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Error field from the tilted median plane in TRIUMF 500 MeV cyclotron

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Abstract: This note summarized the investigations into field survey data of the 500 MeV cyclotron. To correct the disagreement in the redundant field survey data of the median plane error field, we compared the error scale potential map calculated from different survey data. Finally, the azimuthal field B_θ and the axial gradient of the axial field dB_z/dz on the median plane are corrected using the radial field map B_r. The influence of the correction is also studied using CYCLOPS. The result shows a significant vertical displacement of the closed orbits.

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1 Introduction

Cyclotron field survey data is usually taken on the median plane. For the cyclotron with median plane symmetry, there only exits axial magnetic field B_z. However, because of the pole shape geometry error and the unevenly magnetized soft iron, the magnetic plane could be tilted, which could introduce radial and azimuthal fields on the median plane. The error of the tilted median plane could be the driven force when the working point passes through some resonances.

In the gap of the main magnet, the magnetic field can be expressed in terms of a scalar potential Φ. By solving the 3D Laplace's equation using an operator trick, we get the potential Φ expanded in powers of 'z' as follows [1]

$$\begin{aligned} \Psi &= \Psi_{\mathrm{o}} + \Psi_{\mathrm{e}} \\ \Psi_{\mathrm{o}} &= zB - \frac{z^3}{3!}\nabla_2^2 B + \frac{z^5}{5!}\nabla_2^4 B - ... \\ \Psi_{\mathrm{e}} &= C - \frac{z^2}{2!}\nabla_2^2 C + \frac{z^4}{4!}\nabla_2^4 C - ... \end{aligned} \tag{1}$$

Where $B = B(r,\theta)$ and $C = C(r,\theta)$. The odd term Ψ_o produces a field with median plane symmetry, while the even term Ψ_e spoils this symmetry. The magnetic field is given by $\vec{B} = \nabla\Psi$. The magnetic field on the median plane is written as

$$\begin{aligned} B_r &= -\frac{\partial C}{\partial r} \\ rB_\theta &= -\frac{\partial C}{\partial \theta} \\ B_z &= -B \end{aligned} \tag{2}$$

When median plane symmetry is broken, the axial derivative of the axial field dB_z/dz is none zero, which is given as

$$\frac{dB_z}{dz} = \nabla_2^2 C = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2} \tag{3}$$

2 Field map from redundant field survey data

For TRIUMF 500 MeV cyclotron, the tilted error B_r, B_θ and dB_z/dz is measured on a polar grid with 1° intervals in azimuth and 3 inches intervals in radius. It's given as a Fourier serial in azimuth up to 29th harmonics. Since the different harmonics are orthogonal, every single harmonic should satisfy the eqn. 3 and eqn. 4. The n-th harmonic of the error map B_{rn}, $B_{\theta n}$ and dB_{zn}/dz satisfies

$$\begin{aligned}
B_{rn} &= -\frac{dC_n}{dr} \\
B_{\theta n} &= -jn\frac{C_n}{r} \\
\frac{dB_{zn}}{dz} &= \frac{d^2C_n}{dr^2} + \frac{1}{r}\frac{dC_n}{dr} - n^2\frac{C_n^2}{r^2}
\end{aligned} \tag{4}$$

Where C_n is the n-th harmonic of the scale potential of the error field. By solving eqn. 4 numerically, we get the three C map from B_r, B_θ and dB_z/dz respectively. The 1st harmonic of the C map is shown in Fig. 1.

To reduce the error from the calculation of derivatives, we used the compact finite difference method. This method could be used for reconstructing of the error field on the median plane and extrapolating a 3D field map from the median plane using the error potential map. A detailed study of the compact finite difference method is done by Dong-o Jeon, this method was implemented in z4cyclotron code. [2, 3] In this report we directly use the low pass filter, 1st and 2nd order differentiator in Dong-o Jeon's paper. For the given filter and the 1st order differentiator, it is formally sixth-order accurate. The 2nd order differentiator is accurate to the fourth-order. The first harmonic of the error field calculated from different survey data is shown in Fig. 2

Fig. 3 compares the corrected azimuthal field (B_θ) map and the field survey map on the median plane. The corrected field map is calculated from B_r survey data. Fig. 4 compares the corrected dB_z/dz map and the field survey map on the median plane. The corrected field map is also calculated from B_r survey data.

3 Static equilibrium orbit properties

We studied the tune and static equilibrium orbit of the cyclotron using CYCLOPS [4]. Fig. 5 compares the results calculated using the CYC581 field survey map and the corrected map.

4 Conclusion

The redundant field survey maps B_θ, B_r and dB_z/dz, resulting from the median plane tilt error, are compared with each other. A systematical error in the median plane azimuthal field B_θ survey data is found. The median plane error field is corrected using the B_r survey map. The CYCLOPS results show that the correction doesn't affect the tune, but it could significantly change the vertical displacement of the closed orbit.

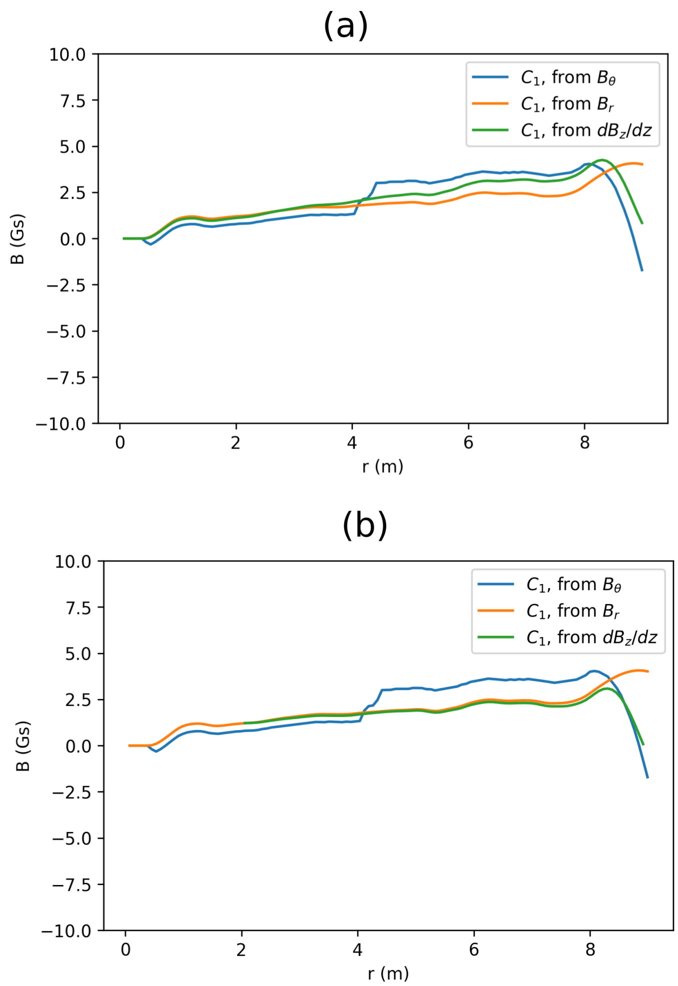

Figure 1: First harmonic of the median plane error field potential map C_1. (a) C_1 calculated using eqn. 4 starting from the center of the cyclotron, where the field is homogeneous. Thus the ODE's initial conditions are $C_1(0) = -jB_\theta(0) = 0$ and $C_1'(0) = -B_r(0) = 0$. (b) C_1 calculated using eqn. 4 starting from the radius of 2 m in the third ODE. The 2 kinks at 0.5 and 4 m in $C1$ calculated from B_θ suggests some systematical error in the measurement of B_θ. The difference between the $C1$ calculated from B_r and dB_z/dz grows with the radius because of the error of the initial conditions. Thus, if starting from a larger radius with relatively constant slope in the field, the difference becomes smaller as shown in (b).

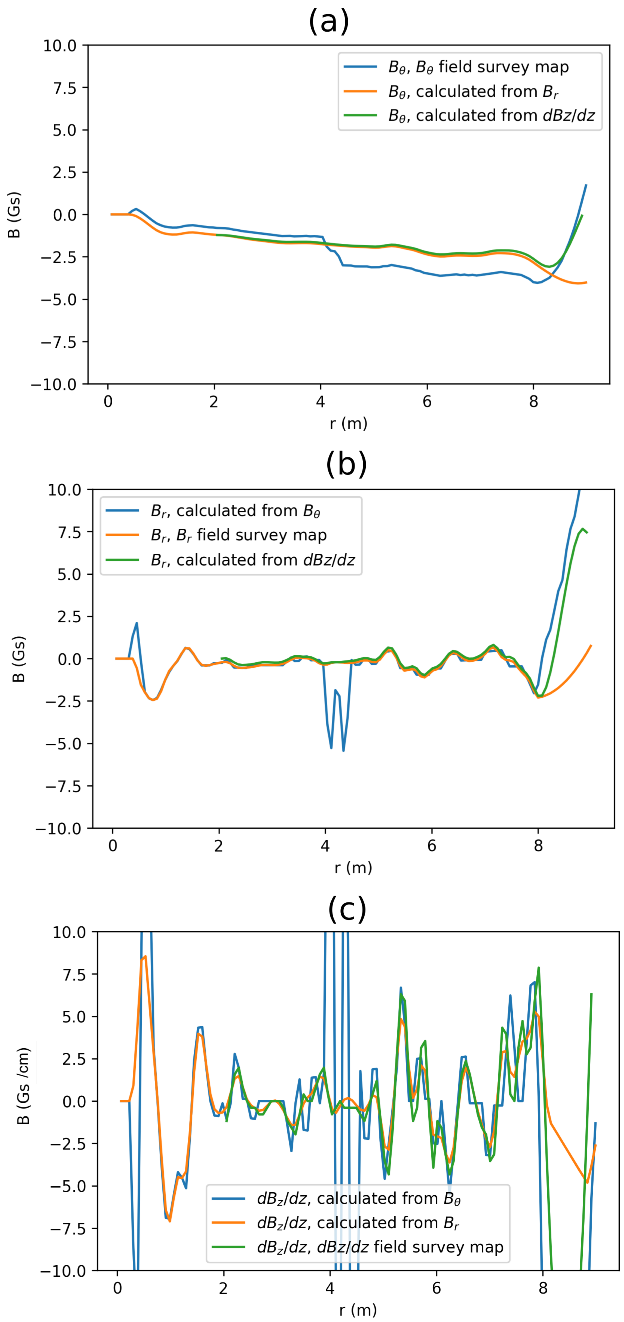

Figure 2: Comparing the First harmonic of the median plane error field B_θ, B_r and dB_z/dz constructed from different potential map C. The first and second-order derivatives are calculated using the compact finite difference method. (a) Azimuthal field B_θ, the B_θ survey data seems to be shifted upward before the radius of 4 m and thereafter downward compared with the other two. (b) Radial field B_r, the calculated B_r from different C map agrees with the B_r survey map except for the spike at 4 m in the one calculated from B_θ. (c) Axial gradient of the axial field, B_θ is too noisy to give a usable dB_z/dz map. dB_z/dz calculated from B_r agrees with the dB_z/dz survey data. It's smoother than the survey data because high-frequency components are filtered by the compact finite difference method.

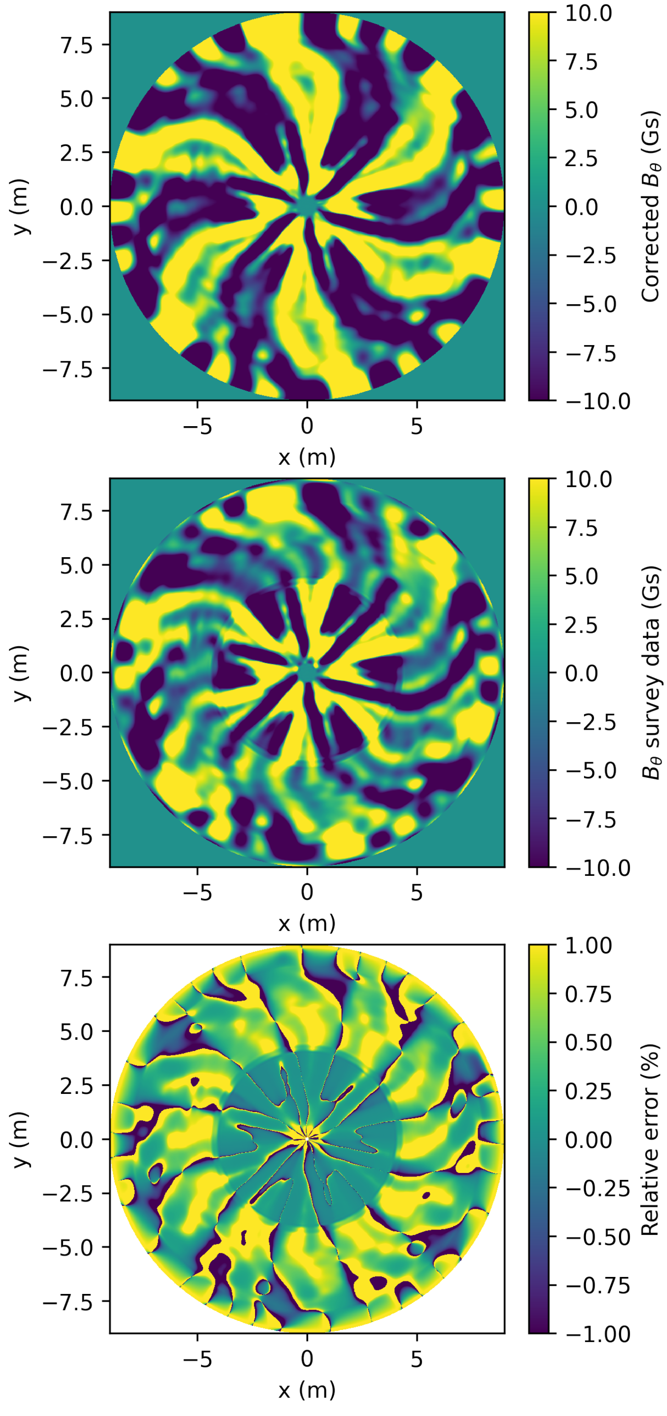

Figure 3: Azimuthal field map on the median plane. The plot on the top is the corrected field map using the B_r map, it shows the edge of the 6 sectors of the main magnet. The middle plot is the B_θ survey map. There is an obvious discontinuity at the radius of 4 m, thereafter the edge of the sector structure becomes blurring. The relative difference also suddenly becomes larger after 4 m, shown by the bottom plot.

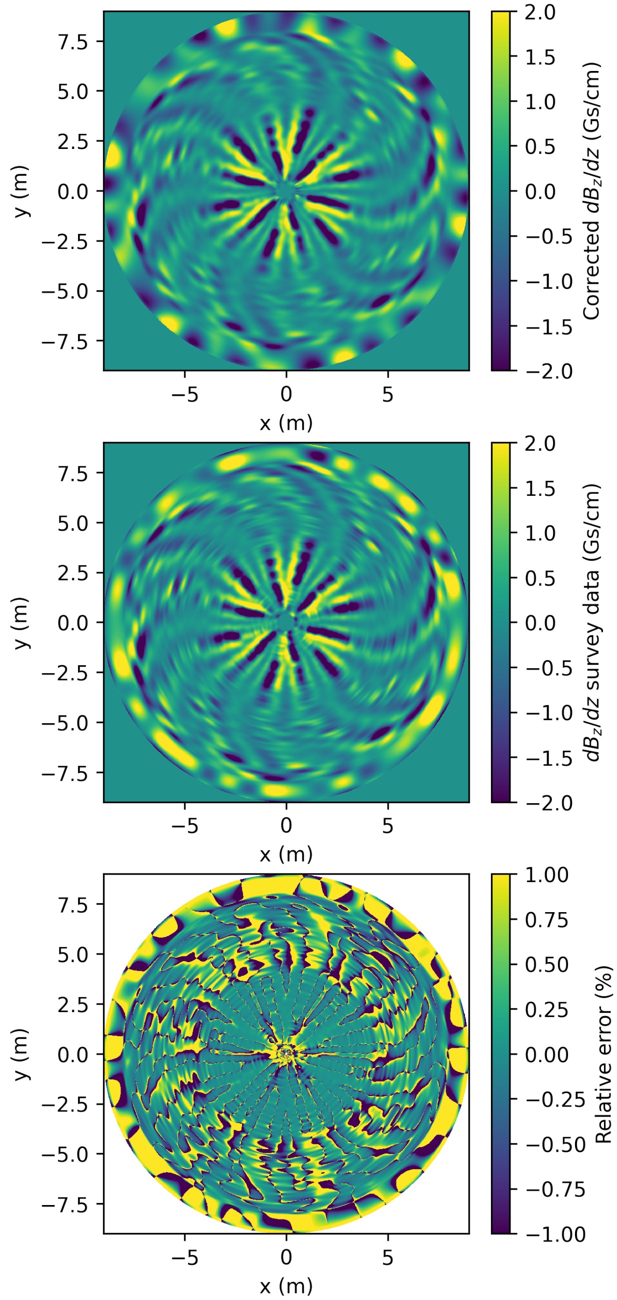

Figure 4: Axial gradient of the axial field. The corrected field map using B_r map (top) and dB_z/dz survey map have the same pattern where the value changes rapidly in the azimuthal direction at the edge of the pole sectors. Thus, small phase noise will make a big difference spike between the 2 results, which is shown in the bottom plot, the maximum error happens at the rapidly falling or rising edges of the dB_z/dz value. The rms of the difference within the radius of 8 m is 0.0013 Gs/cm, which indicates a good agreement between the calculated dB_z/dz and the measured one.

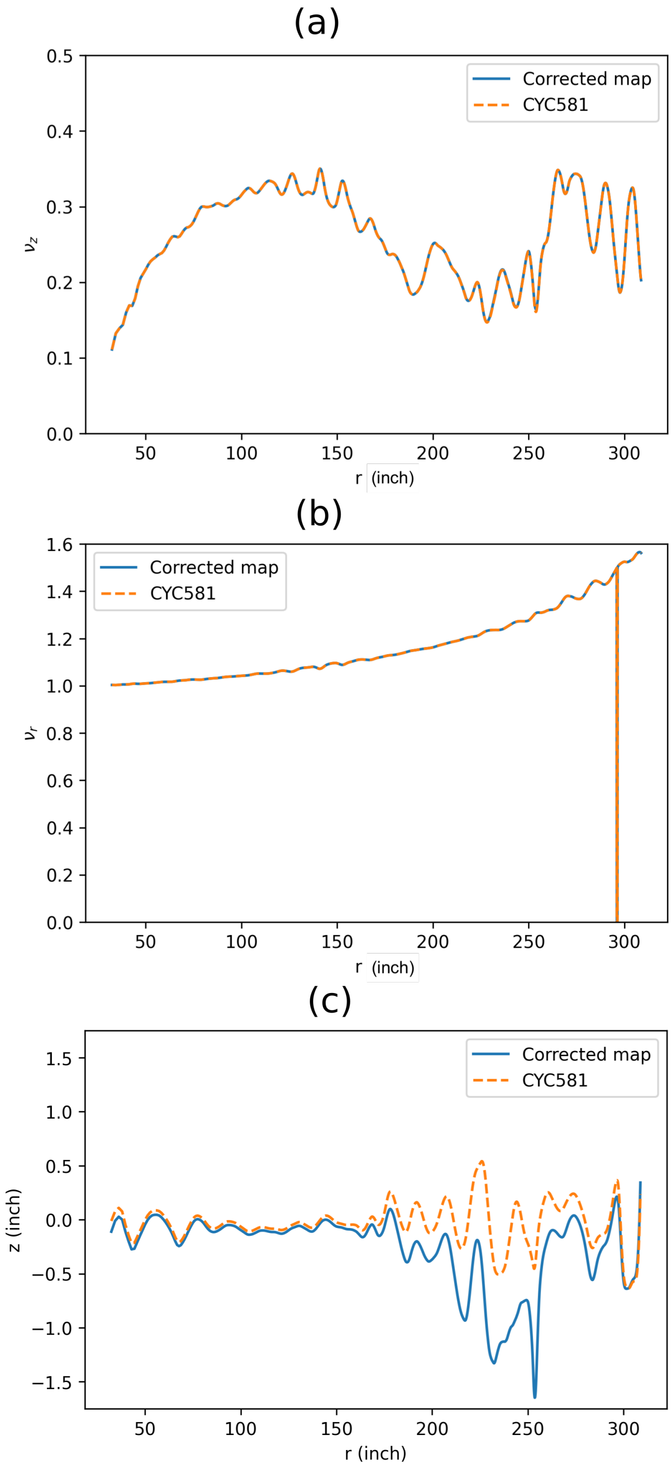

Figure 5: Tune and average vertical coordinates of the closed orbit calculated using CYCLOPS. The corrected field map has the same B_z and B_r median plane map as CYC581 survey map, while the B_θ and dBz/dz is corrected using the potential map calculated from B_r median plane map. The difference between the corrected map and survey map doesn't affect the cyclotron tune. However, it significantly changes the average vertical coordinates of the closed orbit.

References

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[4] M. M. Gordon, Computation of closed orbits and basic focusing properties for sector-focused cyclotrons and the design of 'cyclops', Part. Accel. 16 (1984) 39–62.