Beam Physics Note
TRI-BN-22-22
July 19, 2022

Simple Method for Calculating HRS Multipole Effects Directly from Field Maps

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Abstract: The multipole tuning algorithm for the High Resolution Separator currently uses the ray tracing code Zgoubi to generate matrices that must be recalculated whenever the beam energy is changed. In this Beam Physics Note a simple model that can replicate the Zgoubi output directly from the multipole field maps is presented.

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1 Introduction

Dan Sehayek previously developed an algorithm [1] to tune the multipole corrector of the TRIUMF Canadian Rare isotope facility with Electron Beam ion source (CANREB) High Resolution Separator (HRS). This algorithm takes a horizontal emittance scan at the exit of the HRS and divides the angle range into bins. For each angle bin, the position of the beam centroid is calculated. The ray-tracing code Zgoubi [2] is used to pre-calculate a matrix describing the displacement of each centroid when a 1 V potential is applied to each of the 23 electrode pairs in the multipole. Least-squares is used to determine what voltage to apply to each pole pair to correct the beam.

This algorithm has not yet been tested on the HRS, and errors have previously been found in the Zgoubi simulations [3]. Switching beam energies required re-calculating the displacement matrix with Zgoubi, a time-consuming task [3]. An analytical model of multipole effects would allow Zgoubi outputs to be verified, and could be used to identify methods for scaling the Zgoubi outputs.

2 Simple Model

Here a simple model of the multipole effects is created using linear beam dynamics and the pole pair electric field maps. Relativistic effects are neglected, and it is assumed that the HRS dipoles have been tuned to produce the reference trajectory. Particle longitudinal velocity is assumed to be high enough to be essentially unchanged by longitudinal fields. Vertical effects are not considered.

2.1 Effect at Multipole

Let the subscript 0 be just before the multipole, 1 be just after, and 2 be at the exit emittance scanner. Let a hat indicate that the multipole is on. As standard, a prime indicates angle.

Some obvious relations follow, if drift in the multipole is assumed to be negligible:

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}, \qquad \begin{bmatrix} \hat{x}_0 \\ \hat{x}_0' \end{bmatrix} = \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} \tag{1}$$

With the multipole on, the electric field of the multipole will impart a force on charged particles that pass through it. Due to longitudinal symmetry there is no overall change in the particle longitudinal momentum. The infinitesimal change in x-momentum of the particle as it passes through is given by

$$\mathrm{d}P_x = \frac{\mathrm{d}P_x}{\mathrm{d}t}\,\mathrm{d}t = qE_x(x,s)\frac{1}{v}ds. \tag{2}$$

With a small angle approximation,

$$\Delta x' \approxeq \frac{\Delta P_x}{P} = \frac{q}{mv^2}\int E_x(x,s)\ ds = \frac{q}{2K}\int E_x(x,s)\ ds, \tag{3}$$

where K is the particle kinetic energy $K = 1/2mv^2$. The effect of the multipole is then given by:

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}'_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ x'_0 + \frac{q}{2K} \int E_x(x_0, s)\ ds \end{bmatrix} \tag{4}$$

2.2 Transfer Matrices

Now we construct the overall transfer matrix from multipole to exit emittance scanner:

$$M_{tot} = M_{drift} M_{edge} M_{dipole} M_{edge} M_{drift} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{5}$$

The individual transfer matrices are given below [4], where for the HRS $L = 800\,\text{mm}$, $\rho = 1200\,\text{mm}$, and $\eta = -26.5651°$. The 90° bend angle of the HRS dipole has already been taken into account in the dipole transfer matrix.

$$M_{drift} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}, \quad M_{edge} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{\rho}\tan(\eta) & 1 \end{bmatrix}, \quad M_{dipole} = \begin{bmatrix} 0 & \rho \\ -\frac{1}{\rho} & 0 \end{bmatrix} \tag{6}$$

As a sanity check, the coefficients of the matrix $M_{edge} M_{dipole} M_{edge}$ were compared with those calculated by TRANSOPTR [5] for the dipole. As can be seen in Table 1, other than a difference in units, the coefficients match.

	m_{11}	m_{12}	m_{21}	m_{22}
TRANSOPTR	0.500001	$120\,\text{cm rad}^{-1}$	$-0.006\,249\,99\,\text{rad cm}^{-1}$	0.500001
Calculated Coefficients	0.500001	$1200\,\text{mm rad}^{-1}$	$-0.000\,624\,99\,\text{rad mm}^{-1}$	0.500001

Table 1: Comparison between TRANSOPTR and coefficients of $M_{edge} M_{dipole} M_{edge}$.

The overall matrix, using units mm and rad is:

$$M_{tot} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} 1.775\,25 \times 10^{-6} & 1600 \\ -0.000624999 & 1.775\,25 \times 10^{-6} \end{bmatrix} \tag{7}$$

Angle and position at the emittance scanner can be calculated from the angle and position at the multipole exit:

$$\begin{bmatrix} \hat{x}_2 \\ \hat{x}'_2 \end{bmatrix} = M_{tot} \begin{bmatrix} \hat{x}_1 \\ \hat{x}'_1 \end{bmatrix} \tag{8}$$

2.3 Displacement vs Angle at Emittance Scanner

The desired output is displacement at the emittance scanner with the dipole on, $\hat{x}_2 - x_2$, vs angle at the emittance scanner. To have a common angle, we require $\hat{x}'_2 = x'_2$, or $m_{22} = 0$. The calculated value for m_{22} of $1.775\,25 \times 10^{-6}$ is likely low enough that adjustments with the multipole will cause a minimal amount of angular displacement.

Assuming the angles are equal, we can find:

$$\hat{x}_2 - x_2 = m_{11}\hat{x}_1 + m_{12}\hat{x}'_1 - m_{11}x_1 - m_{12}x'_1 = m_{12}(\hat{x}'_1 - x'_0) = m_{12}\frac{q}{2K}\int E_x(x_0, s)\ ds \tag{9}$$

$$x'_2 = m_{21} x_0 \tag{10}$$

3 Comparisons with Zgoubi

Equations (9) and (10) were used to calculate displacement vs angle for all pole pairs. Figure 1 shows the raw output of this model for Electrode Pair 13. Evidently Zgoubi interpolates the field map; if cubic spline interpolation is used the agreement is quite good (Figure 2).

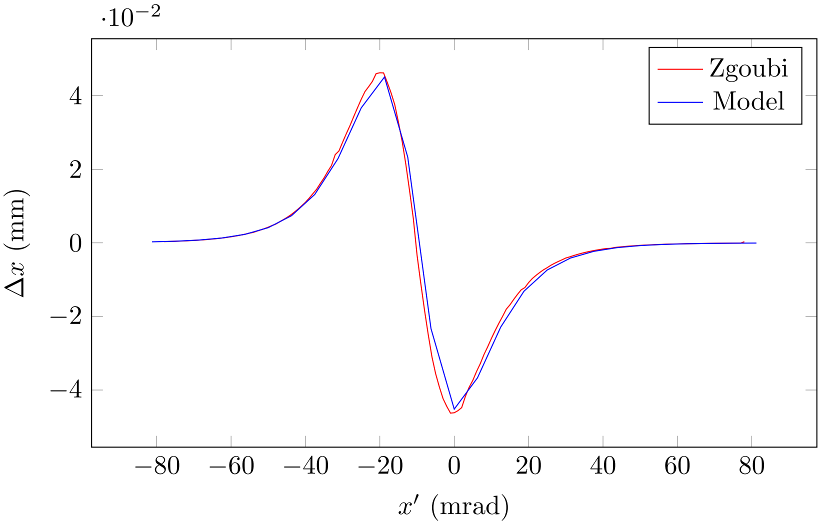

Figure 1: Model vs Zgoubi for Electrode Pair 13

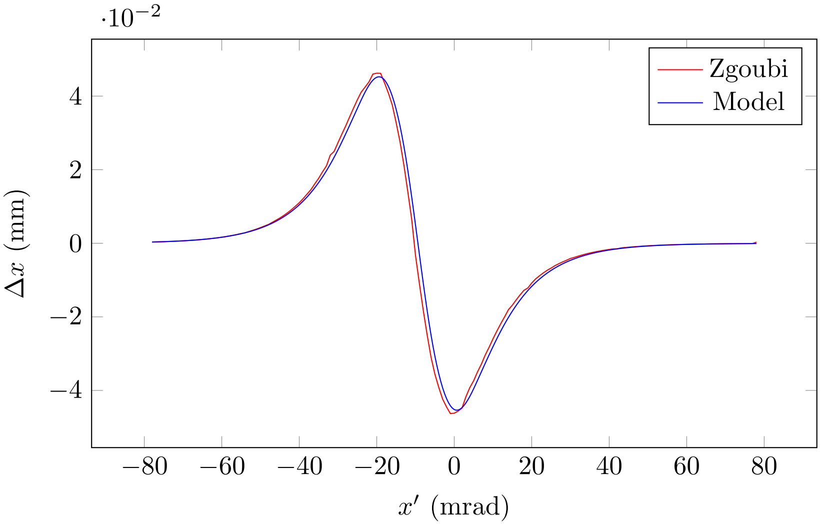

Figure 2: Interpolated Model vs Zgoubi for Electrode Pair 13

The root mean squared deviation and maximum absolute deviation between Zgoubi and the interpolated model are shown for each pole pair in Table 2. Zgoubi's output for $x' = 78\,\mathrm{mrad}$ often deviates significantly from the points surrounding it, so it was excluded and assumed to be a bug in output processing.

Pole Pair	Root Mean Squared Deviation (mm)	Maximum Absolute Deviation (mm)
1	0.000 330	0.002 85
2	0.000 196	0.001 02
3	0.000 317	0.001 72
4	0.000 562	0.002 21
5	0.000 929	0.003 81
6	0.001 36	0.005 64
7	0.001 66	0.006 87
8	0.001 92	0.007 42
9	0.002 08	0.008 72
10	0.002 22	0.008 11
11	0.002 19	0.009 31
12	0.002 10	0.007 58
13	0.001 95	0.008 27
14	0.001 66	0.006 24
15	0.001 32	0.005 74
16	0.000 957	0.003 917
17	0.000 768	0.002 50
18	0.001 14	0.004 90
19	0.001 83	0.007 55
20	0.002 69	0.0158
21	0.000 784	0.005 05
22	0.001 32	0.005 93
23	0.002 26	0.0141

Table 2: Comparison between Zgoubi and interpolated model, excluding $x' = 78\,\mathrm{mrad}$.

The best agreement was for pole pair 2, and the worst was for pole pair 20. Due to the symmetry of the multipole the electric fields for pole pairs 4 and 20 are mirrors of each other, yet the model agrees much better with Zgoubi for pole pair 4 than pole pair 20. This may be caused by nonlinearities of the HRS that are not accounted for in this model.

Pole pair 4 is shown in Figure 3, pair 20 in Figure 4, pair 23 in Figure 5, and pair 16 in Figure 6.

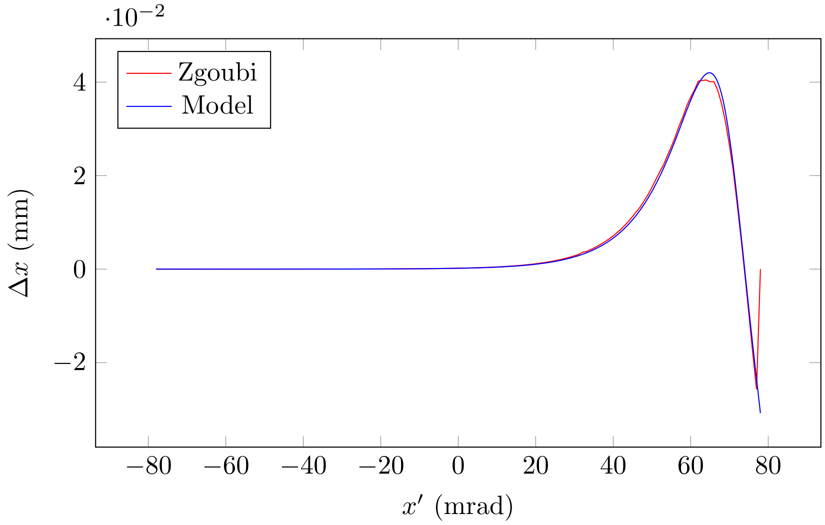

Figure 3: Interpolated Model vs Zgoubi for Electrode Pair 4

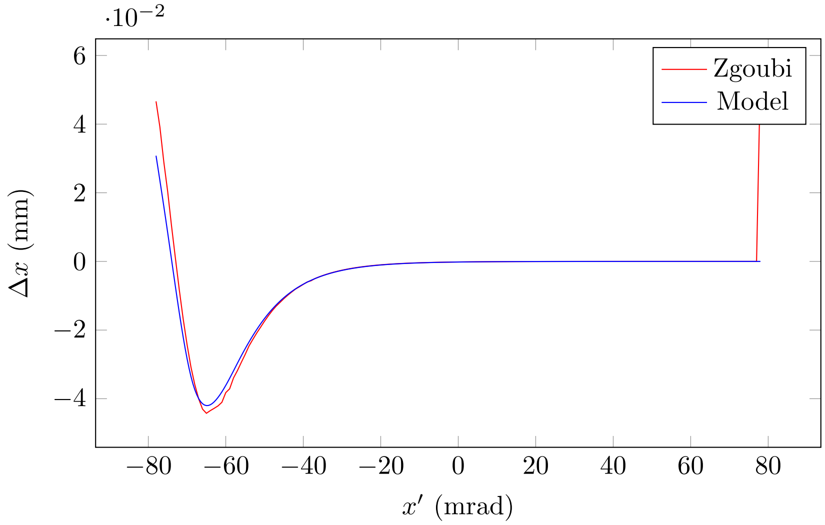

Figure 4: Interpolated Model vs Zgoubi for Electrode Pair 20

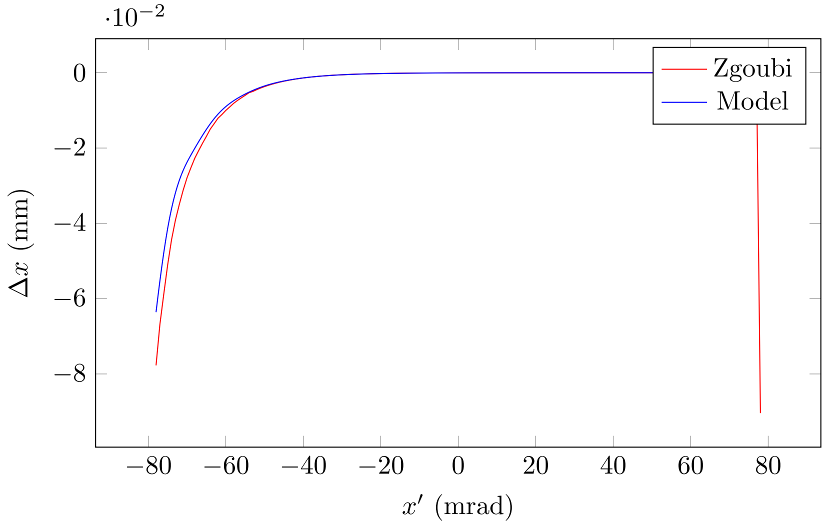

Figure 5: Interpolated Model vs Zgoubi for Electrode Pair 23

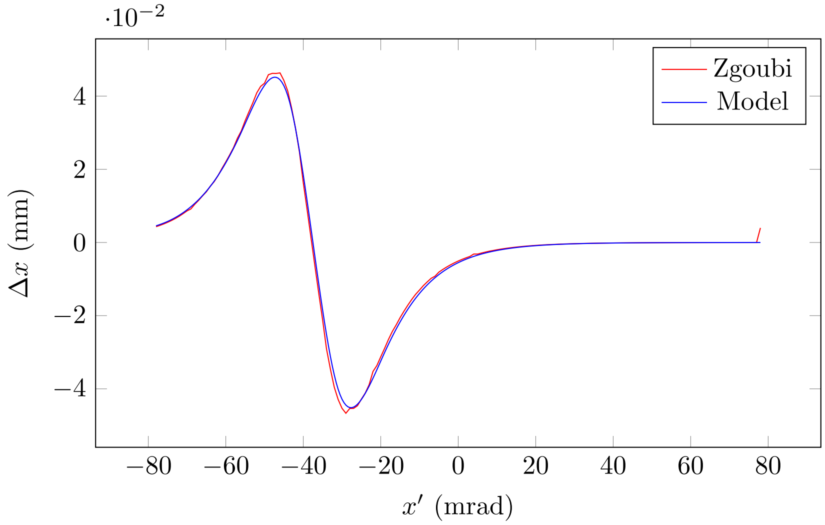

Figure 6: Interpolated Model vs Zgoubi for Electrode Pair 16

4 Conclusion

Despite neglecting nonlinearities, the simple model shown here was able to match Zgoubi outputs quite well, and could be used to entirely replace Zgoubi in the multipole tuning algorithm. The small differences in output that may come from nonlinearities, such as for pole pairs 20 and 23, could easily be compensated for by iterative tuning. Using this model would make changing the beam energy or charge simple, and would be significantly faster than running a Zgoubi simulation each time.

A Source Code

Note that Gitlab repositories may not be visible until you are added to them.

A.1 This Analysis

All the data for the figures in this document was generated by the following Pluto [6] notebook:

`https://gitlab.triumf.ca/hla/atom/-/blob/master/multipole_code_and_docs/multipole_jkraan/multipole.jl`

A.2 Zgoubi Implementation

An archive of Dan Sehayek's original work is located at:

`https://gitlab.triumf.ca/hla/atom/-/tree/master/multipole_code_and_docs/multipole_dsehayek`

Owen Lailey's corrections and modifications of Dan's work are located at:

`https://gitlab.triumf.ca/hla/atom/-/tree/master/multipole_code_and_docs/multipole_olailey`

References

[1] D. Sehayek, R. Baartman, C. Barquest, J. Maloney, M. Marchetto, and T. Planche, "Multipole Tuning Algorithm for the CANREB HRS at TRIUMF," *Proceedings of the 9th Int. Particle Accelerator Conf.*, vol. IPAC2018, 2018, Artwork Size: 3 pages, 1.650 MB ISBN: 9783954501847 Medium: PDF Publisher: JACoW Publishing, Geneva, Switzerland. DOI: `10.18429/JACOW-IPAC2018-THPML079`. [Online]. Available: `http://jacow.org/ipac2018/doi/JACoW-IPAC2018-THPML079.html`.

[2] F. Méot, "The ray-tracing code Zgoubi," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 427, no. 1, pp. 353–356, 1999, ISSN: 0168-9002. DOI: `https://doi.org/10.1016/S0168-9002(98)01508-3`. [Online]. Available: `https://www.sciencedirect.com/science/article/pii/S0168900298015083`.

[3] O. Lailey, "CANREB High-Resolution Separator Commissioning Progress," en, Aug. 2021.

[4] H. Wiedemann, *Particle Accelerator Physics*, en, 4th. New York, NY: Springer Berlin Heidelberg, 2015, ISBN: 978-3-319-18316-9.

[5] R. Baartman, *HRS Zoom Section*. [Online]. Available: `https://gitlab.triumf.ca/beamphys/transoptr/-/tree/master/example/ARIEL/CANREB/HRS_zoom_section`.

[6] F. Van Der Plas, M. Dral, P. Berg, *et al.*, *Fonsp/Pluto.jl: V0.19.9*, Jun. 2022. DOI: `10.5281/ZENODO.4792401`. [Online]. Available: `https://zenodo.org/record/4792401`.