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Design of a 2 GeV Proton Cyclotron with Constant Radial and Vertical Tunes

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Abstract: In this note I present the design of a 2 GeV cyclotron, directly inspired from the CIAE 800 MeV–2 GeV design, that has the particularity of being perfectly isochronous while having rigorously constant radial and vertical tunes.

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1 Introduction

One can produce a perfectly isochronous field distribution by starting from the geometry of the closed orbits. One can use for that purpose realistic non-hard-edge orbits. What is more, one can also calculate the transverse tunes directly from the geometry of these orbits [1]. The advantage of this method is that it allows to produce isochronous field distributions and calculate tunes in a split second. This rapid turn over makes it easier to explore the wide parameter space of possible cyclotron lattice designs.

I have already used this technique to develop designs that, while being perfectly isochronous, have constant radial and vertical tunes over a large energy range [1]. In a previous note [2], I have shown that it is possible to design (using OPERA-3D) a magnet that will produce the corresponding isochronous field distribution.

In this note, I try to tackle the design of a higher-energy cyclotron. I chose to adopt the parameters (number of sectors = 10, $\mathcal{R}_\infty = 28\,\mathrm{m}$, etc) of the 800 MeV to 2 GeV CIAE cyclotron design[3]. Using these basic parameters, I have determined a field distribution with long drift spaces, perfectly isochronous, and with precisely constant radial and vertical tunes over the entire energy range. The purpose of this note is to present this design.

2 Smooth orbits with long straight sections

For such a design to be practical, there needs to be enough space between sectors to place the rf cavities, injection/extraction systems, etc. In a ring-cyclotron-like design, this calls for long drift sections, without magnetic field. In such a drift section, the closed orbits follows straight lines.

I have parametrized the shape of each orbit using only 5 parameters, forcing the orbits to go in a straight line over some distance. Each orbit is described with a spline $r(\theta)$. I choose to use 5 orbits to cover the area between 0.8 GeV and 2 GeV. I then take the Fourier transform or each of these 5 orbits (I used up to j =9 harmonics), and then spline each one of the Fourier harmonics (C_j and S_j) to construct the function:

$$r(\theta, a) = aC_0(a) + a\sum_{j=0}^{9} C_j(a)\cos(j\theta) + S_j(a)\sin(j\theta) \tag{1}$$

which describes the shape of the continuum of closed orbits, where a is the average orbit radius.

The entire field region between 0.8 GeV and 2 GeV is thus described by 5×5 free parameters. I use the 'minimize' function form the package SCIPY.OPTIMIZE to vary these 25 parameters in order to achieve one objective: minimized the RMS variation of both tunes ν_r and ν_z. As a matter of fact I have found numerous solutions, leading to different constant tune values, and I am presenting only one of these solutions in this note.

The shape of the 5 equilibrium orbits are shown in Fig. 1. The corresponding magnetic field distribution is shown with a contour plot is Fig. 2. The corresponding tunes variations is shown in Fig. 3. The tune values calculated directly from geometry matches perfectly the results from particle tracking with the orbit code `CYCLOPS`. The relative variation of the orbit frequency $\frac{\Delta\omega}{\omega}$ is also shown in Fig. 3. Note that I had to blow up $\frac{\Delta\omega}{\omega}$ by a factor 1000 to be able to see any variation at all. Finally I have also plotted the overall tune variation in the tune diagram, see Fig. 4.

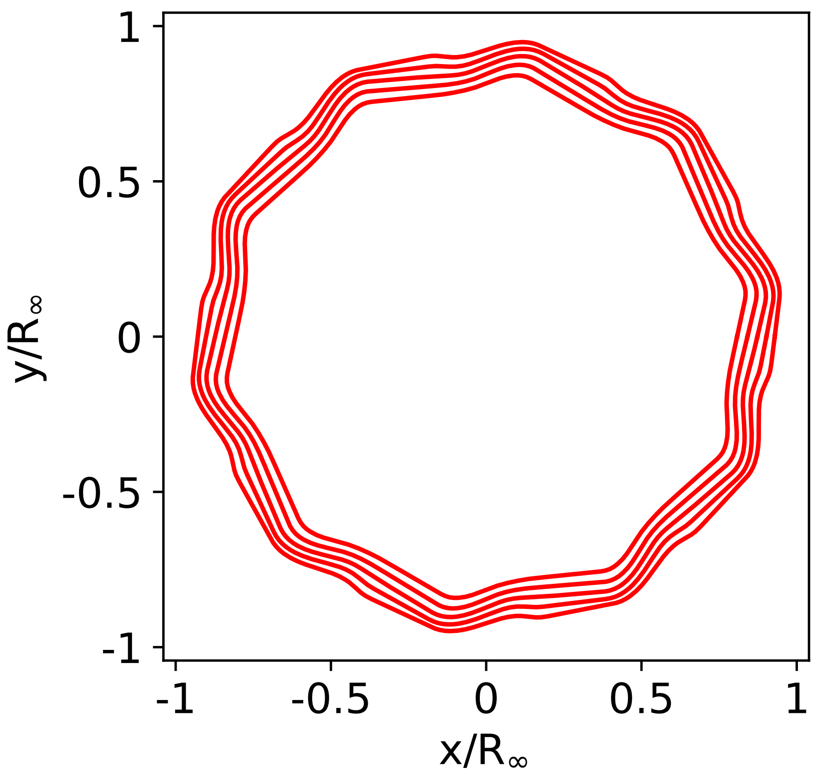

Figure 1: Shape of the equilibrium obits (inner one: 800 MeV, outer one: 2 GeV) corresponding to the design presented in this note.

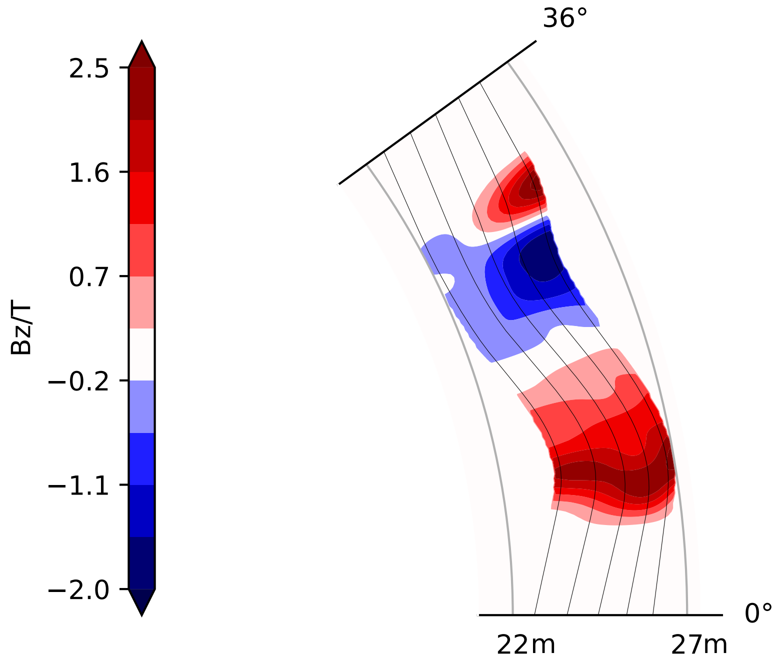

Figure 2: Corresponding field distribution. This is field map is what was used to track particles in using CYCLOPS in Fig. 3.

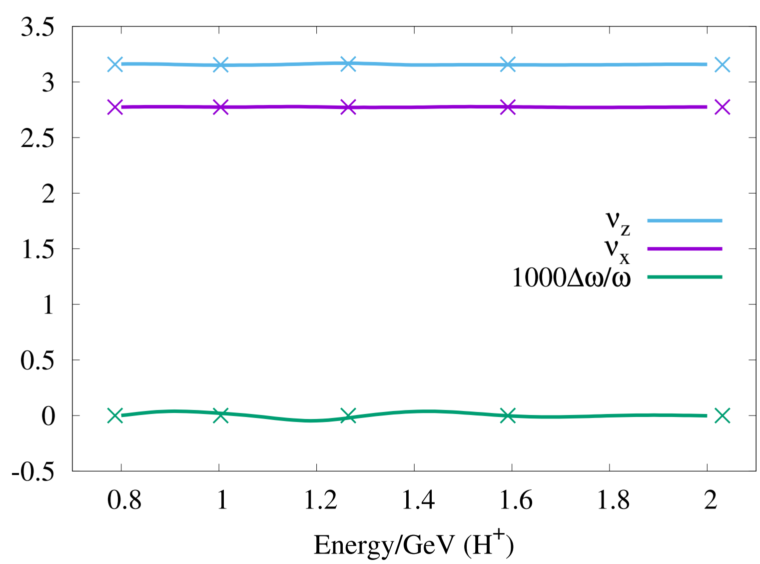

Figure 3: Corresponding tune variation and relative variation of the orbit angular frequency ω. The crosses are calculates from the shape of the orbits, and the solid line are a result of tracking particles in the corresponding magnetic field map using the orbit code CYCLOPS. Tunes are constant, and machine almost perfectly isochronous.

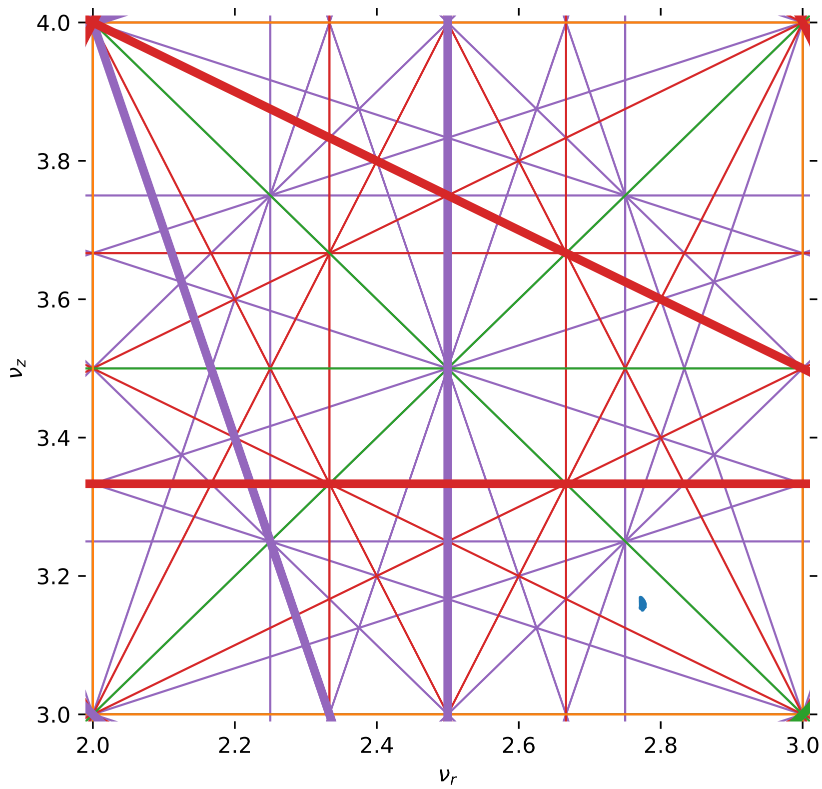

Figure 4: Same data as in Fig. 3, but plotted in the tune diagram, showing structural (thick lines) and non-structural (thin lines) resonances up to order 4 (octupole driven). The entire tune spread between 800 MeV and 2 GeV is the tiny blue spot around ν_z=2.78 and ν_z=3.16.

3 Conclusion

The next step is now to design a magnet (probably 2 separate magnets actually) to produce this field distribution, or one close enough to it to be isochronous with the same sort of extremely limited tune variation.

References

[1] T. Planche, Designing Cyclotrons and Fixed Field Accelerators From Their Orbits, in: Proc. of Int. Conf. on Cyclotrons and their Applications (Cyclotrons'19), 2019, p. FRB01. doi:10.18429/JACoW-Cyclotrons2019-FRB01.

[2] T. Planche, In this Order: Orbits, Tunes, and Magnet Design , Tech. Rep. TRI-BN-21-19, TRIUMF, https://beamphys.triumf.ca/~tplanche/text/note/from-orbit-pop/report/pop.pdf (2021).

[3] T. Zhang, S. An, T. Bian, F. Guan, M. Li, S. Pei, C. Wang, F. Wang, Z. Yin, et al., A new solution for cost effective, high average power (2 GeV, 6 MW) proton accelerator and its R&D activities, in: Proc. Cyclotrons, 2019.