Beam Physics Note
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Bayesian Optimization of Air-core Superconducting Coils

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Abstract: This work investigates the feasibility of applying Bayesian Optimization to configure any random magnetic field distribution required to customize a constant-tune cyclotron.

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1 Objective

An air-core superconducting coil has a linear dependence between the coil current and magnetic field. This direct relationship simplify the optimization process for any magnetic field distribution of a cyclotron. Such an air-coil superconducting coils is especially well-suited for a more sophisticated cyclotron such as the Constant Tune Cyclotron (CTC) [1]. CTC has a fixed tune from injection until extraction. However, in order to ensure contribution of tune from higher order terms, this special type of cyclotron has a rather complex magnetic field distribution. A grid-like multi-coil configuration was previously proposed to produce such a complex magnetic field [2]. In this work, another kind of coil configuration with manipulated coil shapes are proposed. Bayesian Optimization (BO) is the main methodology used in this work.

2 Methodology

2.1 Parametrization of Coils

Assuming the n^{th} coil can be parametrized by the current density j_n and a truncated Fourier series that defines the coordinates of the coil:

$$x = \sum_{i=0}^{N} a_i \cos i(\theta + \psi_i)$$
$$y = \sum_{i=0}^{N} c_i \cos i(\theta + \phi_i), \ \theta \in [0, 2\pi) \tag{1}$$

2.2 BO for Fine Tuning

2.2.1 Objective Function

The details of BO can be found in [3]. The objective function, O, that describes the “goodness” of fit is defined as follows:

$$O = \exp\left[-\frac{\sqrt{\sum_k (B_{\text{each ite}} - B_{\text{ideal}})_k^2}}{5}\right] \tag{2}$$

2.2.2 Kernel

2.2.3 Acquisition Function

Overall, the Upper Confidence Bound (UCB) shows the best estimation of the next term using:

$$\alpha = \mu(x) - \beta\sigma(x) \tag{3}$$

where $\beta > 0$ is a tradeoff parameter and $\sigma(x)$ is the marginal standard deviation of the main function $f(x)$.

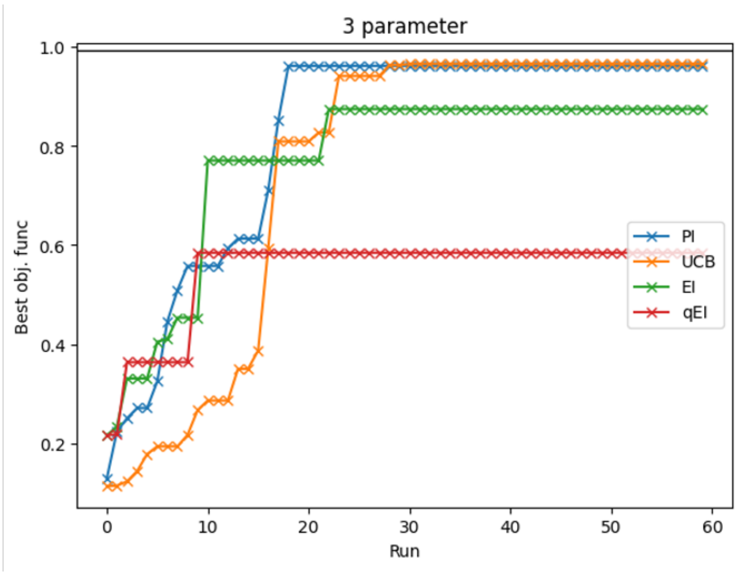

Figure 1: The comparison of the best objective function for up to 60 runs using different acquisition in BO of a circular coil.

3 Results and Discussions

3.1 Toy model

In order to test the feasibility of the BO routine, a toy model with a circular coil was created. The corresponding magnetic field distribution is as shown in Fig. 2. The X and Y of the coil are defined by

$$x = 22 + \cos(\theta)$$
$$y = 8 + \cos(\theta + \frac{\pi}{2}), \ \ \theta_i \in [0, 2\pi) \tag{4}$$

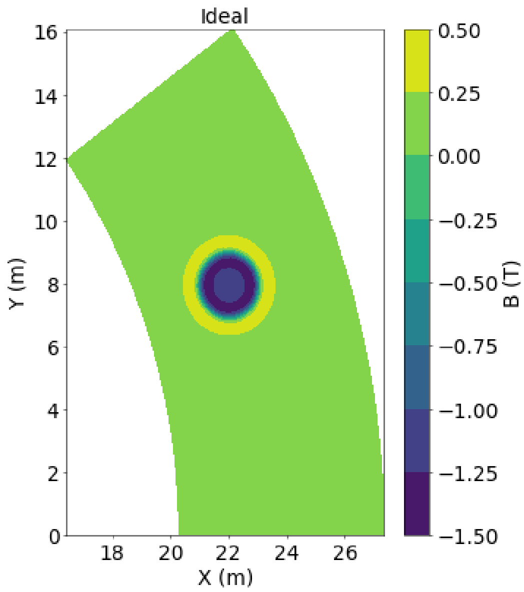

Figure 2: The ideal toy-model magnetic field distribution to be achieved by the coil

The final goal is to reproduce a coil configuration that generates a field map as close as possible to this toy model. As for this toy model, eqn is truncated at the third order. There are a total of 11 input parameters to be optimized:

$$X : a_0, a_1, a_2, \psi_1, \psi_2$$
$$Y : c_0, c_1, c_2, \phi_1, \phi_2$$
$$J : q \text{ (coil current density)}$$

The BO uses a maximum of beta of 0.5 and a default Matern kernel. The change of the field map during different runs of the BO process is shown in Fig. 3. The corresponding change of the best objective function is also given in Fig. 4

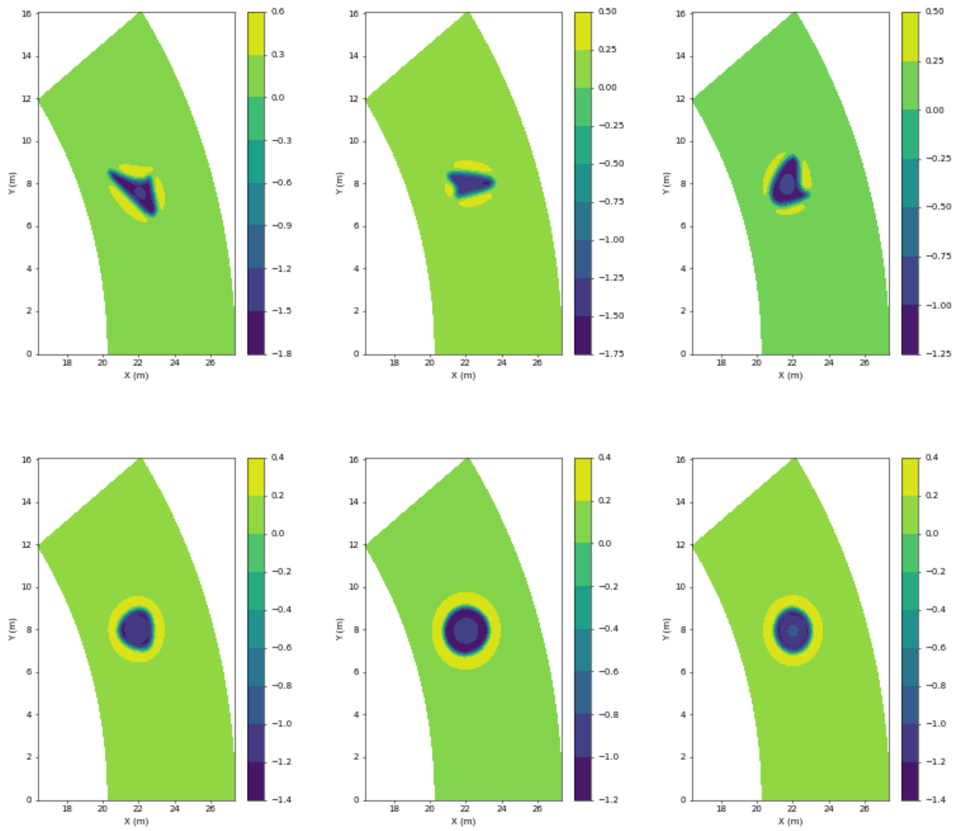

Figure 3: The change of magnetic field distribution during the BO for run (left to right) 10, 20, 30, 40, 50 and 100.

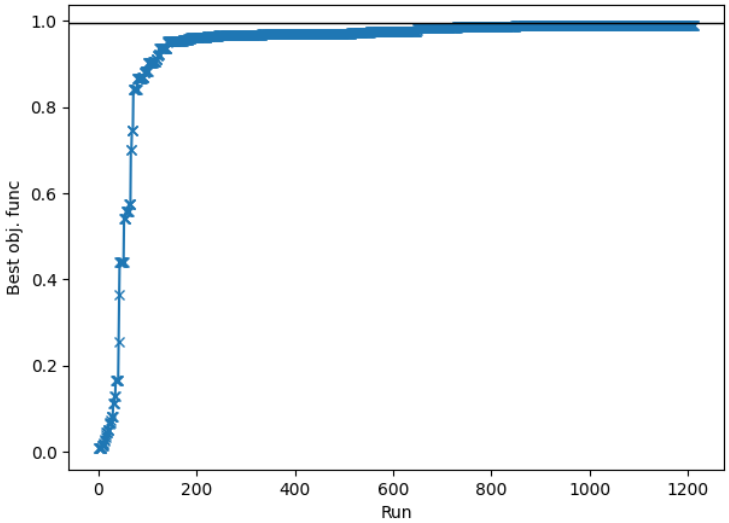

Figure 4: The change of the best objective function with runs for the toy model described in Fig. 3.

3.2 CTC

The ultimate goal is to achieve a CTC field map as shown below:

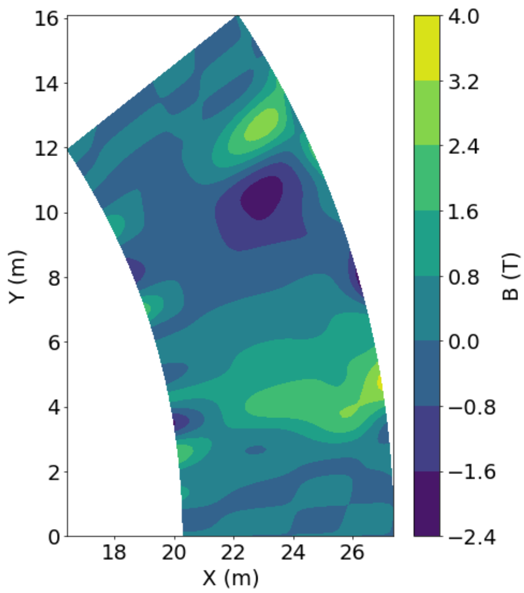

Figure 5: The ideal CTC magnetic field distribution to be achieved by the coil

3.2.1 Start from scratch

1. The initial field map and the initial objective function (O_i) are set to zero.

2. One coil is added. The coil is described up to the fifth order. This means a total of 19 tuning parameters (9 for X, 9 for Y, 1 for J) for N=36 coils:

$$X_n = a_0 + \sum_{i=1}^{5} a_i \cos i(\theta + \psi_i)$$
$$Y_n = c_0 + \sum_{i=1}^{5} c_i \cos i(\theta + \phi_i), \ n \in N \quad (5)$$

   where a_i, c_i, ϕ_i, ψ_i and the current density j are all tunable parameters.

3. Each parameter is set to have the following range:

   - Range of a_0 = [16,27]
   - Range of c_0 = [0,16]
   - Range of a_i and c_i= [-3,3] , $i \in [1, 5]$
   - Range of ψ_i and ϕ_i = [-$\frac{\pi}{2}$,$\frac{\pi}{2}$]
   - Range of j_n = [-100,100] A/mm^2

4. It is iterated over 200 runs. The best objective functions (O_f) throughout the run are recorded.

5. If $O_f > O_i$, then the latest coil configuration is kept, else the coil is rejected.

6. Steps 4-5 are repeated to add another coil up to the 36$^{\text{th}}$ coil. The final coil configurations are given in Fig. 6.

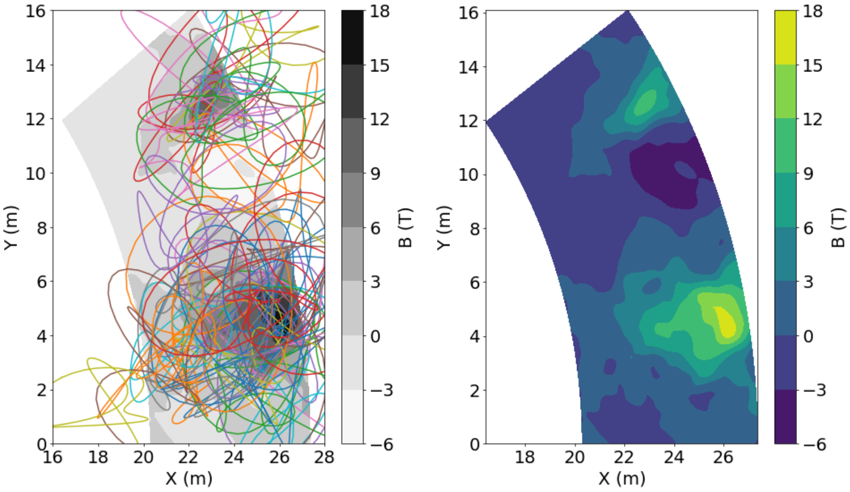

Figure 6: The 35 coil configurations with the corresponding magnetic field distribution.

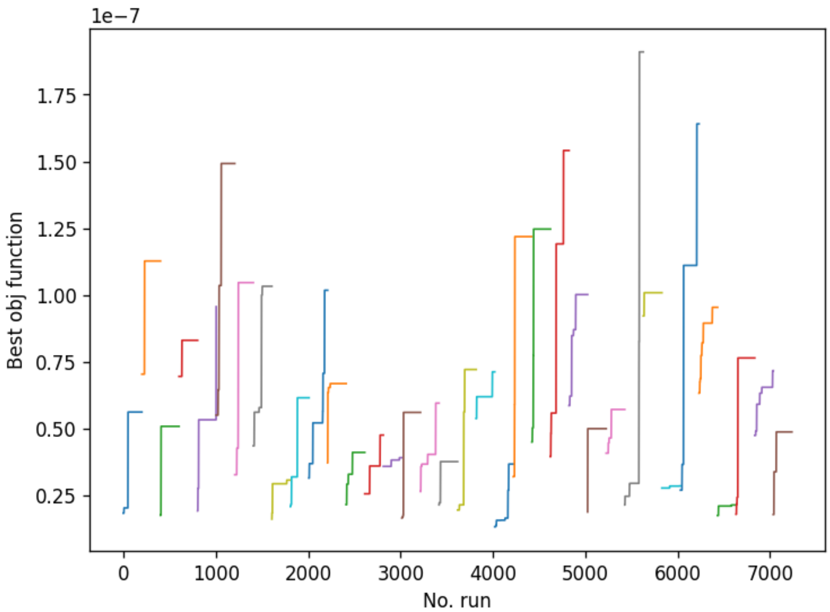

Figure 7: The change of the best objective function with runs for the CTC. Each line represents each coil is added and optimized by BO.

3.2.2 Start from a base model

1. The initial coil configuration is obtained from analytical fitting using least-squared method (LSM) as reported in [2].

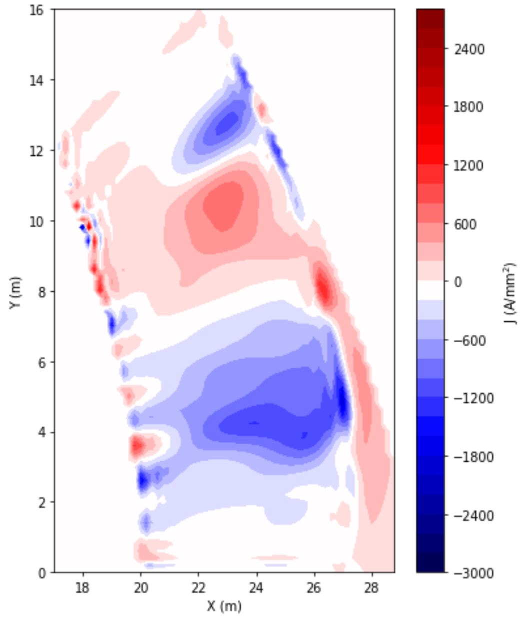

Figure 8: The current density plot from [2].

2. The initial field map and the corresponding objective function (O_i) are recorded and the parameters are as listed below:

$$
\begin{aligned}
X_n &= a'_0 + \sum_{i=1}^{5} a'_i \cos i(\theta + \psi'_i) \\
Y_n &= c'_0 + \sum_{i=1}^{5} c'_i \cos i(\theta + \phi'_i) \\
\text{where } a'_i &= a_i + \Delta a_i \\
c'_i &= c_i + \Delta c_i \\
\psi'_i &= \psi_i + \Delta \psi_i \\
\phi'_i &= \phi_i + \Delta \phi_i \\
j' &= j + \Delta j
\end{aligned} \tag{6}
$$

where a_i, c_i, ϕ_i and ψ_i are obtained from the base model obtained in step 1.

3. Each parameter is set to have a range as follow:
   - Range of Δa_i and Δc_i= [-3,3]
   - Range of $\Delta\psi_i$ and $\Delta\phi_i = [-\frac{\pi}{2}, \frac{\pi}{2}]$
   - Range of Δj = [-50, 50] A/mm^2
4. Keep the remaining coils untouched, optimizes only the first coil (n=1) using the BO by fine tuning 19 parameters mentioned.
5. It is iterated over 200 runs. The best objective functions (O_f) throughout the run are recorded.
6. If $O_f > O_i$, then the latest coil configuration is kept, else the initial configuration before BO is kept (i.e. $a_i' = a_i$, $c_i' = c_i$ etc ...)
7. Steps 4-6 are repeated for the next coil up to the 36$^{\text{th}}$ coil. The final coil configurations are given in Fig. 9.

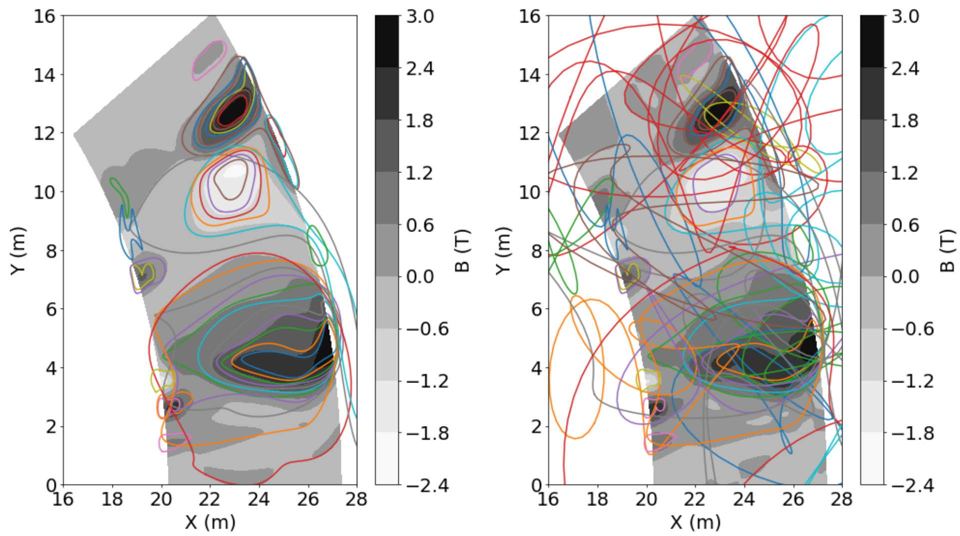

Figure 9: The coil configuration before(left) and after(right) the BO routine of the 35 pairs of coils.

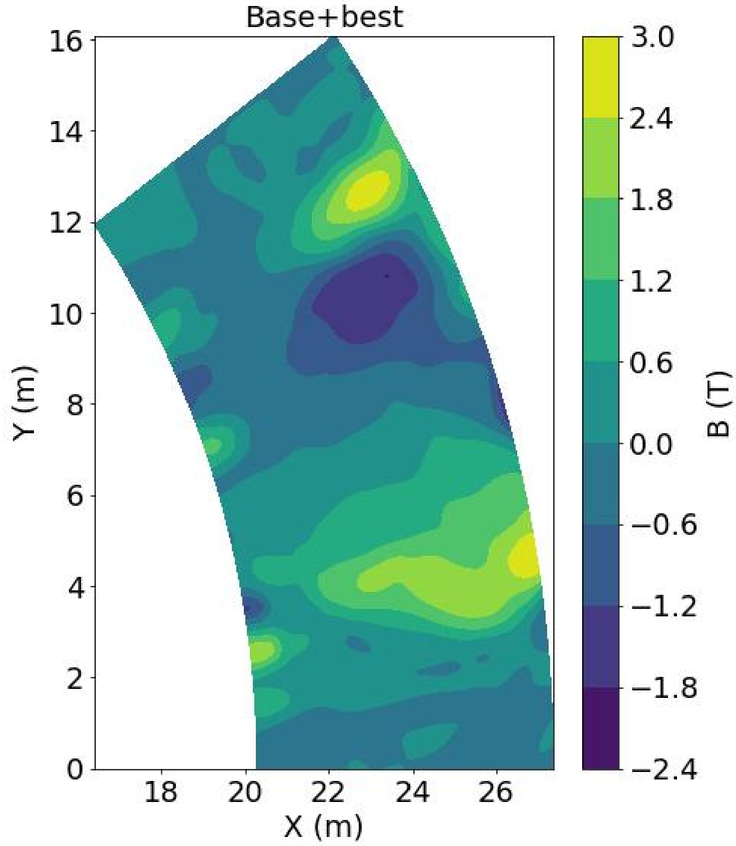

Figure 10: The magnetic field distribution of the CTC from 35 pairs of coils.

4 Summary

This is a work under progress due to the poor performance when the number of coils increased to 35. One of the main reasons of the poor performance is related to the objective function, as the value of the objective function dropped when a new coil is added. Possible solutions to improve this includes changing the objective function, introducing restriction to the input parameters to avoid the crossing of coils, and configure a new way to parametrize the coil.

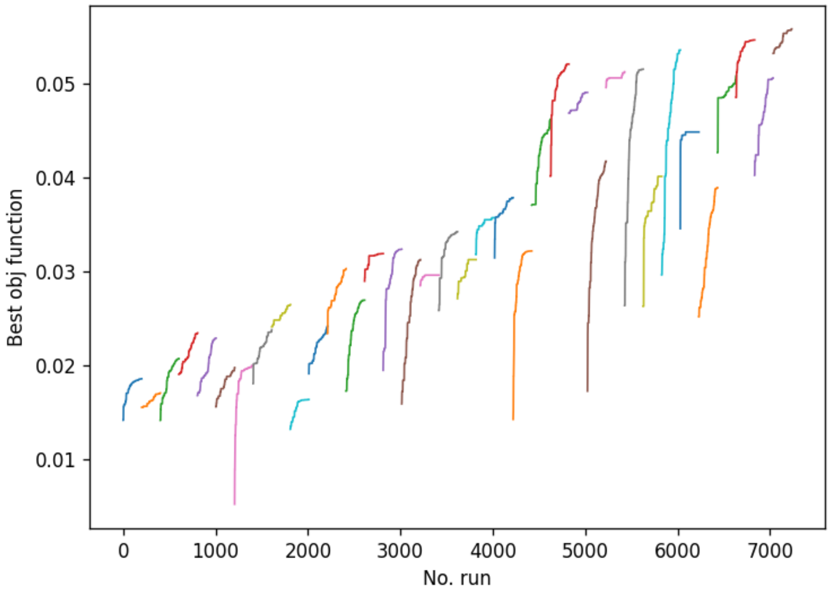

Figure 11: The change of the best objective function with runs for the CTC. Each line represents each coil when it is optimized by BO.

References

[1] Thomas Planche. "Constant-tune cyclotrons". In: *Journal of Instrumentation* 18.03 (Mar. 2023), P03019. DOI: 10.1088/1748-0221/18/03/P03019. URL: https://dx.doi.org/10.1088/1748-0221/18/03/P03019.

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[3] Maximilian Balandat et al. "BoTorch: A Framework for Efficient Monte-Carlo Bayesian Optimization". In: *Advances in Neural Information Processing Systems 33*. 2020. URL: http://arxiv.org/abs/1910.06403.