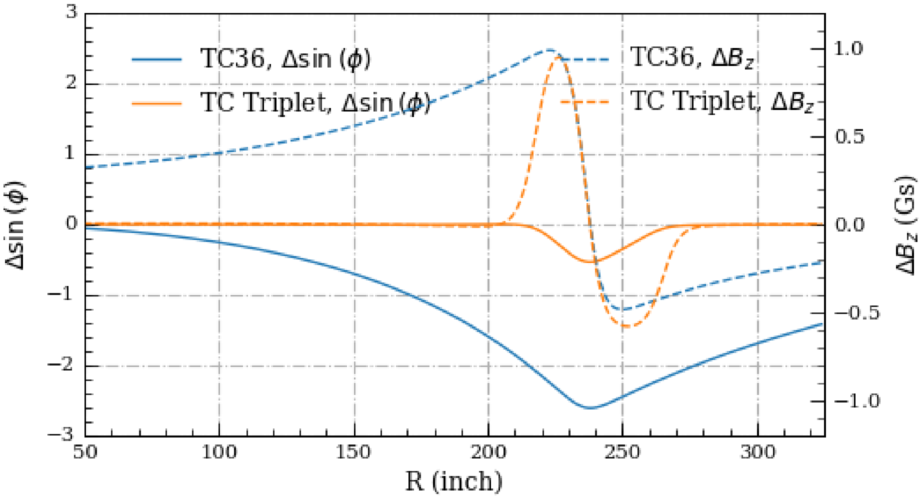
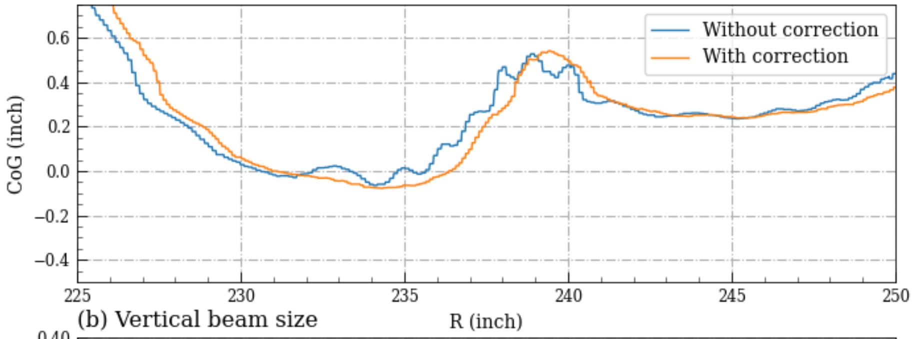


Figure 10: Axial field profile and $\Delta sin(\phi)$values vs. radius. The blue line is the results of the single TC36. The orange line is the results of the TC33 36 41 triplet. The triplet axial field is optimized to minimize the tails on on both sides of its $\Delta sin(\phi)$ peak.

3.3 Coherent oscillation

There exists radial centering error under the cyclotron production setting. So the HE measurement without correction, shown by the blue line in figure 11, directly shows the effect of the passage through the coupling resonance.

(a) Vertical beam center of gravity



(b) Vertical beam size

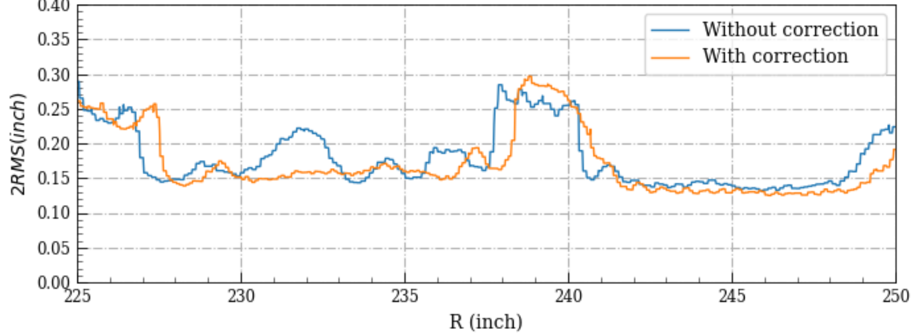
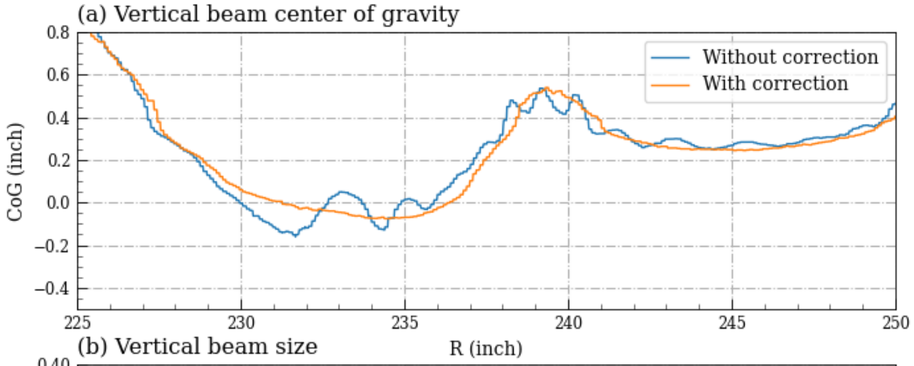


Figure 11: Coherent oscillation and beam size under cyclotron production setting. Before correction, shown by the blue line, the CoG result shows the vertical coherent oscillation after 230”. The incoherent oscillation shown by the vertical beam size happens at the same radius. After correction, the oscillation is removed.

(a) Vertical beam center of gravity



(b) Vertical beam size

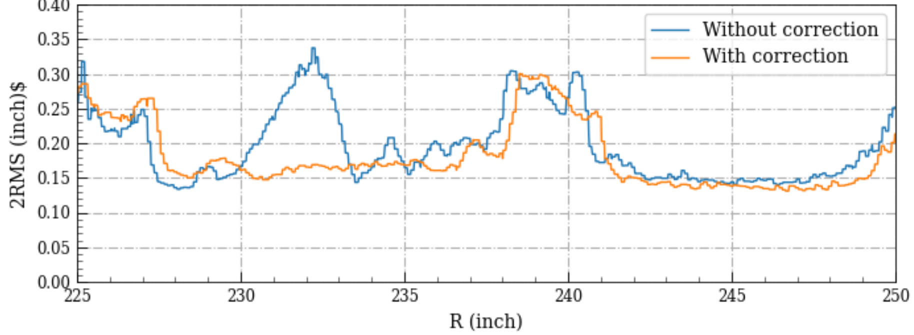


Figure 12: Coherent oscillation and beam size with deflector detuned. A larger radial centering error is created by detuning the deflector. The coupling resonance before correction makes the vertical oscillation amplitude larger comparing with the figure 11 blue line. The same TC36 correction as in figure 11 still works well to remove the vertical oscillation, which is shown by the orange line.

4 Conclusion

A peak of vertical tune at cyclotron radius of around 235" observed experimentally confirms an extra $\nu_r - \nu_z = 1$ coupling resonance passage which is not shown by the tune from historical field survey data. After correcting the vertical tune, both the coherent and the incoherent oscillation in the vertical direction at 235" disappear, which leads to a reduction of maximum vertical beam size at this radius. Eventually, it is expected to reduce the beam loss in the cyclotron.

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