TRIUMF Beam Physics Note
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Primary beamline vacuum requirement from Rutherford scattering

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Abstract: Mathematica notebook for calculating Rutherford scattering.

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Introduction:

Please use link to Mathematica notebook in abstract to calculate fraction of beam scattered. Customize by choosing your own beamline length, proton energy, scatter angle, temperature, etc. The example given uses 10^{-5} Torr, 0.1 mrad scatter angle, 80 m beamline length, 480 MeV protons on diatomic oxygen. In this example, the fraction scattered is 13 parts per million.

Initialize constants:

$\mathbf{cm = 0.01metre; inch = 2.54cm; Torr = 133.Pascal;}$
$\mathbf{Pascal = Joule/metre^3 ; c = 3. \times 10^8 metre/sec;}$
$\boldsymbol{E_p = 938. \times 10^6 eV; e_- = 1.6 \times 10^{-19} coulomb;}$
$\boldsymbol{r_p = e_- c\ (c/10^7)\ sec^2/metre\, eV/coulomb/E_p ;}$
$\boldsymbol{k_B = 1.38 \times 10^{-23}\ Joule/Kelvin;}$

Kinematic:

$$\boldsymbol{E_E = 480.\ 10^6 eV; Z = 8; \gamma{:=}1 + \frac{E_E}{E_p}; \beta{:=}\frac{\sqrt{\gamma^2-1}}{\gamma};}$$

Classical proton radius: (1.535×10^{-18}) metre

$\beta = 0.7499$

$\sigma_{\rm tot}$ is the total cross section of scattering to angle larger than θ_0

$$\boldsymbol{\sigma_{tot} = 4\pi \left(\frac{r_p Z}{\gamma\beta^2\theta_0}\right)^2;}$$

Total Cross Section of scattering to angles larger than θ_0: (2.62×10^{-25}) metre^2

NN is number density according to ideal gas law.

$\mathbf{NN{:=}\frac{PV}{k_B T}2;}$ **(*assumes diatomic molecules*)**

Ambient conditions:

$\boldsymbol{L = 80metre; T = 300Kelvin; P = 10^{-5}Torr; \theta_0 = 0.00010;}$

"Thickness" of total beamline length is τ:

$$\boldsymbol{\tau = L\frac{NN}{V};}$$

Thickness (nuclei per unit area)= $\frac{5.14\times10^{19}}{\rm metre^2}$

Fraction scattered:

$\boldsymbol{f_p = \sigma_{\mathbf{tot}} \tau;}$

For a proton beam of length 80 metre, the fraction scattered with angles larger than 0.1 milliradian is

$$f_p = 0.0000135.$$