



Design Note TRI-DN-15-28
Electro-mechanical Aspects of the
ARIEL-II HRS Dipole Magnet

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Author(s): George S. Clark, P.Eng.

	Name:	Review Scope	Signature:	Date:
Author:	George Clark, P.Eng			
Reviewed By:	Marco Marchetto			
	Jim Maloney			
	Rick Baartman			
	Doug Preddy			
	Franco Mammarella, P.Eng.			
	Isaac Earle, P.Eng.			
Accepted By:	Reiner Kruecken			
Approved By:	George Clark, P.Eng.			

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1 Abstract

This Design Note describes the electrical and mechanical aspects of the ARIEL High Resolution Separator/Spectrometer [HRS] Dipole Magnet.

Note: Do not rely on the information contained in this note without first contacting the author to discuss your use of the information.

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2 Introduction

The radioactive ion beams from the ARIEL targets are to be analyzed, purified, and separated using two nearly identical dipole magnets. The beam optics of the spectrometer and the magnetic aspects of the magnet are described in [1]. The goal is to have a mass resolution of 1/20000.

Figure 1 shows a section through the magnet with its' vacuum chamber.

3 Base Specification

Bend Angle:	90 degrees horizontally
Reference trajectory radius:	1200 mm
Maximum Field:	4.7 kGauss
Main Gap:	70.0 mm (includes a vacuum chamber allowance)
Uniformity:	+/- 5E-5 on the integrals thru the magnet +/- 180 mm from the reference trajectory
Pole Shape:	Symmetrical sector with curved entrance and exit. Pole has a 6.7 mm Purcell Filter ¹ .
Magnet Form:	H-Frame
Coil Size constraint	< 185 mm vertically and < 81 mm horizontally
Operating Frequency:	DC
Power Supply constraint:	< 200 A and <~100 V
Environment:	North Basement of the ARIEL Building, standard epoxy fibreglass coils may be used.
Other Concerns:	provisions for temperature control/stability should be provided. The magnet must be small enough to be lowered down the shaft that leads to the ARIEL North Basement [7].

¹ A Purcell filter is an air gap or void between the pole and the yoke through which the flux must pass. The gap height is usually constant. It is used to make the field in the main gap more uniform.

4 Ampere-Turn Calculation

From Banford [2, p.83]

$$4 \cdot \pi \cdot N \cdot I = 10 \cdot H \left(g + \left(\frac{s}{\mu} \right) \right)$$

where H is the field in gauss, g is the gap in cm, s is the path length in the steel (cm), and μ is the permeability. N is the number of turns of conductor in the magnet and I is the current (ampere) in the conductor. If we assume s/μ is 0.02 g and re-arrange we get:

$$N \cdot I = \frac{1.02 \cdot 10 \cdot H \cdot g}{4 \cdot \pi}$$

With the Purcell Filter $g = \sim (70 + 6.7 + 6.7) \text{ mm} = \sim 8.34 \text{ cm}$. $H = 5000 \text{ gauss}$;
 $N \cdot I = \sim 33850 \text{ amp} \cdot \text{turns}$ for 2 coils, $N \cdot I = \sim 16925 \text{ amp} \cdot \text{turns}$ per coil.

In choosing the number of turns in the coil we refer to the following table.

Table 1 For 5 kGauss

I (Amperes)	200	192.3	188	176.3	173
N (Turns)	84.6	88	90	96	98

We choose a 7 turn wide by 14 turn tall coil giving 5 kGauss at about 173 A. The coil would be made up of 7 double pancakes each two layers thick.

5 Conductor Information

We choose a 10 mm x 10 mm square outside dimension hollow copper conductor. The specification for the conductor is

Table 2 Conductor size

Outside Dim.	10 ± 0.1	mm
Inside Diameter	4 ± 0.1	mm
Corner radius	1	mm

Copper alloy 102 dead soft fully annealed temper. This is a standard size [3]. Later we will see that this size of conductor is reasonable for these magnets.

6 The Insulation System

These dipoles will operate in a low radiation field. Therefore standard epoxy-fibreglass insulation will be used.

Turn-to-Turn Insulation: Wrap the conductor with one winding of 0.23 mm thick open weave E-type fibreglass tape half-lapped. The result is each conductor is wrapped with about 0.5 mm insulation and the turn-to-turn insulation is about 1 mm.

Ground Wrap Insulation: Wrap the coil with two windings of 0.23 mm thick open weave E-type fibreglass tape half-lapped. Avoid build-up of tape in corners. The nominal thickness is 0.92 mm. For coil sizing we will use 1 mm thick ground wrap.

Encapsulation: Vacuum Impregnation in a mould using a clear bisphenol A epoxy resin system.

7 Coil Cross-section Width

For most of its circumference the coil will be 7 turns wide.

Table 3: Coil Width

		Min mm	Nominal mm	Max mm
Conductor	$7 \times (10 \pm 0.1) =$	69.3	70.0	70.7
Turn Insulation	$7 \times 2 \times 0.5 =$	7.0	7.0	7.0
Gaps	$(7-1) \times 0.1 =$	0.0	0.3	0.6
Ground Wrap	2×1	2.0	2.0	2.0
		————	————	————
Total		78.3	79.3	80.3

We will use 79.3 ± 1 mm as the coil width except in the transition region. In the transition region some layers may be 8 turns wide (91.5 mm max. width.).

See Section 12 for a discussion of the coil height.

8 Magnet Size

Figures 2, 3, 4, and 5 show some of the dimensions of the magnet. Figure 2 shows a mid-plane section, Figure 3 shows a vertical section, and Figure 4 shows some details of the pole. Figure 5 shows how the pole is bolted to the yoke.

9 Coil Cooling Calculations

Figures 6 and 7 show the coil. The mean turn length of the coil is about 5553 mm. Appendix 1 shows calculations of the following quantities for the coil operating at 173 A.

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Resistance per coil (at 20C)	0.11	ohm
Resistance per coil (warm)	0.12	ohm
Inlet water temperature	30	C
Water Temperature Rise	20	C
Power per coil	3.54	kW
Voltage per coil (warm)	20.5	V
Cooling circuits per coil	7	
Cooling water flow per coil	2.54	litre/min
Pressure drop	~17	psi.
Coil weight	~941	lb.

The two magnets have a total of 4 coils = ~3800 lb.

Figure 9 is a schematic of the coil connections.

The coil tails could go at any of the four corners of the magnet. The “outer” corners are better suited to mounting the vacuum chamber supports, than the “inner” corners. Placing the coil tails at an inner corner gives more clearance when the magnet is lowered down the hatch. We will place the tails at the entrance “inner” corner of the magnet. This is near the small end of the vacuum chamber on the first magnet and near the big end of the vacuum chamber on the second magnet.

10 Extra Cooling

The possibility of extra cooling has been added to the coil to reduce the heat flow to the magnet steel. If the heat flow to the steel is reduced, we expect that thermal distortions of the magnet will be reduced.

Figure 7 shows a cross-section of the coil. A row of extra cooling tubes has been added to the coil between the conductors and the magnet steel. The tubes are 8 mm OD x 1 mm wall. Stainless steel was chosen for its low thermal conductivity. The water flowing through these extra cooling tubes is on a separately chilled loop.

Appendix 2 shows that with a flow of about 1.9 liters/min per coil of 17.5 C water, the heat flow to the steel is reduced from about 167 Watts to 2.4 Watts. This is almost a 99% reduction.

Details:

Tube size:	8 mm OD x 1 mm wall 316 Stainless
Water Flow:	1.9 liters/min per coil
Inlet Temp:	17.5 C
Outlet Temp:	26.5 C
Pressure Drop:	~26 psi.

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The cooling tubes will be grounded and should be insulated similar to the main conductor. Wrap the tube with one winding of 0.23 mm thick open weave E-type fibreglass half-lapped. Then wrap the tube pancake with one windings of 0.23 mm thick open weave E-type fibreglass half-lapped.

The cooling tube tails should be shorted together through a 1000 ohm 1 watt resistor. One tail should be shorted to ground.

In the Mass Separator room the inlet water temperature for the main coil will probably be about 30C held constant to about +- 0.25C. The room air temperature is probably at about 22C held constant to about +-2C [4]. We expect the inlet water temperature to the extra cooling tubes to be controlled to keep the magnet steel temperature constant.

Two other measures may be necessary to keep the magnet temperature constant—enclose the magnets in a tent and blow air through the magnet aperture.

11 Stored Energy and Inductance

At 158.4 A (0.458 T) the energy stored in the magnet (model 23C62eng) is 10827 Joules [5].

$$\text{Inductance } L = \frac{2 \times \text{Stored_Energy}}{I^2} \quad [6, p. 317]$$

$L = 0.86$ Henry.

The coil connections and coil tails shall be insulated where possible. Any bare live parts shall be protected to prevent anyone from inadvertently contacting them. The installer of the magnet shall provide and install a suitable enclosure to protect the coil connections.

The magnet's time constant is

$$\frac{L}{R_{\text{magnet}}} = \frac{0.86 \text{ H}}{0.22 \text{ ohm}} = \sim 4 \text{ seconds} \quad [8, p. 265]$$

12 Sense Coil

Four measurement sources are under consideration for providing feedback signals to the power supply: a direct current current-transformer, a hall probe, an NMR probe, and a sense coil. A 20 turn sense coil may be used to sense fast changes in the magnetic field. The emf in a single turn sense coil is

$$\text{emf} = \int_S \frac{\partial B}{\partial t} dS \quad [6, p. 332]$$

If we assume dB/dt is uniform over surface S, then

$$emf = - \frac{dB}{dt} Ac$$

where Ac is the area inside the coil. If $B = 0.46$ Tesla and we want to measure a change of $B/10000$ in 0.1 seconds then $dB/dt = 4.6E-5$ T/s. Instead of the area of the coil, we will use the area of the base of the pole (1.41 m^2 for model 23C62eng9 [14]). For a one turn sense coil

$$V = 4.6 \times 10^{-5} \frac{T}{s} \times 1.41 \text{ m}^2 = \sim 6.5 \times 10^{-5} \text{ volt.}$$

For a 20 turn coil $V = \sim 0.0013$ volt.

The change in the power supply current to correct $dB/dt = 4.6E-5$ T/s would be

$$4.6 \times 10^{-5} \frac{T}{s} \times \frac{158.4 \text{ A}}{0.458 \text{ T}} = 0.016 \frac{\text{A}}{\text{s}}$$

The change in the power supply voltage would be

$$\Delta V = L \times \frac{di}{dt} = 0.86 \text{ H} \times 0.016 \frac{\text{A}}{\text{s}} = 0.014 \text{ volt.}$$

Sense coil details

Conductor:	14 AWG square copper
Bare conductor size	1.628 mm square [9, p.40]
Maximum Insulated size:	1.781 mm square heavy build [9, p.43]
Resistance	6.872 ohms/km (nominal) [9, p. 41]
Mean turn	~ 5500 mm
Length (20 turns)	~ 110 meters
Resistance (20 turns)	0.76 ohms

If the magnet were to trip off, a voltage would be induced in the sense coil. If no additional resistance was added, then the main coil current would decay like

$$I(t) = I_0 e^{-\frac{R_{magnet}}{L} t}$$

The maximum rate of change would be

$$- I_0 \frac{R_{magnet}}{L}$$

The maximum rate of change of B would be

$$-\frac{0.458 T}{158.4 A} I_0 \frac{R_{magnet}}{L} = -\frac{0.458 T}{158.4 A} \times 173 A \times \frac{0.22 ohm}{0.86 H} = -0.128 \frac{T}{s}$$

A magnet trip would induce about

$$V = 20 \times 0.128 \frac{T}{s} \times 1.41 m^2 = 3.6 volt.$$

If the sense coil is not used, it should be shorted with a 1000 ohm resistor with power rating greater than 1 watt. One end of the sense coil should be shorted to ground.

The sense coil will be built into the lower coil, see figure 7.

13 Coil Height

The upper coil will be 14 layers high plus the extra cooling tubes.

Table 4: Upper Coil Height

		Min mm	Nominal mm	Max mm
Conductor	14 x (10 ± 0.1)	138.6	140.0	141.4
Turn Insulation	14 x 2 x 0.5	14.0	14.0	14.0
Gaps	(14-1) x 0.1	0.0	0.7	1.3
Keystoning	14 x 0.4	0.0	5.6	5.6
Ground Wrap	2 x 1	2.0	2.0	2.0
X-Cooling Tubes		8.0	8.0	8.0
X-Cooling Turn Insulation	1 x 2 x 0.5	1.0	1.0	1.0
X-Cooling Ground wrap	2 x 0.5	1.0	1.0	1.0
		————	————	————
Total		164.6	172.3	174.3
Rubber Pad		6.0	6.0	6.0
		————	————	————
Total + rubber		170.6	178.3	180.3

Keystoning of 0.4 mm per layer is based on a 40 mm minimum bend radius [10]. There will be a 6 mm thick rubber pad between each coil and the yoke. We will use 172.3 ± 2 mm for the height for the upper coil.

The lower coil also includes the sense coil.

Table 5: Lower Coil Height

		Min mm	Nominal mm	Max mm
Conductor	14 x (10 ± 0.1)	138.6	140.0	141.4
Turn Insulation	14 x 2 x 0.5	14.0	14.0	14.0
Gaps	(14-1) x 0.1	0.0	0.7	1.3
Keystoning	14 x 0.4	0.0	5.6	5.6
Ground Wrap	2 x 1	2.0	2.0	2.0
X-Cooling Tubes		8.0	8.0	8.0
X-Cooling Turn Insulation	1 x 2 x 0.5	1.0	1.0	1.0
X-Cooling Ground wrap	2 x 0.5	1.0	1.0	1.0
Sense coil	14 AWG heavy build	1.7	1.7	1.8
0.5 mat		0.5	0.5	0.5
		_____	_____	_____
Total		166.8	174.5	176.6
Rubber Pad		6.0	6.0	6.0
		_____	_____	_____
Total + rubber		172.8	180.5	182.6

We will use 174.5 ± 2 mm for the height of the lower coil.

The pole height (pole face to yoke distance) is 190 mm. The pole height is 7 mm taller than the maximum size of the bottom coil+rubber.

14 Insulation Rating

The highest voltages the coils are likely to see are from the “High-Pot” test during manufacture. This test will be in the 1000 to 2000 Volt range. For the purposes of this section we will assume 2000 Volts. The coil design is more complicated than usual. There are three distances between the main conductor and ground:

Main conductor to the yoke	1.5 mm
Main conductor to extra cooling tube	1.5 mm
Main conductor to sense coil	1.07 mm = 0.042 inch

The insulation between the conductor and ground will experience a voltage stress of

$$2000 \text{ V} / 0.042 \text{ inch} = \sim 47,500 \text{ volt/inch} = \sim 48 \text{ volt/mil}$$

The coils are wound as double pancakes, 7 two layer double pancakes per coil, with 7 turns per layer. If an open circuit develops while the magnet is operating, it is possible that an arc will develop. If we assume the arc voltage is 2000 Volt, then the maximum inter-turn stress would be

$$2000 \text{ V} \times (14 / 98) / 0.039 \text{ inch} = 7,326 \text{ volt/inch} = \sim 8 \text{ volt/mil.}$$

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NEMA grade G-10 and G-11 are epoxy-fiberglass insulation systems similar to the coil's insulation system. G-10 has a short time dielectric strength (0.063 inch) of 500 volts/mil [11, p.275]. The insulation specified should be sufficient.

15 Temperature Switches

Each coil shall be protected by normally closed temperature switches interlocked to the power supply. The switches shall open at 71C and re-close on falling temperature at about 60C. Radiation hard, hermetically sealed, switches like the KLIXON 4344-13 (stud mount) style [12] are recommended.

16 Magnet Weight

We calculate the magnet's weight below, based on OPERA-3D [13] model 23C62eng21 [14].

Table 6. Magnet Weight

Part	Quantity	Weight each Lb.	Weight Lb.
Poles	2	4360	8720
Horizontal Yokes	2	8725	17450
Outer Side Yoke	1	2770	2770
Inner Side Yoke	1	2100	2100
Field Clamp	4	110	440
Coils	2	950	1900
Vacuum Box	1	200	200
Miscellaneous	1	120	120
Total			33700

17 Lifting the magnet

If we mount 3 lifting devices on the top yoke, equidistant from and centred on the center of gravity, then each device must be rated for at least

$$34000 \text{ lb} / (3 \times \sin 45) = 16030 \text{ lb. [7290 kg]}$$

The ADB 34402 Heavy Duty Hoist Ring is rated at 11000 kg and attaches with one M36x4 socket head bolt [15]. If the rigging angle is 45° or less, the horizontal load seen by each bolt will be less than

$$34000 \text{ lb} / 3 = 11333 \text{ lb.}$$

Appendix 3 shows the tensile stress area for M36x4 bolts is 795 mm² [1.233 inch²], so the shear stress due to the side load (ignoring thread stresses) is 11333 lb/1.233 inch² = 9192 psi.

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The vertical rigging load per bolt would be about

$$34000 \text{ lb} / 3 = 11333 \text{ lb.}$$

ADB recommends a preload torque of 1085.5 N.m. This provides a preload force of about 33897 lb. which is about 3 times the load lifted. The direct stress in the bolt would be about $33897 \text{ lb} / 1.233 \text{ inch}^2 = 27491 \text{ psi}$. Using a Mohr's circle type analysis (Fig. 10) the maximum shear and direct stresses would be

$$\tau_{max} = \sqrt{9192^2 + \left(\frac{27491}{2}\right)^2} = 16536 \text{ psi.}$$

$$\sigma_{max} = \frac{27491}{2} + \tau_{max} = 30300 \text{ psi}$$

The bolt has an ultimate tensile strength of 180000 psi. The bolt's factor of safety = $180000/30300 = \sim 5.9$.

We specify a thread engagement of 90 mm. This thread engagement makes the hoist ring a special order. The standard engagement is only 67 mm. Appendix 3 shows the static strength of the Nut threads to be 169000 lb. The nut's factor of safety = $169000 \text{ lb} / 33897 \text{ lb} = \sim 5$.

The magnet will be constructed from ArcelorMittal USA HP Magnet Plate Steel [1]. In calculating the strength of the nut threads, we use the Yield Strength (25,000 psi.) and Tensile Strength (35,000 psi.) supplied by ArcelorMittal. These properties are not guaranteed; they "have been obtained through testing." [17]

18 Structural Calculations

The magnet is bolted together. When the magnet is lifted, the bolts taking the largest load, other than the hoist ring bolts, are the bolts connecting the top yoke to the side yokes. We assume the load is carried by 8 bolts, 4 to the outer side yokes, 4 to the inner side yokes. These are specified as M20 x 2.5 pitch x 150 long A4 (316 stainless) class 70 socket head capscrews. The thread engagement is 43 mm.

$$\text{Load per bolt} = \sim (34000 - 4360 - 8725) \text{ lb} / 8 = 2615 \text{ lb.}$$

Appendix 4 shows the static strength of the bolt is about 37000 lb and the static strength of the nut threads is about 43600 lb. Torque these bolts to 200 N.m.

The upper pole is suspended from the top yoke with ten M12 x 1.75 pitch x 150 long A4 (316 stainless) class 70 socket head capscrews. See figure 5. The thread engagement is 18 mm. Load per bolt = $4360 \text{ lb} / 10 = \sim 440 \text{ lb}$. Appendix 5 shows the static strength of the bolt is about 12635 lb and the static strength of the nut threads is about 10000 lb. Torque these bolts to 24 N.m.

19 Magnetic Forces and Deflections

OPERA-2D [13] was used to investigate the magnetic forces on the poles based on the 23C48 design. See Figure 8. The *net* magnetic force on each pole is $8.204 \text{ N/mm} \times 1200 \text{ mm} \times \pi/2 = 15464 \text{ N} = 3477 \text{ lb.}$ towards the closest horizontal yoke.

The attractive magnetic force between the poles is about $64.2 \text{ N/mm} \times 1200 \text{ mm} \times \pi/2 = 121 \text{ kN} = 27214 \text{ lb.}$

For the upper yoke, the magnetic force and the gravity force act in the same direction. For the lower yoke, the magnetic force opposes the gravity force.

The top and bottom yokes are treated as fixed end beams. The magnetic forces at the joint with the side yokes will clamp the yokes together acting like a fixed end beam. The complex shapes of these yokes are simplified as shown in Figure 11.

To calculate the pole deflection, the pole was straightened out and treated as a rectangular beam. Each pole is supported at 10 places. Deflection calculations were made in both the long and short directions. See Figure 12.

The tables below summarize the deflection calculations:

Description	Deflection meters + = up	Details in
Top Yoke deflection	-1.68E-6	Appendix 6
Top Pole deflection—Long direction	-1.37E-9	Appendix 7
Top Pole deflection—Short direction	-1.74E-8	Appendix 8
Total	-1.7E-6	

Description	Deflection meters + = up	Details in
Bottom Yoke deflection	7.15E-7	Appendix 6
Bottom Pole deflection—Long direction	-8.85E-8	Appendix 7
Bottom Pole deflection—Short direction	-1.18E-6	Appendix 8
Total	-5.5E-7	

These deflections should be compared to the flatness tolerance on machining the poles (10 micrometer flatness over +/- the 200mm good field region). The flatness tolerance was determined by negotiation [16].

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20 Segmenting the Side Yokes

To save material costs, the side yokes will be segmented as shown in figure 13. If the side yokes are not segmented, the steel needed is thicker than the mill can produce. Segmenting reduces the thickness, and reduces the volume of steel needed by about 43%.

21 Field Mapping

Each magnet is to be field mapped twice as part of acceptance tests at the magnet factory [18]. The first mapping is to be fast with a coarse spacing (20 mm) between measurements, the second mapping is to have a smaller spacing (5 mm) between measurements. The coarse map is expected to take less than a day and avoid any drift problems that might show up in the fine map.

Figure 14 shows the region to be mapped and a typical mapping/integration path. The size and shape of the magnet makes it difficult to map from one side, so mapping from two sides with overlap is expected. Integrals of B along each path are compared to the expected values for the path. For acceptance the Field Integral Error must be $< 1E-4$.

$$\text{Field Integral Error} = \left| \frac{[\int B_{meas} dl] - [\int B dl]_{expected}}{[\int B dl]_{expected}} \right|$$

$$\left[\int B dl \right]_{expected} = B(0,1200) * Li(Ri)$$

$$Li(Ri) = \text{Hard Edge Length of the magnet at radius } Ri$$

The beam physics group is considering developing a more accurate method of measuring the field integrals once the magnets arrive at TRIUMF.

22 Notes

This design is to be used at TRIUMF's Main Wesbrook Site only. During operation the yoke shall be grounded in accordance with the Canadian Electrical Code. The coil connections shall be guarded as noted above.

The power supply and the current leads should be designed to deliver at least 200 A and 48 volts at the magnet.

Both the overall assembly and the coil drawings should note that

1. the maximum current is 200A
2. a cooling water flow of at least 2.54 litre/minute per coil is needed at 173 A. (3.4 litre/min at 200 A).

23 Review

Franco Mammarella, P.Eng, reviewed the electrical aspects of this design. Isaac Earle, P.Eng, reviewed the mechanical and structural aspects of the design.

24 Summary

Air Gap	70.0	mm
Maximum Field	0.47	Tesla
Effective Length at 1200 mm R	1.885	m
Bend Angle	90.0	degrees
Bend Radius	1.2	meter
Pole Shape	symmetrical sector with curved entrance and exit	
Coil mean turn length	5.553	m
Turns/coil	98	
Coil Array (W x H)	7 x 14	
Cooling	7	cct./coil
Top Yoke thickness	205	mm
Weight per coil	950	lb.
Weight steel assembly	31540	lb.
Weight overall assembly	34000	lb.
Maximum DC Current	200	A
Average Coil Temp at 173 A	40	C
Magnet Resistance at 40C	0.24	ohm
Magnet Resistance at 20C	0.22	ohm
Magnet Water Flow at 173 A	5.1	liter/minute
Water Pressure Drop	18	psi.
Magnet Power at 173A	7.1	Kilowatt
Magnet Inductance	0.86	H
Magnet Voltage at 173 A	41	V

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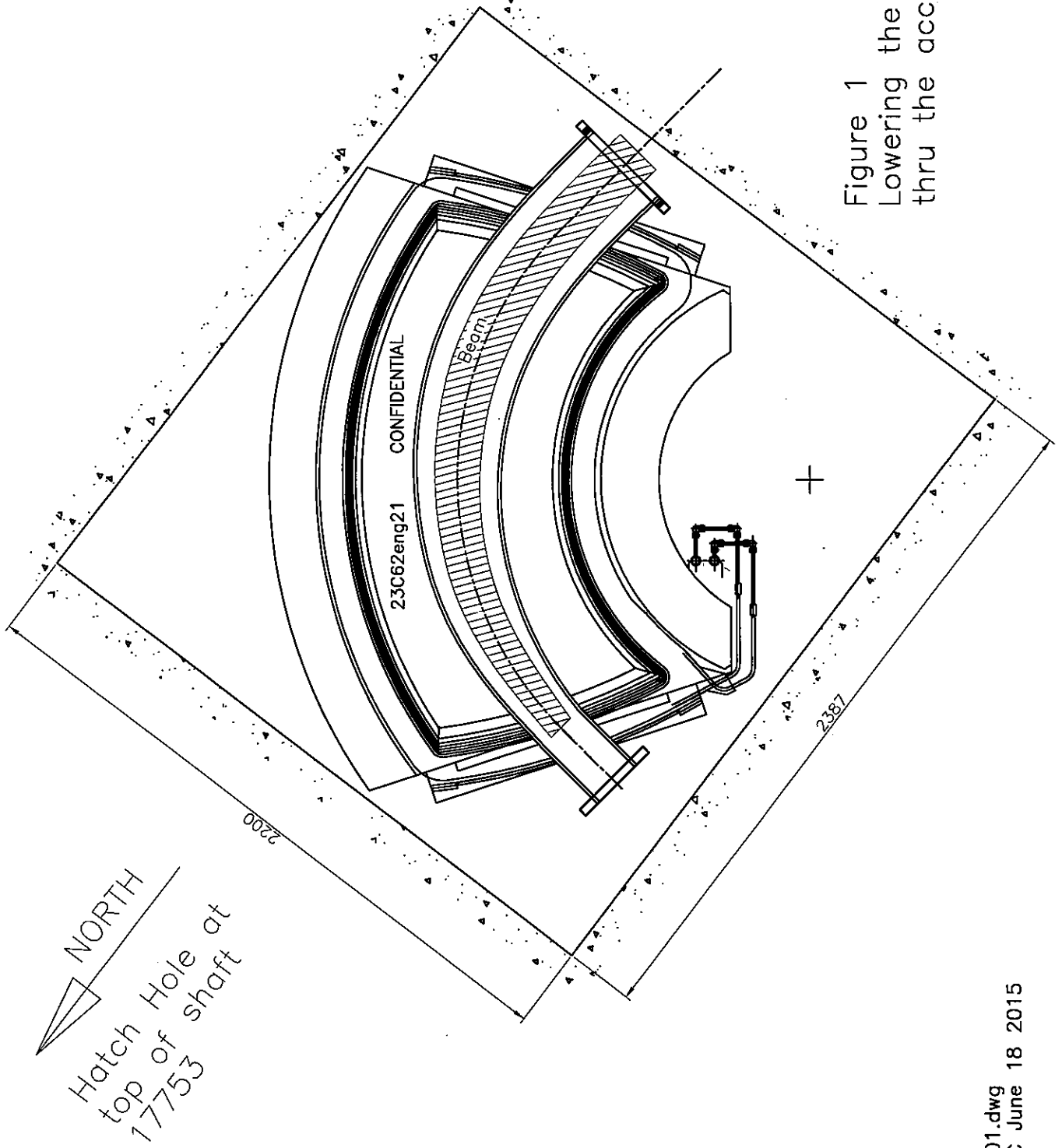
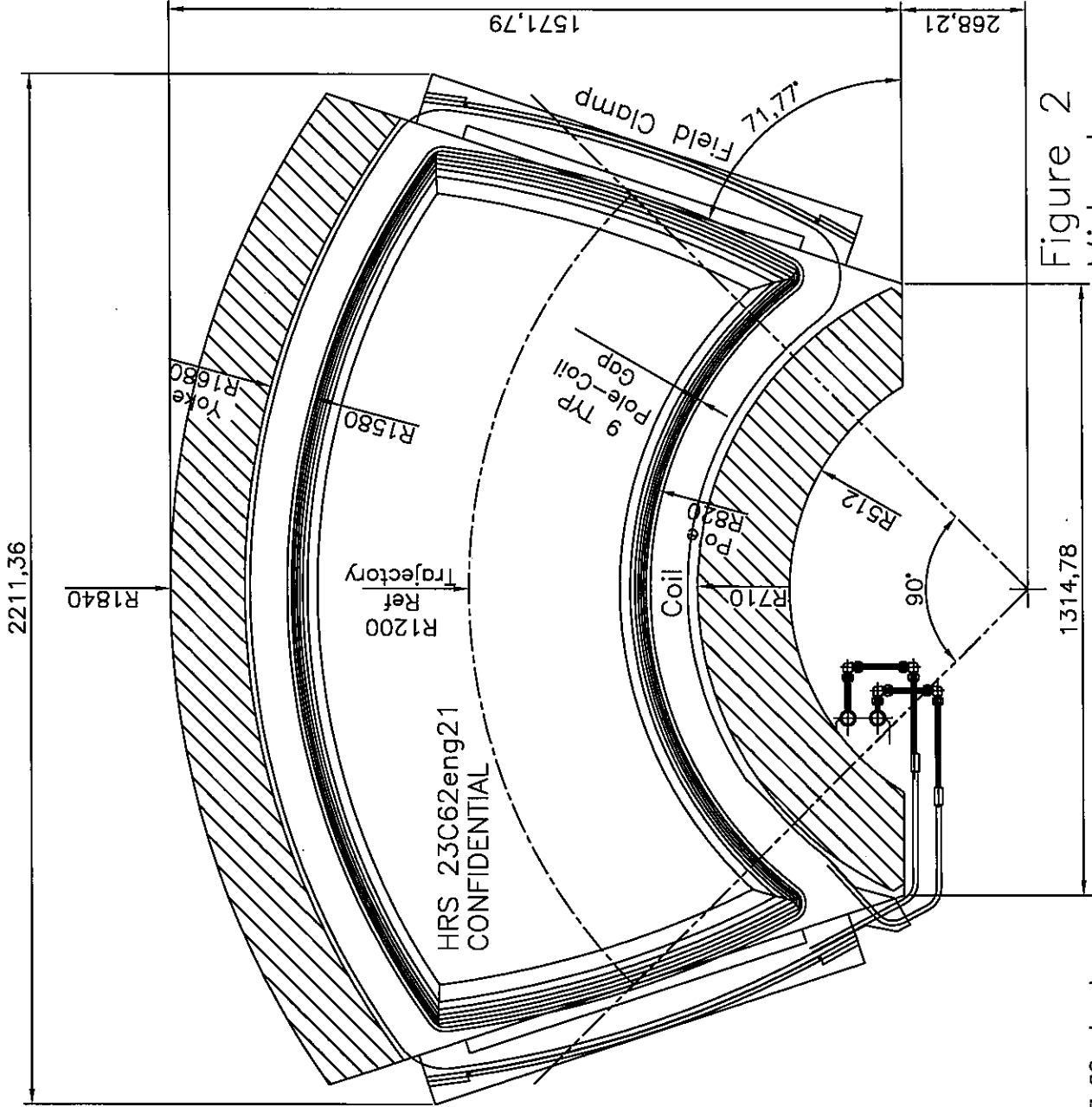


Figure 1
Lowering the HRS Magnet
thru the access hatch.



JUN 18 20

Figure 2
Mid-plane section

Fig02_plan.dwg
GSC June 18 2015

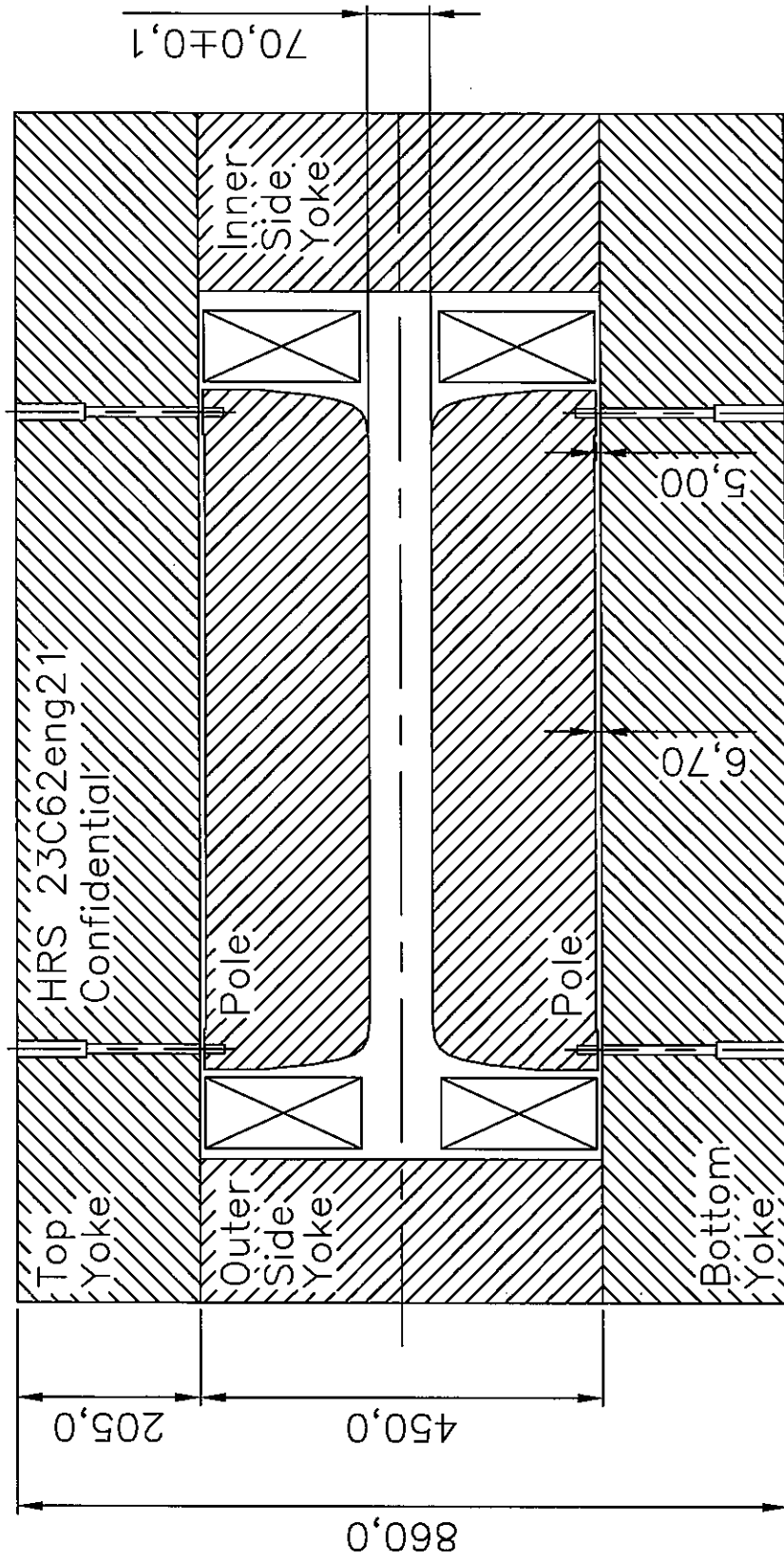


Figure 3. Vertical Section

Fig03_section.dwg
GSC May 28 2015

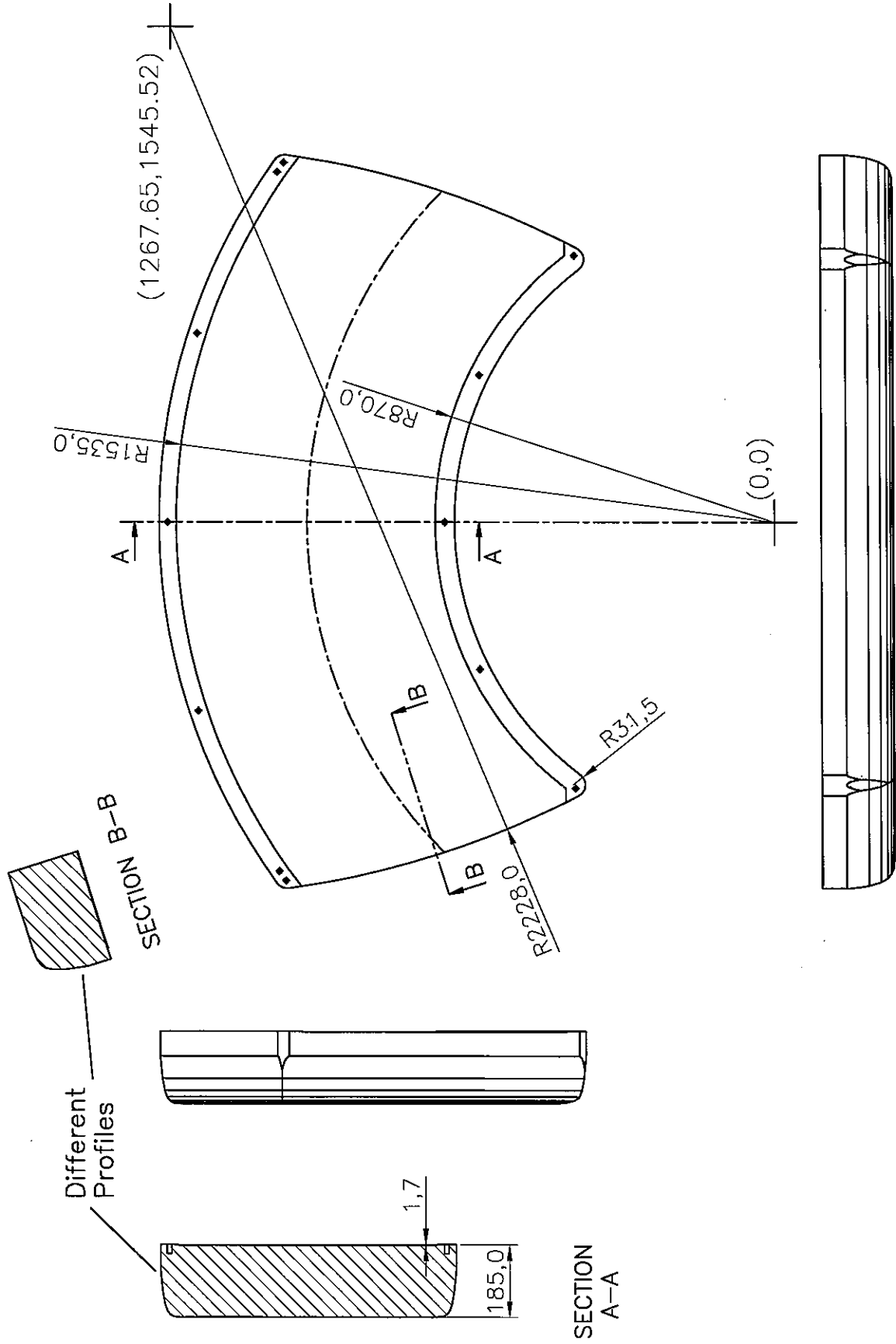
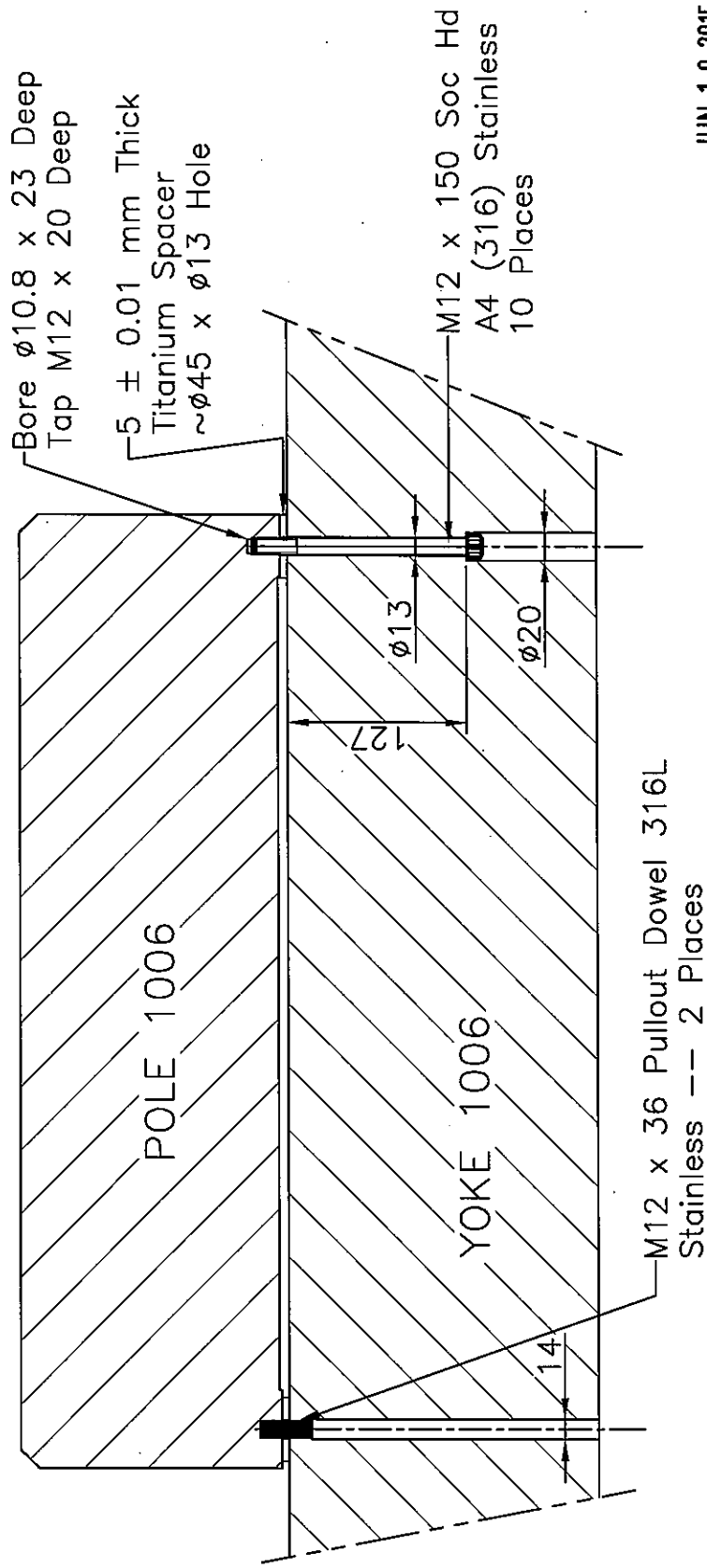


Fig04_Pole.dwg
GSC June 18 2015

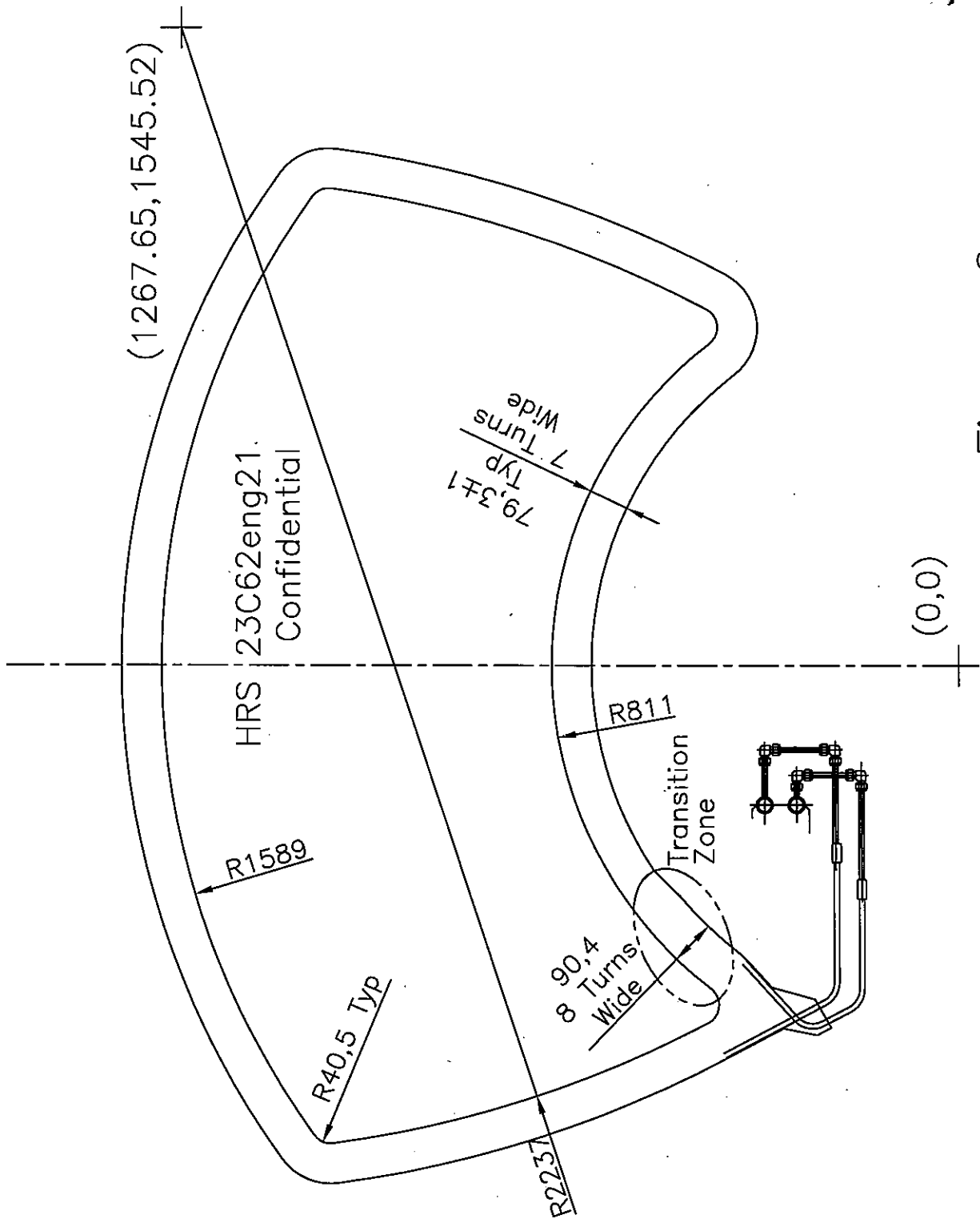
Figure 4. Pole details



JUN 19 2015

1/5 Scale
 GSC 150610
 Fig05PoleMount.dwg

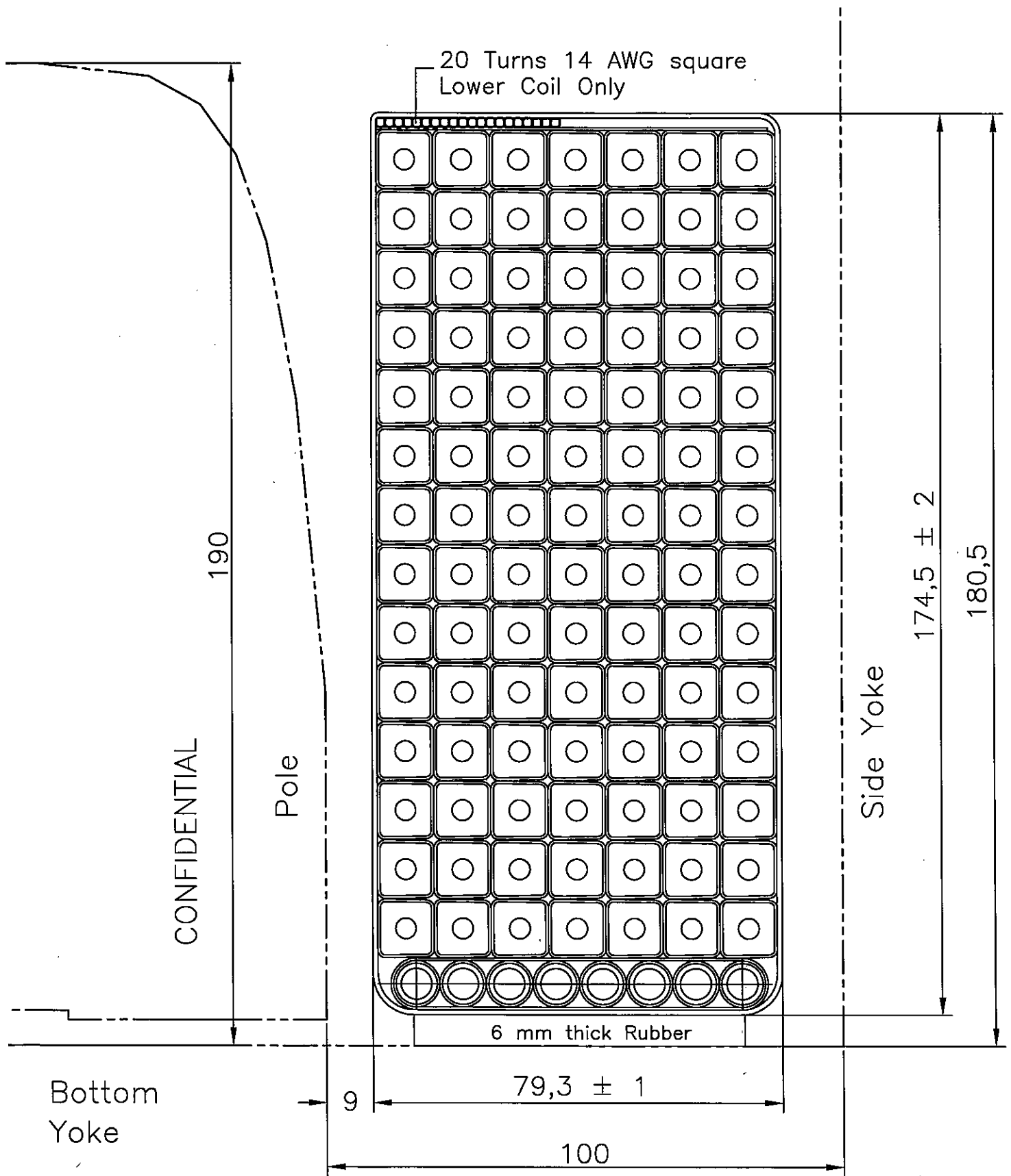
Figure 5. Pole Support



JUN 19 2015

Figure 6.
Coil Plan View

Fig06_CoilPlan.dwg
GSC June 18, 2015



JUN 19 2015

Figure 7. Coil Cross-section

Fig07_Coil_CS.dwg GSC June 19, 2015

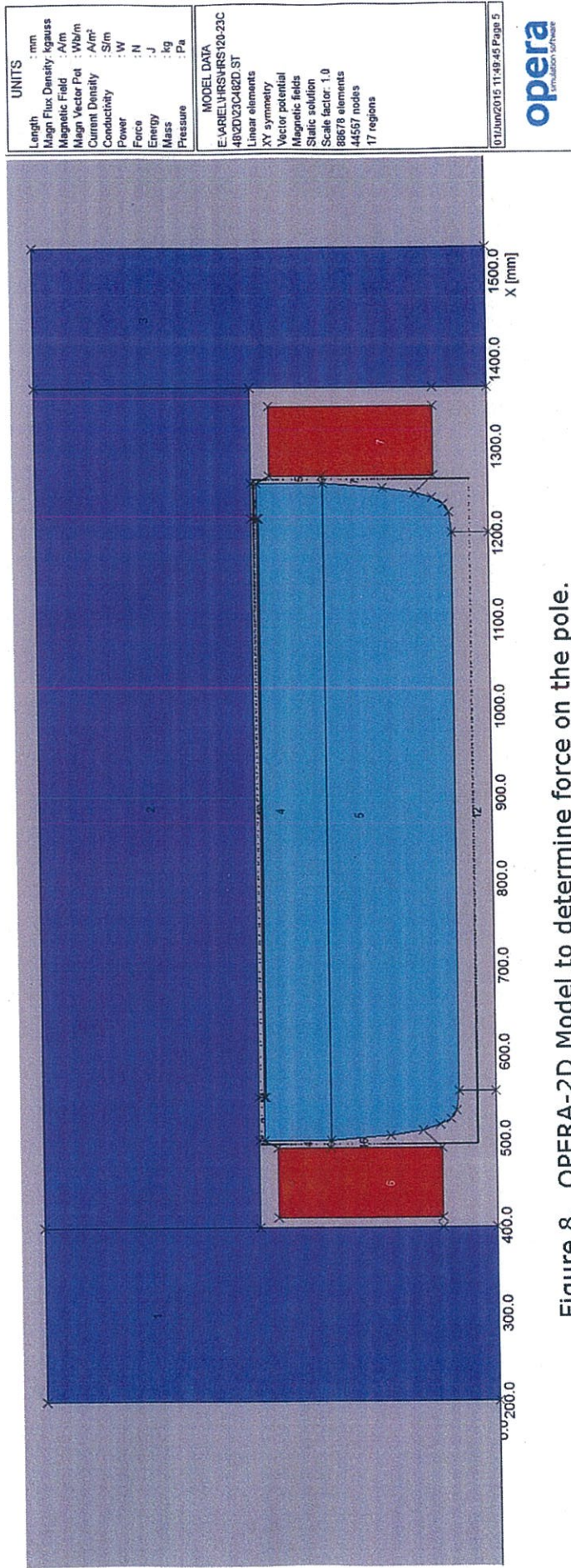


Figure 8. OPERA-2D Model to determine force on the pole.

CONFIDENTIAL

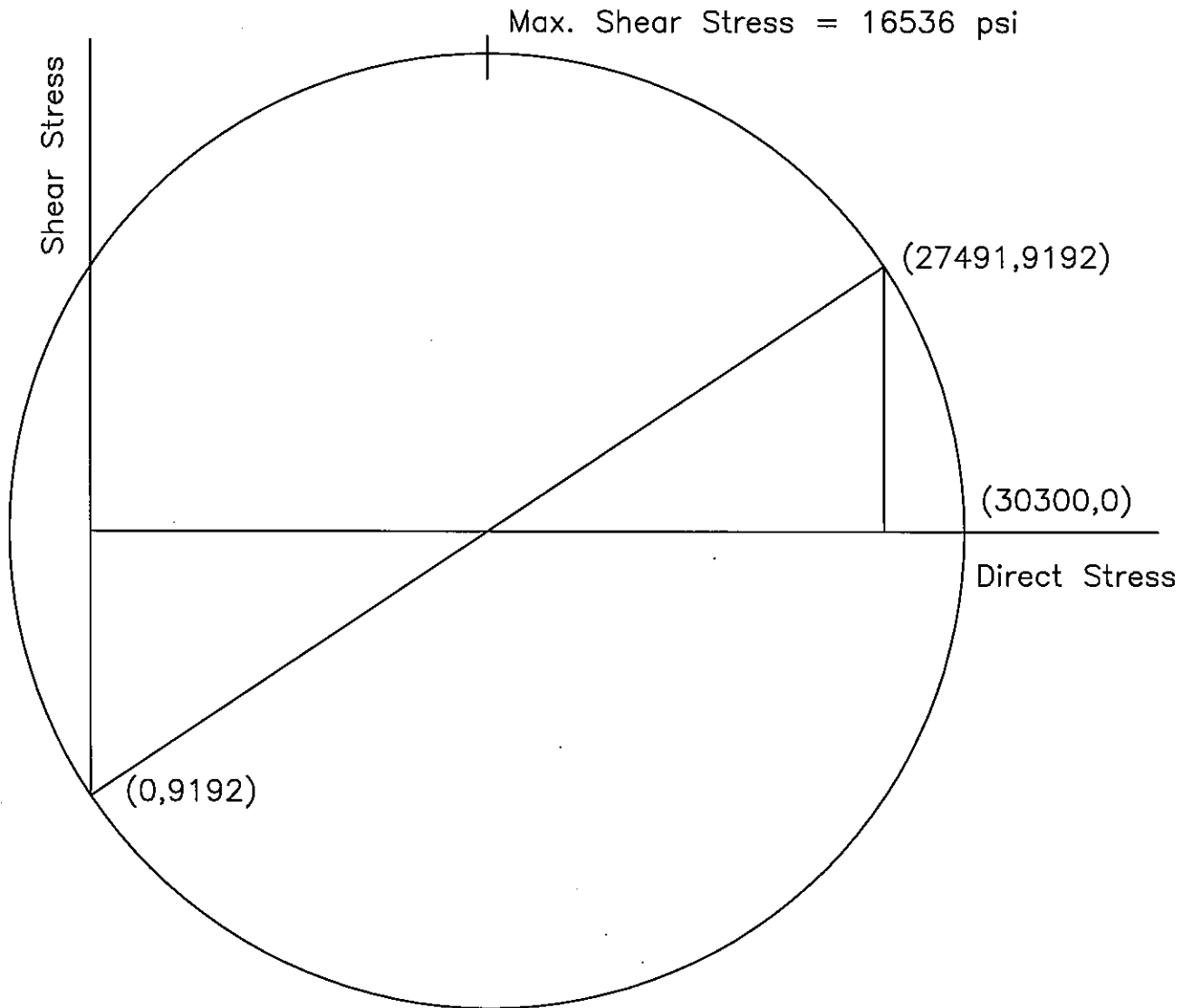


Figure 10. Mohr's Circle for the Hoist Ring Bolt.

Fig10.dwg GSC June 1, 2015

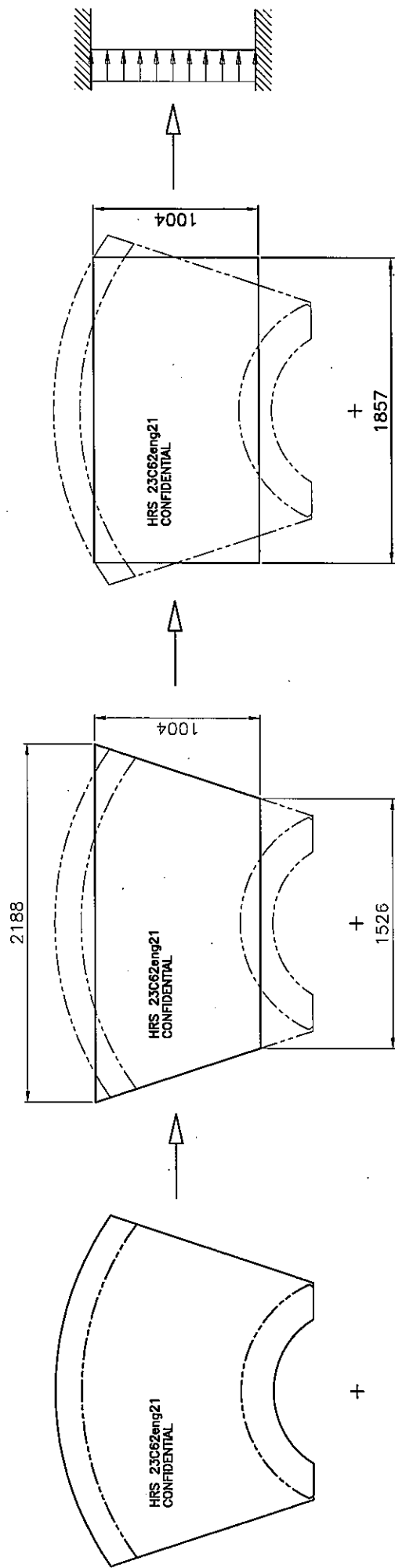


Figure 11. Simplifying the Top/Bottom Yoke shape.

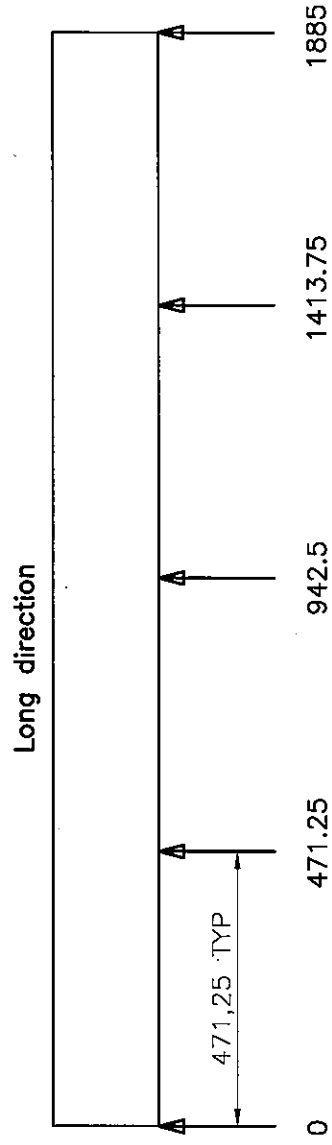
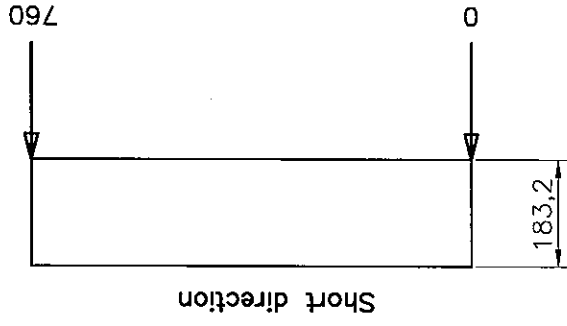
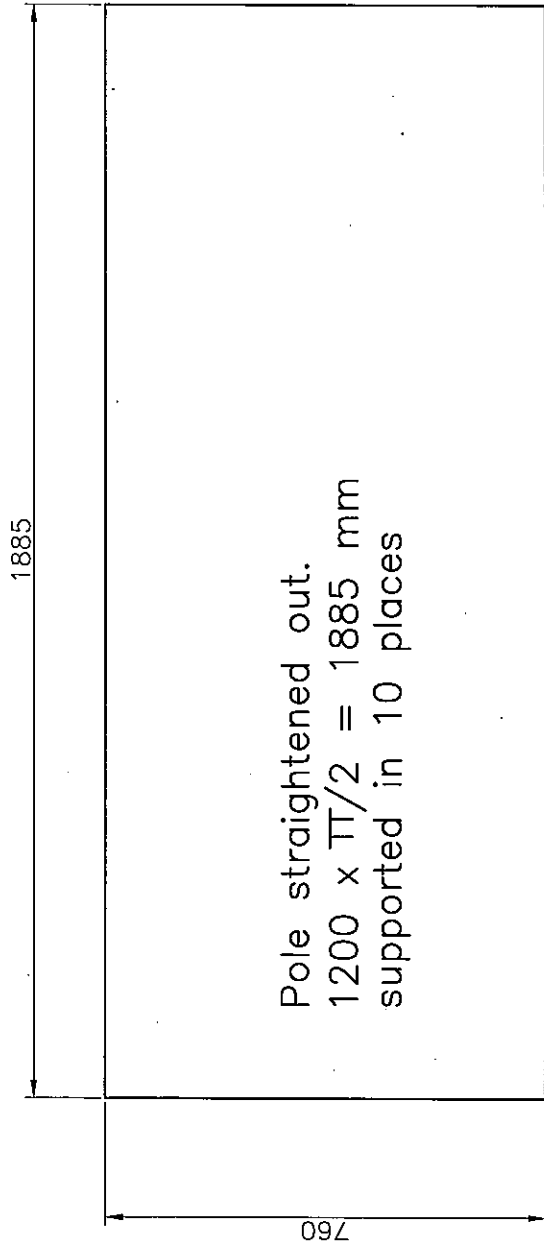
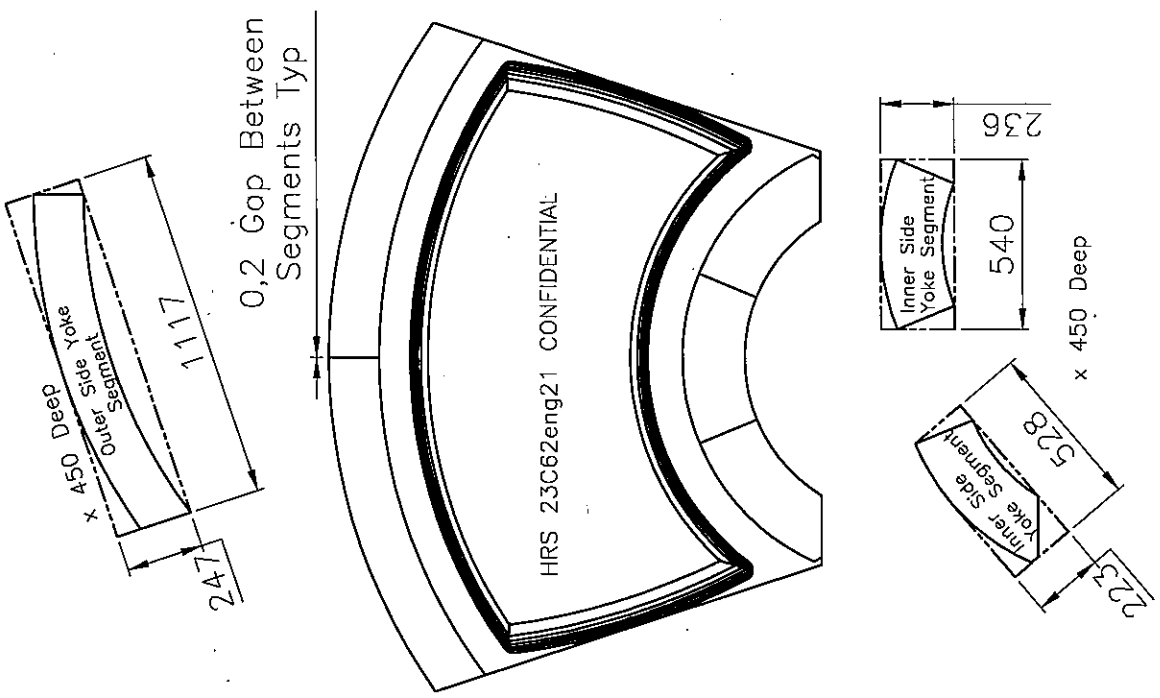


Figure 12. Simplified Pole for deflection calculations

Fig12_StraightPole.dwg GSC June 2, 1025

JUN 19 2015



Aprox. blank size with segmented side yokes

Aprox. blank size without segmented side yokes

Figure 13. Segmenting Side Yokes

JUN 19 2015

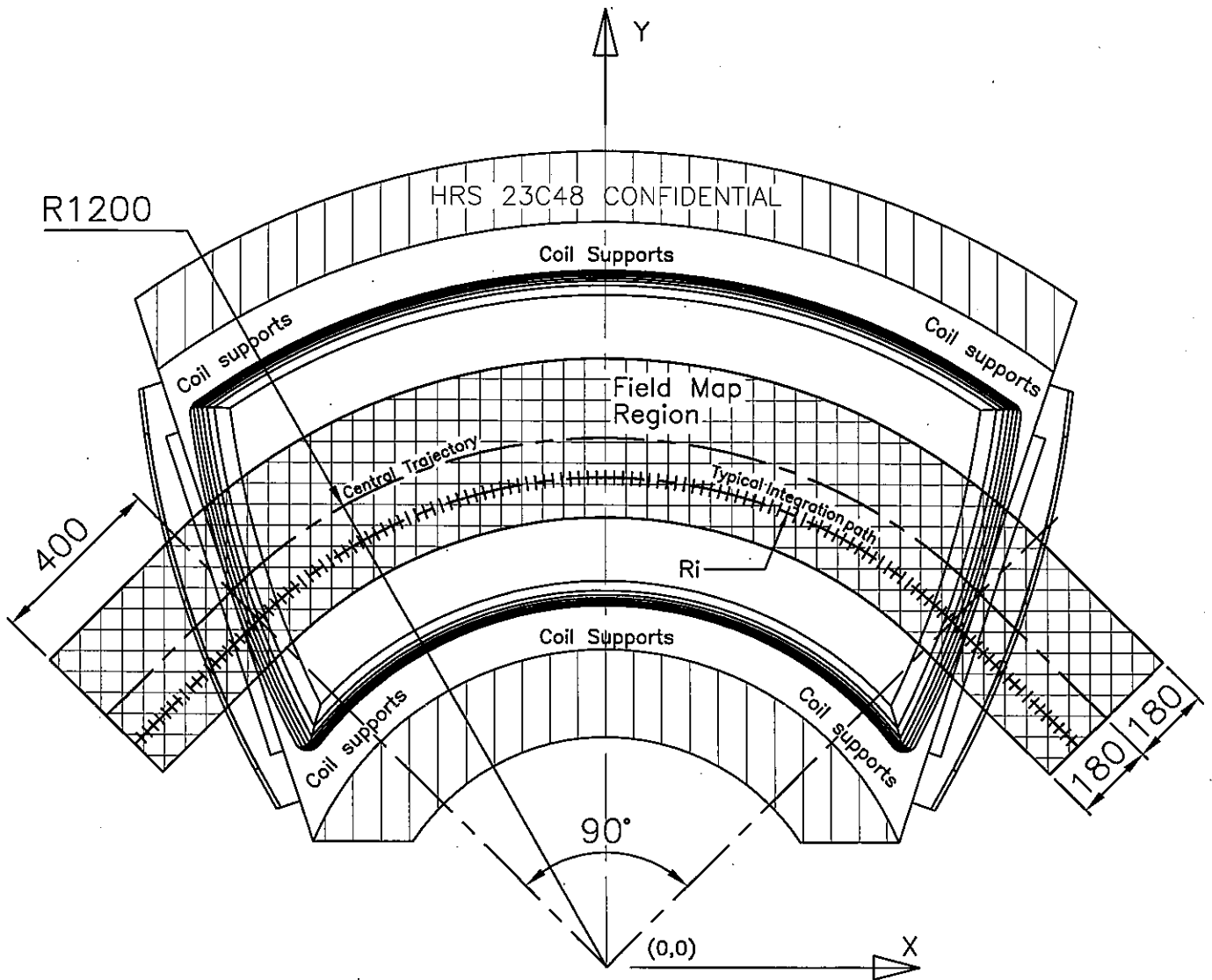


Figure 14. Field Map Region

Fig14_FieldMap.dwg GSC June 10, 2015

JUN 19 2015

Appendix 1. HRS Coil Calculations CONFIDENTIAL

Given:

Version = 4.9s

$N_{\text{CoolingCcts}} := 7$	Number of cooling circuits per coil	
$N_{\text{Turns}} := 7 \cdot 14 = 98$	Number of turns per coil	
$N_{\text{Corners}} := 4$	Number of corners per turn	
$I := 173 \cdot \text{amp}$	Current in coil	
$L_{\text{MeanTurn}} := 5553 \cdot \text{mm}$	Length of mean turn in coil	$L_{\text{MeanTurn}} = 5.553 \cdot \text{m}$
$L_{\text{tail}} := 14 \cdot 500 \cdot \text{mm}$	Coil tail Allowance	
$C_w := 10 \cdot \text{mm}$	Conductor width	$C_w = 10 \cdot \text{mm}$
$C_h := C_w$	Conductor height	
$C_r := 1 \cdot \text{mm}$	Conductor Corner Radius	$C_r = 1 \cdot \text{mm}$
$D_{\text{hole}} := 4 \cdot \text{mm}$	Diameter of cooling channel hole	$D_{\text{hole}} = 4 \cdot \text{mm}$
$K_{\text{Anaconda}} := 0.0$	Anaconda flow factor; use 0.0 if unknown	
$f_{\text{water}} := 1$	Fraction of heat going into water	
$T_{\text{in}} := 30 \cdot \text{C}$	Inlet water temperature	
$\Delta T := 20 \cdot \text{C}$	Cooling water temperature rise	

Constants:

$C \equiv 1$		
$\rho_{20} := 1.7241 \cdot 10^{-6} \cdot \text{ohm} \cdot \text{cm}$	Volume resistivity of IACS 100% Copper at 20C	[1,p.4-4]
$\alpha_{20} := 0.00393$	Temperature-Resistance Co-eff. IACS 100% Cu 20C	[1,p.4-9]
$\rho_{\text{cu}} := 8.94 \cdot \frac{\text{gm}}{\text{cm}^3}$	Density of Copper [11, p.275]	

Calculated Values:

$A_c := C_w \cdot C_h - (4 - \pi) \cdot C_r^2 - \frac{\pi \cdot D_{\text{hole}}^2}{4}$	Conductor copper c/s area	$A_c = 0.866 \cdot \text{cm}^2$
$w := \rho_{\text{cu}} \cdot A_c$	Conductor weight/unit length	$w = 0.52 \cdot \frac{\text{lb}}{\text{ft}}$
$L_c := N_{\text{Turns}} \cdot L_{\text{MeanTurn}} + L_{\text{tail}}$	Conductor Length per coil	$L_c = 5.512 \times 10^4 \cdot \text{cm}$
$W_{\text{coil}} := L_c \cdot w$	Weight of conductor in coil	$W_{\text{coil}} = 940.524 \cdot \text{lb}$

$$T_{avg} := T_{in} + \frac{\Delta T}{2} \quad \text{Average water temperature} \quad T_{avg} = 40 \cdot C$$

$$\rho_{T_{avg}} := \rho_{20} \cdot [1 + \alpha_{20} \cdot (T_{avg} - 20)] \quad [1,p.4-8] \quad \rho_{T_{avg}} = 1.86 \times 10^{-6} \cdot \text{ohm} \cdot \text{cm}$$

$$R_{T_{avg}} := \rho_{T_{avg}} \cdot \frac{L_c}{A_c} \quad \text{Coil Resistance at } T_{avg} \quad R_{T_{avg}} = 0.118 \cdot \text{ohm}$$

$$R_{20} := \rho_{20} \cdot \frac{L_c}{A_c} \quad \text{Coil Resistance at } 20C \quad R_{20} = 0.11 \cdot \text{ohm}$$

$$\text{Power} := I^2 \cdot R_{T_{avg}} \quad \text{Coil Power} \quad \text{Power} = 3543.4 \cdot \text{watt}$$

$$\text{Voltage} := I \cdot R_{T_{avg}} \quad \text{Coil Voltage} \quad \text{Voltage} = 20.48 \cdot \text{volt}$$

Density of Water [2,p.F-5]:

$$\rho_{\text{water}}(T) := 1.00 \cdot \frac{\text{gm}}{\text{cm}^3}$$

Specific Heat of water [2,p.D-158]:

$$C_p(T) := 4.18 \cdot \frac{\text{joule}}{\text{gm} \cdot C}$$

$$\text{Flow} := \frac{\text{Power} \cdot f_{\text{water}}}{C_p(T_{avg}) \cdot \Delta T \cdot \rho_{\text{water}}(T_{avg}) \cdot N_{\text{CoolingCcts}}} \quad \text{Required cooling water flow per cooling circuit}$$

$$\text{Flow} = 6.055 \cdot \frac{\text{cm}^3}{\text{sec}} \quad \text{Flow} = 0.363 \cdot \frac{\text{liter}}{\text{min}} \quad \text{Flow} = 0.096 \cdot \frac{\text{gal}}{\text{min}}$$

$$A_{\text{hole}} := \frac{\pi \cdot D_{\text{hole}}^2}{4} \quad \text{Area of cooling channel hole} \quad A_{\text{hole}} = 0.126 \cdot \text{cm}^2$$

$$\text{Velocity} := \frac{\text{Flow}}{A_{\text{hole}}} \quad \text{Water velocity} \quad \text{Velocity} = 0.482 \cdot \frac{\text{m}}{\text{sec}} \quad \text{Velocity} = 1.581 \cdot \frac{\text{ft}}{\text{sec}}$$

Tanabe [12, p. 118] recommends that velocity should be less than or equal to 4 m/sec "to avoid flow vibration and erosion of the conductor coolant passage."

The Canadian Copper & Brass Development Association recommends flow velocities less than 1.5 m/sec for hot water up to 60C and less than 1.2 m/sec over 60C. Ref CCBDA-IS 97-02

$$\text{FlowPerCoil} := \text{Flow} \cdot N_{\text{CoolingCcts}} = 2.543 \cdot \frac{\text{liter}}{\text{min}}$$

Water Viscosity [2,p.F-49]:

$$\mu_{\text{water}}(T) := \begin{cases} 0 & \text{if } T < 20 \\ 0.01002 \cdot 10 \frac{1.3272 \cdot (20-T) - [0.001053 \cdot (T-20)^2]}{(T+105)} \cdot \frac{\text{gm}}{\text{cm} \cdot \text{sec}} & \text{otherwise} \end{cases}$$

$$\mu_{\text{water}}(\text{Tavg}) = 6.53 \times 10^{-3} \cdot \frac{\text{gm}}{\text{cm} \cdot \text{sec}}$$

$$\text{Re} := \frac{\rho_{\text{water}}(\text{Tavg}) \cdot \text{Velocity} \cdot D_{\text{hole}}}{\mu_{\text{water}}(\text{Tavg})} \quad \text{Reynolds Number} \quad \text{Re} = 2.952 \times 10^3 \quad [3,p.40]$$

$$\text{Drawn tubing roughness: } \epsilon_r := 5 \cdot 10^{-6} \cdot \text{in} \quad \text{Relative roughness: } \text{RR} := \frac{\epsilon_r}{D_{\text{hole}}} \quad [3,p.363]$$

$$f_0 := 0.25 \cdot \log \left(\frac{\text{RR}}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right)^{(-2)} \quad \text{Miller's initial estimate of friction factor} \quad [3,p.364]$$

$$f_c := \left(-2 \cdot \log \left(\frac{\text{RR}}{3.7} + \frac{2.51}{\text{Re} \cdot f_0^{0.5}} \right) \right)^{-2} \quad \text{Colebrook formula for friction factor} \quad [3,p.364]$$

$f_0 = 0.045 \quad f_c = 0.044$

$$f(\text{Re}) := \begin{cases} \frac{64}{\text{Re}} & \text{if } (\text{Re} < 2000) \text{ laminar flow friction factor} \\ f_c & \text{otherwise turbulent flow friction factor} \end{cases} \quad f(\text{Re}) = 0.044$$

$$L_w := L_c + N_{\text{Turns}} \cdot N_{\text{Corners}} \cdot 30 \cdot D_{\text{hole}} \quad \text{Effective water length} \quad L_w = 598.234 \cdot \text{m}$$

$$h_L := \frac{f(\text{Re}) \cdot L_w \cdot \text{Velocity}^2}{D_{\text{hole}} \cdot 2 \cdot N_{\text{CoolingCts}}} \quad \text{head loss [3,p.361]} \quad h_L = 1.081 \times 10^6 \cdot \frac{\text{cm}^2}{\text{sec}^2}$$

$$\Delta_p := h_L \cdot \rho_{\text{water}}(\text{Tavg}) \quad \text{Pressure drop} \quad \Delta_p = 15.7 \cdot \text{psi}$$

Calculate the pressure drop the Anaconda way [4]

$$K_{\text{Otter}} := 0.0033605 \cdot \left(\frac{D_{\text{hole}}}{\text{in}} \right)^{-1.2119} \quad [5] \quad K_{\text{Otter}} = 0.032$$

$$K := \text{if}(K_{\text{Anaconda}} = 0, K_{\text{Otter}}, K_{\text{Anaconda}}) \quad K = 0.032$$

$$\Delta_{p2} := \begin{cases} K \cdot \left[\frac{\text{Velocity}}{\left(\frac{\text{ft}}{\text{sec}} \right)} \right]^{1.79} \cdot \left(\frac{L_c}{\text{ft} \cdot N_{\text{CoolingCts}}} \right) \cdot \text{psi} & \text{if } \text{Re} \geq 2320 \quad \text{Pressure drop} \\ (0 \cdot \text{psi}) & \text{otherwise} \end{cases} \quad \Delta_{p2} = 18.5 \cdot \text{psi}$$

$$L_c = 1.808 \times 10^3 \cdot \text{ft}$$

$$Re = 2.952 \times 10^3$$

$$\Delta_{p2} = 1.277 \times 10^5 \cdot \frac{\text{newton}}{\text{m}^2}$$

Heat Transfer to Water

$$\text{Power/Area} \quad \text{Pow_per_area} := \frac{\text{Power}}{\pi \cdot D_{\text{hole}} \cdot L_c} \quad \text{Pow_per_area} = 0.051 \cdot \frac{\text{watt}}{\text{cm}^2}$$

Ref[8, p. 39-40] reports conductor burnout at a flux of about 1 kW/cm².
 Ref[8, p.42] reports that "For pool boiling in water the critical flux is about 120 W/cm²....In narrow channels...under forced convection and with sub-cooling, higher fluxes can be attained, perhaps by a factor of 10 or so with water."

Thermal Conductivity of Water (from 0C to 100C) based on [2,p.E-11]

$$k(T) := (0.56049 + 0.001989 \cdot T - 7.7765 \cdot 10^{-6} \cdot T^2) \cdot \frac{\text{watt}}{\text{m} \cdot \text{C}} \quad k(T_{\text{avg}}) = 0.628 \cdot \frac{\text{watt}}{\text{m} \cdot \text{C}}$$

$$Pr := \frac{C_p(T_{\text{avg}}) \cdot \mu_{\text{water}}(T_{\text{avg}})}{k(T_{\text{avg}})} \quad Pr = 4.349 \quad [6,p.239]$$

$$Nu_{L2} := 4.36 \quad Nu \text{ at } Re=2000 \quad Nu_{L2} = 4.36 \quad [6,p.225]$$

$$Nu_{T8} := 0.023 \cdot 8000^{0.8} \cdot Pr^{0.33} \quad Nu \text{ at } Re=8000 \quad Nu_{T8} = 49.53 \quad [6,p.241]$$

$$C_{tr} := 1.33 - \frac{Re}{6000} \quad \text{Transitional Flow Co-efficient} \quad C_{tr} = 0.838 \quad [7,p.472]$$

$$Re = 2.952 \times 10^3$$

$$C_{PR} := \begin{cases} 0 \\ 1 \text{ if } (Pr > 0.5) = (Pr < 100) \end{cases} \quad C_{PR} = 1$$

Nusselt Number Nu

$$Nu := \begin{cases} 0 \\ C_{PR} \cdot (0.023 \cdot Re^{0.8} \cdot Pr^{0.33}) \text{ if } Re \geq 8000 \\ C_{tr} \cdot Nu_{L2} + (1 - C_{tr}) \cdot Nu_{T8} \text{ if } Re < 8000 \\ 4.36 \text{ if } Re < 2000 \end{cases} \quad [6,p.241]$$

$$[7,p.472]$$

$$[6,p.225]$$

$$Nu = 11.675$$

$$hbar := Nu \cdot \frac{k(T_{\text{avg}})}{D_{\text{hole}}} \quad \text{Average convection heat-transfer coefficient at fluid to solid interface.} \quad [6,p.223]$$

$$hbar = 1831.9 \cdot \frac{\text{watt}}{\text{m} \cdot \text{C}}$$

According to [6, p.15 table 1-2] Approximate Values of hbar (Forced convection, water) is in the range 50 to 10,000 w/(m² C). Higher values are associated with boiling water.

Delta T between Water and Copper

$$DT := \frac{\text{Pow_per_area}}{\text{hbar}} \qquad DT = 0.279 \qquad [6, p.15]$$

Current Density $\frac{I}{A_c} = 1.998 \cdot \frac{\text{amp}}{\text{mm}^2}$

Ref [9, p.67] recommends that "in water cooled hollow copper conductors, the current density should not exceed the order of 30 A/mm²."

References:

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Appendix 2. Extra Cooling coil for HRS magnet coil CONFIDENTIAL
 8 mm OD
 6 mm ID 8 tubes

revised July 3, 2015

These calculations assume some temperatures and calculate heat flows. The temperature are re-calculated and then iteratively adjusted by hand to be consistent.

The assumed temperatures are

Air temperature set from discussions with Bill Richert
 Copper temperature based on coil cooling. We assume a coil cooling water inlet temp = 30C and a delta T of 20C in a double pancake. The average temperature of the cold pancake should be $30 + (20/4) = 35C$.

Stainless tube OD temperature. Guess and iterate.

Inlet water temp The extra cooling will be on a separate cooling loop from the coil. It will be chilled water with inlet temperature $\geq 13C$. Set so that the average water temperature = the Air temperature. [In reality, one would control the water flow and/or temperature to hold the magnet steel temperature constant.]

Change in water temperature...Set so that the water flow and pressure drop are acceptable.

See figures 6 and A2_1.

$$T_{air} := 22 \text{ } ^\circ\text{C}$$

Air Temperature

$$T_{copper} := 35 \text{ } ^\circ\text{C}$$

Copper temperature

$$T_{SSod} := 22.46 \text{ } ^\circ\text{C}$$

Stainless tube OD temperature -- Iterated
 See NT.SSod on page 8.

$$T_{Win} := 17.5 \text{ } ^\circ\text{C}$$

Inlet Water Temperature $\geq 13 \text{ } C$

$$\Delta T_w := 9 \cdot K$$

Change in water temperature

$$T_{w_{avg}} := T_{Win} + \frac{\Delta T_w}{2} = 22 \cdot ^\circ\text{C}$$

Average Water Temp; want = Tair

$$k_{G10} := 0.288 \cdot \frac{\text{watt}}{\text{m} \cdot \Delta^{\circ}\text{C}} \quad \text{Thermal conductivity of G10 from matweb.com}$$

$$k_{SS} := 14.4 \cdot \frac{\text{watt}}{\text{m} \cdot \Delta^{\circ}\text{C}} \quad \text{Thermal conductivity of SS304 [6, p.511]}$$

$$k_{\text{rubber}} := 0.465 \cdot \frac{\text{W}}{\text{m} \cdot \Delta^{\circ}\text{C}} \quad \text{Thermal conductivity of Rubber [6, p.512]}$$

$$x1 := 5.5 \cdot \text{mm} \quad \text{Tube centerline depth, Distance from copper conductor to tube centerline. see sketch}$$

$$D := 8 \cdot \text{mm} \quad \text{Tube OD}$$

$$ID := 6 \cdot \text{mm} \quad \text{Tube ID}$$

$$N_{\text{tubes}} := 8$$

$$L_{\text{turn}} := 5553 \cdot \text{mm} \quad \text{Water flow length per turn}$$

$$L_{\text{contact}} := 3830 \cdot \text{mm} \quad \text{estimated Coil Steel contact length based on 23C62eng21 model (revised 150703)}$$

$$t_{\text{rubber}} := 6 \cdot \text{mm} \quad \text{Rubber thickness}$$

$$w_{\text{rubber}} := 66.6 \cdot \text{mm} \quad \text{Rubber width}$$

For heat flow thru the G10 to the SS tube use
Equations for heat loss from buried objects and Cavities
in Handbook of Heat Transfer, Edited by Rohsenow & Hartnett
McGraw-Hill 1973. [1, Table 7, p. 3-121]
Infinite circular hole in semi-infinite solid.
Use the "x is about equal to D" formula
qprime is q per unit length

$$q_{\text{primeG10}} := \frac{2 \cdot \pi}{\text{acosh}\left(\frac{2 \cdot x1}{D}\right)} \cdot k_{G10} \cdot (T_{\text{copper}} - T_{\text{SSod}}) = 27 \cdot \frac{\text{W}}{\text{m}}$$

Heat flow thru G10

$$q_{\text{primeG10}} \cdot L_{\text{turn}} \cdot N_{\text{tubes}} = 1198.62 \text{ W}$$

Heat Flow through the stainless tube wall is based on Kreith & Black [6, p.54-55]

$$q := \frac{(T_i - T_o) \cdot 2 \cdot \pi \cdot k \cdot l}{\ln\left(\frac{r_o}{r_i}\right)} \quad [6, \text{eqn 2-34, p.55}]$$

In our case q is reduced by 2 because only the top half of the tube is working/active. $q_{\text{prime}} = q/l$. Re-arrange to get:

$$\Delta T_{SS} := \frac{q_{\text{prime}} G_{10} \cdot 2 \cdot \ln\left(\frac{D}{ID}\right)}{2 \cdot \pi \cdot k_{SS}} = 0.172 \text{ K}$$

$$T_{SSid} := T_{SSod} - \Delta T_{SS} = 22.288 \cdot ^\circ\text{C} \quad \text{Temperature of tube's inside wall}$$

Heat Flow to magnet steel from the tube OD to the steel, thru the G10 and rubber pad. Use the electric analog:
 $q = \text{dellta}T / \text{Sum_Thermal_Resistances}$

$$T_{\text{Steel}} := T_{\text{air}} = 22 \cdot ^\circ\text{C}$$

$$x2 := 6 \cdot \text{mm} \quad \text{Tube centerline to outside surface distance}$$

$$q_{\text{primeSteel}} := \frac{(T_{SSod} - T_{\text{Steel}})}{\left(\frac{\text{acosh}\left(\frac{2 \cdot x2}{D}\right)}{2 \cdot \pi \cdot k_{G10}} \right) + \frac{t_{\text{rubber}}}{k_{\text{rubber}} \cdot w_{\text{rubber}}}} = 0.634 \cdot \frac{\text{W}}{\text{m}}$$

$$\text{HeatFlowToSteel} := q_{\text{primeSteel}} \cdot L_{\text{contact}} = 2.4 \text{ W}$$

Water Calculations

Power is power into the water.

$$\text{Power} := q_{\text{primeG10}} \cdot (\text{Lturn} \cdot \text{Ntubes}) - \text{HeatFlowToSteel} = 1196 \text{ W}$$

$$\rho_{\text{water}} := 1.00 \cdot \frac{\text{gm}}{\text{cm}^3} \quad \text{Density of Water}$$

$$C_p := 4.18 \cdot \frac{\text{joule}}{\text{gm} \cdot \Delta^\circ\text{C}} \quad \text{Specific Heat of water}$$

$$\text{Flow} := \frac{\text{Power}}{C_p \cdot \Delta T_w \cdot \rho_{\text{water}}} = 3.18 \times 10^{-5} \frac{\text{m}^3}{\text{s}} \quad \text{Required cooling water flow per cooling circuit}$$

$$\text{Flow} = 31.797 \cdot \frac{\text{cm}^3}{\text{sec}} \quad \text{Flow} = 1.908 \cdot \frac{\text{liter}}{\text{min}} \quad \text{Flow} = 0.504 \cdot \frac{\text{gal}}{\text{min}}$$

$$D_{\text{hole}} := \text{ID}$$

$$A_{\text{hole}} := \frac{\pi \cdot \text{ID}^2}{4} \quad \text{Area of cooling channel hole} \quad A_{\text{hole}} = 0.283 \cdot \text{cm}^2$$

$$\text{Velocity} := \frac{\text{Flow}}{A_{\text{hole}}} \quad \text{Water velocity} \quad \text{Velocity} = 1.12 \cdot \frac{\text{m}}{\text{sec}}$$

$$\text{Velocity} = 3.69 \cdot \frac{\text{ft}}{\text{sec}}$$

Tanabe [10, p. 118] recommends that velocity should be less than or equal to 4 m/sec "to avoid flow vibration and erosion of the conductor coolant passage."

The Canadian Copper & Brass Development Association recommends flow velocities less than 1.5 m/sec for hot water up to 60C and less than 1.2 m/sec over 60C. Ref CCBDA-IS 97-02

Water Viscosity based on [2,p.F-49]:

$$\mu_{\text{water}}(T) := \begin{cases} T2 \leftarrow \frac{T}{K} - 273.15 \\ 0 & \text{if } T2 < 20 \\ 0.01002 \cdot 10 \frac{1.3272 \cdot (20 - T2) - [0.001053 \cdot (T2 - 20)^2]}{(T2 + 105)} \cdot \frac{\text{gm}}{\text{cm} \cdot \text{sec}} & \text{otherwise} \end{cases}$$

$$\mu_{\text{water}}(T_{w_{\text{avg}}}) = 0.00955 \cdot \frac{\text{gm}}{\text{cm} \cdot \text{sec}} \quad T_{w_{\text{avg}}} = 22 \cdot ^\circ\text{C}$$

$$\text{Re} := \frac{\rho_{\text{water}} \cdot \text{Velocity} \cdot D_{\text{hole}}}{\mu_{\text{water}}(T_{w_{\text{avg}}})} \quad \text{Reynolds Number} \quad \text{Re} = 7066.5 \quad [3, \text{p.40}]$$

Drawn tubing roughness: $er := 5 \cdot 10^{-6} \cdot \text{in}$ [3,p.363]

Relative roughness: $RR := \frac{er}{D_{\text{hole}}}$ [3,p.363]

$$f_0 := 0.25 \cdot \log \left(\frac{RR}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right)^{(-2)}$$

Miller's initial estimate of friction factor [3,p.364]

$$f_c := \left(-2 \cdot \log \left(\frac{RR}{3.7} + \frac{2.51}{\text{Re} \cdot f_0^{0.5}} \right) \right)^{-2}$$

Colebrook formula for friction factor [3,p.364]
 $f_0 = 0.034 \quad f_c = 0.034$

$$f(\text{Re}) := \begin{cases} \frac{64}{\text{Re}} & \text{if } (\text{Re} < 2000) \\ f_c & \text{otherwise} \end{cases}$$

laminar flow friction factor
 turbulent flow friction factor

$$f(\text{Re}) = 0.034$$

$$L_w := N_{\text{tubes}} \cdot L_{\text{turn}} \quad \text{water length} \quad L_w = 44.424 \cdot \text{m}$$

$$h_L := \frac{f(\text{Re}) \cdot L_w \cdot \text{Velocity}^2}{D_{\text{hole}} \cdot 2} \quad \text{head loss} \quad h_L = 1.588 \times 10^6 \cdot \frac{\text{cm}^2}{\text{sec}^2}$$

[3, p.361]

$$\Delta_p := h_L \cdot \rho_{\text{water}} \quad \text{Pressure drop} \quad \Delta_p = 23 \cdot \text{psi}$$

[3, p.360]

Calculate the pressure drop the Anaconda way [4]

$$K_{\text{Otter}} := 0.0033605 \cdot \left(\frac{D_{\text{hole}}}{\text{in}} \right)^{-1.2119} \quad [5] \quad K_{\text{Otter}} = 0.019$$

$$\Delta_{p2} := \begin{cases} K_{\text{Otter}} \cdot \left[\frac{\text{Velocity}}{\left(\frac{\text{ft}}{\text{sec}} \right)} \right]^{1.79} \cdot \left(\frac{L_w}{\text{ft}} \right) \cdot \text{psi} & \text{if } \text{Re} \geq 2320 \\ (0 \cdot \text{psi}) & \text{otherwise} \end{cases} \quad \text{Pressure drop}$$

$$\Delta_{p2} = 29.1 \cdot \text{psi}$$

$$\text{Re} = 7066.5$$

Heat Transfer to Water

$$\text{Power/Area } \text{Pow_per_area} := \frac{\text{Power}}{\pi \cdot D_{\text{hole}} \cdot L_w} \quad \text{Pow_per_area} = 0.143 \cdot \frac{\text{watt}}{\text{cm}^2}$$

Ref[8, p. 39-40] reports conductor burnout at a flux of about 1 kW/cm².
 Ref[8, p.42] reports that "For pool boiling in water the critical flux is about 120 W/cm²....In narrow channels...under forced convection and with sub-cooling, higher fluxes can be attained, perhaps by a factor of 10 or so with water."

Thermal Conductivity of Water (from 0C to 100C) based on [2,p.E-11]

$$k(T) := \left[-0.550582 + 0.006165 \cdot \frac{T}{K} - 7.668998 \cdot 10^{-6} \cdot \left(\frac{T}{K} \right)^2 \right] \cdot \frac{W}{m \cdot K}$$

$$k(Tw_{avg}) = 0.601 \frac{1}{K} \cdot \frac{watt}{m} \quad Tw_{avg} = 295.15 K$$

$$Pr := \frac{C_p \cdot \mu_{water}(Tw_{avg})}{k(Tw_{avg})} = 6.642 \quad [6,p.239]$$

$$Nu_{L2} := 4.36 \quad Nu \text{ at } Re=2000 \quad [6,p.225]$$

$$Nu_{T8} := 0.023 \cdot 8000^{0.8} \cdot Pr^{0.33} = 56.957 \quad Nu \text{ at } Re=8000 \quad [6,p.241]$$

$$C_{tr} := 1.33 - \frac{Re}{6000} = 0.152 \quad \text{Transitional Flow Co-efficient} \quad [7,p.472]$$

$$Re = 7.067 \times 10^3$$

$$C_{PR} := \begin{cases} 0 \\ 1 \text{ if } (Pr > 0.5) = (Pr < 100) \end{cases} \quad C_{PR} = 1$$

Nusselt Number Nu

$$Nu := \begin{cases} 0 \\ C_{PR} \cdot (0.023 \cdot Re^{0.8} \cdot Pr^{0.33}) \text{ if } Re \geq 8000 & [6,p.241] \\ C_{tr} \cdot Nu_{L2} + (1 - C_{tr}) \cdot Nu_{T8} \text{ if } Re < 8000 & [7,p.472] \\ 4.36 \text{ if } Re < 2000 & [6,p.225] \end{cases}$$

$$Nu = 48.95$$

$$hbar := Nu \cdot \frac{k(T_{w_{avg}})}{D_{hole}} \quad \text{Average convection heat-transfer coefficient at fluid to solid interface.} \quad [6, p.223]$$

$$hbar = 4902.7 \frac{1}{K} \cdot \frac{\text{watt}}{m^2}$$

According to [6, p.15 table 1-2] Approximate Values of hbar (Forced convection, water) is in the range 50 to 10,000 w/(m² C). Higher values are associated with boiling water.

Delta T between Water and Tube ID

$$DT := \frac{Pow_per_area}{hbar} \quad DT = 0.291 \text{ K} \quad [6, p.15]$$

$$NT_{SSod} := Tw_{avg} + DT + \Delta T_{SS} = 22.463 \cdot ^\circ C \quad \text{FEEDBACK to TSSod on page 1.}$$

$$DT = 0.291 \text{ K}$$

$$\Delta T_{SS} = 0.172 \text{ K}$$

Heat Flow to steel from page 3

$$\text{HeatFlowToSteel} = 2.4 \text{ W}$$

Without extra cooling the heat flow to the steel would be

$$t_{G10} := 2 \cdot \text{mm}$$

$$q_{primeSteelNoExtra} := \frac{(T_{copper} - T_{Steel})}{\frac{t_{G10}}{k_{G10} \cdot w_{rubber}} + \frac{t_{rubber}}{k_{rubber} \cdot w_{rubber}}} = 43.622 \cdot \frac{W}{m}$$

$$q_{primeSteelNoExtra} \cdot L_{contact} = 167 \text{ W}$$

References:

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3. Robert Fox & Alan McDonald, Introduction to Fluid Mechanics 3rd Ed., John Wiley & Sons, 1985.
4. Anaconda Technical Publication 56, Anaconda American Brass Company, 1968
5. Alan J Otter, Private Communication, 1995
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9. Mathcad 15.0, Parametric Technology Corporation, Needham, MA, USA.
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Appendix 3.
Analysis of a metric sized Bolted Joint based on
An Introduction to the Design and Behavior of Bolted Joints 2nd Ed.
by John Bickford [1] p.23-29

09 Apr 2015
METRIC version

Assume: M36x4.0 6g class12.9 socket head capscrew
M36x4.0 6H C1006 HP Magnet Steel NUT

Given:

$BoltS_{ult} := 1220 \cdot \frac{N}{mm^2}$	Ultimate Tensile Strength of bolt	$BoltS_{ult} = 176946 \cdot psi$
		$BoltS_{ult} = 1220 \cdot MPa$

$BoltSy := 0.9 \cdot BoltS_{ult} = 1098 \cdot MPa$	Bolt Yield Strength
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$NutS_{ult} := 35000 \cdot psi$	Ultimate Tensile Strength of nut UTS_1006 35KSI; Yield_1006 =25KSI [2]
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$NutSy := 25000 \cdot psi$

$D_{nom} := 36 \cdot mm$	Nominal Bolt diameter
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$P := 4 \cdot mm$	Nominal Pitch
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$E_{forD2} := 0 \cdot mm$	Fundamental Deviation on D2 the pitch diameter Internal Thrd	[3] MH29ed p.1886 Table 6
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$es_{FORd2} := -0.060 \cdot mm$	Fundamental Deviation on d2 the pitch diameter Ext Thrd	[3] MH29ed p.1886 Table 6
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$TD1 := 0.6 \cdot mm$	Tol on D1 Internal Thrd	[3] MH29ed p.1889 Table 8
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$Td := 0.475 \cdot mm$	Tol on d the Major Dia Ext Thrd	[3] MH29ed p.1890 Table 9
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$TD2 := 0.3 \cdot mm$	Tol on D2 the Pitch Dia Internal Thrd	[3] MH29ed p.1890 Table 10
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$Td2 := 0.224 \cdot mm$	Tol on d2 the Pitch Dia Ext. Thrd	[3] MH29ed p.1891 Table 11
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$ALe := 90 \cdot mm$	ACTUAL Thread Engagement
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Calculated values:

$E_p := D_{nom} - 0.649515 \cdot P = 33.402 \cdot mm$	Nominal Bolt Pitch diameter [3] MH29ed p.1529
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$BoltMaxPitchDia := D_{nom} - 0.6495191 \cdot P - es_{FORd2} = 33.342 \cdot mm$	[3] MH29ed p.1889
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$E_{Smin} := BoltMaxPitchDia - Td2 = 33.118 \cdot mm$	Min. Bolt Pitch diameter [3] MH29ed p.1889
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$es := es_{FORd2} = 0.06 \cdot mm$

$$\text{BoltMaxMajorDia} := D_{\text{nom}} - e_s = 35.94 \cdot \text{mm} \quad [3] \text{ MH29ed p.1889}$$

$$D_{\text{smin}} := \text{BoltMaxMajorDia} - T_d = 35.465 \cdot \text{mm} \quad \text{Bolt Min Major Dia} = \text{Min. Bolt Threads O.Dia} \\ [3] \text{ MH29ed p.1889.}$$

$$\text{NutMinPitchDia} := D_{\text{nom}} - 0.6495191 \cdot P + E_{\text{forD2}} = 33.402 \cdot \text{mm} \quad [3] \text{ MH29ed p.1889}$$

$$E_{\text{nmax}} := \text{NutMinPitchDia} + TD2 = 33.702 \cdot \text{mm} \quad \text{Max Nut Pitch diameter} [3] \text{ MH29ed p.1889}$$

$$\text{NutMinMajorDia} := D_{\text{nom}} + E_{\text{forD2}} = 36 \cdot \text{mm}$$

$$\text{NutMinMinorDia} := \text{NutMinMajorDia} - 1.0825318 \cdot P = 31.67 \cdot \text{mm}$$

$$K_{\text{nmax}} := \text{NutMinMinorDia} + TD1 = 32.27 \cdot \text{mm} \quad \text{Nut Max Minor dia} = \text{Max. Nut I.Dia.}$$

$$\text{TPI} := \frac{1}{P} = 0.25 \cdot \text{mm}^{-1} \quad \text{Threads per inch}$$

$$\text{BSS}_{\text{ult}} := 0.577 \text{BoltSy} \quad \text{Bolt Ultimate Shear Stress using Maximum Shear Stress Theory. Shigley [5] p.169} \quad \text{BSS}_{\text{ult}} = 91888 \cdot \text{psi}$$

$$\text{BSS}_{\text{ult}} = 634 \cdot \text{MPa}$$

$$\text{NSS}_{\text{ult}} := 0.577 \text{NutSy} \quad \text{Nut Ultimate Shear Stress using Distortion-energy Stress Theory. Shigley [5] p.171} \quad \text{NSS}_{\text{ult}} = 14425 \cdot \text{psi}$$

$$\text{NSS}_{\text{ult}} = 99 \cdot \text{MPa}$$

$$A_S := \begin{cases} 0.785 \cdot \left(D_{\text{nom}} - \frac{0.9743}{\text{TPI}} \right)^2 & \text{if BoltS}_{\text{ult}} < 100000 \cdot \text{psi} \\ \left[\pi \cdot \left(\frac{E_{\text{Smin}}}{2} - \frac{0.16238}{\text{TPI}} \right)^2 \right] & \text{otherwise} \end{cases} \quad \text{Bolt [Tensile] Stress Area} \\ [3] \text{ MH29ed page 1529} \\ [1] \text{ Bickford p.23}$$

$$A_S = 795.17 \cdot \text{mm}^2$$

$$A_S = 1.233 \cdot \text{in}^2$$

Shear Area of bolt threads: [1] Bickford eqns 2.9 and 2.13

$$\text{BoltATS} := \begin{cases} \pi \cdot \text{TPI} \cdot A_{\text{Le}} \cdot K_{\text{nmax}} \cdot \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (E_{\text{Smin}} - K_{\text{nmax}}) \right] & \text{if } \text{BSS}_{\text{ult}} \neq \text{NSS}_{\text{ult}} \\ \pi \cdot E_p \cdot \frac{A_{\text{Le}}}{2} & \text{otherwise} \end{cases}$$

$$\text{BoltATS} = 5678.9 \cdot \text{mm}^2$$

Shear Area of nut threads: See Bickford [1] eqns 2.11 and 2.13

$$\text{NutATS} := \begin{cases} \pi \cdot \text{TPI} \cdot A_{Le} \cdot D_{smin} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right] & \text{if } BSS_{ult} \neq NSS_{ult} \\ \pi \cdot E_p \cdot \frac{A_{Le}}{2} & \text{otherwise} \end{cases}$$

$$\text{NutATS} = 7565.5 \cdot \text{mm}^2$$

Length of thread engagement required to develop full strength of the threads:

$$L_e := \begin{cases} \frac{2 \cdot A_S}{\pi \cdot \text{TPI} \cdot K_{nmax} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (E_{Smin} - K_{nmax}) \right]} & \text{if } NSS_{ult} > BSS_{ult} \\ \frac{4 \cdot A_S}{\pi \cdot E_p} & \text{if } NSS_{ult} = BSS_{ult} \\ \frac{\text{BoltS}_{ult} \cdot 2 \cdot A_S}{\text{NutS}_{ult} \cdot \pi \cdot \text{TPI} \cdot D_{smin} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right]} & \text{otherwise} \end{cases}$$

$$L_e = 95.646 \cdot \text{mm}$$

$$F_{\text{BoltBody}} := \text{BoltS}_{ult} \cdot A_S \quad \text{Static Strength of Bolt Body} \quad F_{\text{BoltBody}} = 218089 \cdot \text{lbf}$$

$$F_{\text{BoltThreads}} := BSS_{ult} \cdot \text{BoltATS} \quad \text{Static Strength-Bolt Threads} \quad F_{\text{BoltThreads}} = 808825 \cdot \text{lbf}$$

Bickford [1] eqn 2.8

$$F_{\text{NutThreads}} := NSS_{ult} \cdot \text{NutATS} \quad \text{Static Strength-Nut Threads} \quad F_{\text{NutThreads}} = 169156 \cdot \text{lbf}$$

Bickford [1] eqn 2.8

$$F_0 := F_{\text{BoltBody}} \quad F_1 := F_{\text{BoltThreads}} \quad F_2 := F_{\text{NutThreads}}$$

The lowest static strength is $\min(F) = 169156 \cdot \text{lbf}$

$$F_{\text{NutThreads}} = 752 \cdot \text{kN} \quad \frac{F_{\text{NutThreads}}}{9.8 \cdot \frac{\text{m}}{\text{sec}^2}} = 76780 \cdot \text{kg} \quad \frac{F_{\text{NutThreads}}}{5.98 \cdot \frac{\text{m}}{\text{sec}^2}} = 15356 \cdot \text{kg}$$

$$\text{TorquePerPreload} := \frac{1014 \cdot \text{N} \cdot \text{m}}{140850 \cdot \text{N}} = 7.199 \times 10^{-3} \text{ m} \quad [1] \text{ Bickford p.673}$$

$$\text{Preload} := \frac{1085.5 \cdot \text{N} \cdot \text{m}}{\text{TorquePerPreload}} = 33897.1 \cdot \text{lbf}$$

References:

1. John Bickford, An Introduction to the design and behavior of bolted joints, 2nd Ed., Marcel Dekker, Inc. New York, 1990.
2. ArcelorMital USA HP Magnet Plate Steel Brochure, http://usa.arcelormittal.com/globalassets/arcelormittal-usa/what-we-do/steel/plate/plate-product-brochures/201309_magnet.pdf , accessed: May 29, 2015.
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4. Mathcad 15.0, Parametric Technology Corporation, Needham, MA, USA.
5. Joseph Shigley, Mechanical Engineering Design, 3rd Ed., McGraw-Hill, New York, 1977.

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Appendix 4.
Analysis of a metric sized Bolted Joint based on
An Introduction to the Design and Behavior of Bolted Joints 2nd Ed.
by John Bickford [1] p.23-29

09 Apr 2015
METRIC version

Assume: M20x2.5 6g A4 Stainless class 70 Socket Head Capscrew
M20x2.5 6H C1006 HP Magnet Steel NUT

Given:

$BoltS_{ult} := 700 \cdot \frac{N}{mm^2}$ Ultimate Tensile Strength of bolt
Fabory [5, p.15-40-3]. BoltS_{ult} = 101526·psi
BoltS_{ult} = 700·MPa

$BoltS_y := 0.9 \cdot BoltS_{ult} = 630 \cdot MPa$ Bolt Yield Strength. consider using Proof stress.

$NutS_{ult} := 35000 \cdot psi$ Ultimate Tensile Strength of nut UTS_1006 35KSI; Yield_1006 =25KSI
[2]

$NutS_y := 25000 \cdot psi$

$D_{nom} := 20 \cdot mm$ Nominal Bolt diameter

$P := 2.5 \cdot mm$ Nominal Pitch

$E_{forD2} := 0 \cdot mm$ Fundamental Deviation on D2
the pitch diameter Internal Thrd [3] MH29ed p.1886 Table 6

$esFORd2 := -0.042 \cdot mm$ Fundamental Deviation on d2
the pitch diameter Ext Thrd [3] MH29ed p.1886 Table 6

$TD1 := 0.45 \cdot mm$ Tol on D1 Internal Thrd [3] MH29ed p.1889 Table 8

$Td := 0.335 \cdot mm$ Tol on d the Major Dia Ext Thrd [3] MH29ed p.1890 Table 9

$TD2 := 0.224 \cdot mm$ Tol on D2 the Pitch Dia Internal Thrd [3] MH29ed p.1890 Table 10

$Td2 := 0.170 \cdot mm$ Tol on d2 the Pitch Dia Ext. Thrd [3] MH29ed p.1891 Table 11

$ALe := 43 \cdot mm$ ACTUAL Thread Engagement

Calculated values:

$E_p := D_{nom} - 0.649515 \cdot P = 18.376 \cdot mm$ Nominal Bolt Pitch diameter [3] MH29ed p.1529

$BoltMaxPitchDia := D_{nom} - 0.6495191 \cdot P - |esFORd2| = 18.334 \cdot mm$ [3] MH29ed p.1889

$E_{Smin} := BoltMaxPitchDia - Td2 = 18.164 \cdot mm$ Min. Bolt Pitch diameter [3] MH29ed p.1889

$es := |esFORd2| = 0.042 \cdot mm$

BoltMaxMajorDia := $D_{nom} - es = 19.958 \cdot \text{mm}$ [3] MH29ed p.1889

$D_{smin} := \text{BoltMaxMajorDia} - Td = 19.623 \cdot \text{mm}$ Bolt Min Major Dia = Min. Bolt Threads O.Dia
 [3] MH29ed p.1889.

NutMinPitchDia := $D_{nom} - 0.6495191 \cdot P + E\text{for}D2 = 18.376 \cdot \text{mm}$ [3] MH29ed p.1889

$E_{nmax} := \text{NutMinPitchDia} + TD2 = 18.6 \cdot \text{mm}$ Max Nut Pitch diameter [3] MH29ed p.1889

NutMinMajorDia := $D_{nom} + E\text{for}D2 = 20 \cdot \text{mm}$

NutMinMinorDia := $\text{NutMinMajorDia} - 1.0825318 \cdot P = 17.294 \cdot \text{mm}$

$K_{nmax} := \text{NutMinMinorDia} + TD1 = 17.744 \cdot \text{mm}$ Nut Max Minor dia = Max. Nut I.Dia.

$TPI := \frac{1}{P} = 0.4 \cdot \text{mm}^{-1}$ Threads per inch

$BSS_{ult} := 0.577 \text{BoltSy}$ Bolt Ultimate Shear Stress using Maximum Shear Stress Theory. Shigley [6] p.169 $BSS_{ult} = 52723 \cdot \text{psi}$

$BSS_{ult} = 364 \cdot \text{MPa}$

$NSS_{ult} := 0.577 \text{NutSy}$ Nut Ultimate Shear Stress using Distortion-energy Stress Theory. Shigley [6] p.171 $NSS_{ult} = 14425 \cdot \text{psi}$

$NSS_{ult} = 99 \cdot \text{MPa}$

$A_S := \begin{cases} 0.785 \cdot \left(D_{nom} - \frac{0.9743}{TPI} \right)^2 & \text{if BoltS}_{ult} < 100000 \cdot \text{psi} \\ \left[\pi \cdot \left(\frac{E_{Smin}}{2} - \frac{0.16238}{TPI} \right)^2 \right] & \text{otherwise} \end{cases}$ Bolt [Tensile] Stress Area
 [3] MH29ed page 1529
 [1] Bickford p.23

$A_S = 236.485 \cdot \text{mm}^2$

$A_S = 0.367 \cdot \text{in}^2$

Shear Area of bolt threads: See Bickford [1] eqns 2.9 and 2.13

$\text{BoltATS} := \begin{cases} \pi \cdot TPI \cdot ALe \cdot K_{nmax} \left[\frac{1}{2 \cdot TPI} + 0.57735 \cdot (E_{Smin} - K_{nmax}) \right] & \text{if } BSS_{ult} \neq NSS_{ult} \\ \pi \cdot E_p \cdot \frac{ALe}{2} & \text{otherwise} \end{cases}$

$\text{BoltATS} = 1431.3 \cdot \text{mm}^2$

Shear Area of nut threads: See Bickford [1] eqns 2.11 and 2.13

$$\text{NutATS} := \begin{cases} \pi \cdot \text{TPI} \cdot A_{Le} \cdot D_{smin} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right] & \text{if } BSS_{ult} \neq NSS_{ult} \\ \pi \cdot E_p \cdot \frac{A_{Le}}{2} & \text{otherwise} \end{cases}$$

$$\text{NutATS} = 1951.6 \cdot \text{mm}^2$$

Length of thread engagement required to develop full strength of the threads:

$$L_e := \begin{cases} \frac{2 \cdot A_S}{\pi \cdot \text{TPI} \cdot K_{nmax} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (E_{Smin} - K_{nmax}) \right]} & \text{if } NSS_{ult} > BSS_{ult} \\ \frac{4 \cdot A_S}{\pi \cdot E_p} & \text{if } NSS_{ult} = BSS_{ult} \\ \frac{BoltS_{ult} \cdot 2 \cdot A_S}{\text{NutS}_{ult} \cdot \pi \cdot \text{TPI} \cdot D_{smin} \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right]} & \text{otherwise} \end{cases}$$

$$L_e = 30.23 \cdot \text{mm}$$

$$F_{BoltBody} := BoltS_{ult} \cdot A_S \quad \text{Static Strength of Bolt Body} \quad F_{BoltBody} = 37215 \cdot \text{lbf}$$

$$F_{BoltThreads} := BSS_{ult} \cdot BoltATS \quad \text{Static Strength-Bolt Threads} \quad F_{BoltThreads} = 116964 \cdot \text{lbf}$$

Bickford eqn 2.8

$$F_{NutThreads} := NSS_{ult} \cdot NutATS \quad \text{Static Strength-Nut Threads} \quad F_{NutThreads} = 43635 \cdot \text{lbf}$$

Bickford eqn 2.8

$$F_0 := F_{BoltBody} \quad F_1 := F_{BoltThreads} \quad F_2 := F_{NutThreads}$$

The lowest static strength is $\min(F) = 37214.8 \cdot \text{lbf}$

$$F_{NutThreads} = 194 \cdot \text{kN} \quad \frac{F_{NutThreads}}{9.8 \cdot \frac{\text{m}}{\text{sec}^2}} = 19806 \cdot \text{kg} \quad \frac{F_{NutThreads}}{5 \cdot 9.8 \cdot \frac{\text{m}}{\text{sec}^2}} = 3961 \cdot \text{kg}$$

$$\text{TorquePerPreload} := \frac{168.96 \cdot \text{N} \cdot \text{m}}{42240 \cdot \text{N}} = 4 \times 10^{-3} \text{ m} \quad [1, \text{p.673}]$$

$$\text{Preload} := \frac{200 \cdot \text{N} \cdot \text{m}}{\text{TorquePerPreload}} = 11240.4 \cdot \text{lbf}$$

References:

1. John Bickford, An Introduction to the design and behavior of bolted joints, 2nd Ed., Marcel Dekker, Inc. New York, 1990.
2. ArcelorMital USA HP Magnet Plate Steel Brochure, http://usa.arcelormittal.com/globalassets/arcelormittal-usa/what-we-do/steel/plate/plate-product-brochures/201309_magnet.pdf , accessed: May 29, 2015.
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Appendix 5.
Analysis of a metric sized Bolted Joint based on
An Introduction to the Design and Behavior of Bolted Joints 2nd Ed.
by John Bickford [1] p.23-29

09 Apr 2015
METRIC version

Assume: M12-1.75 6g A4 Stainless class 70 Bolt
M12-1.75 6H C1006 HP Magnet Steel NUT

Given:

$BoltS_{ult} := 700 \cdot \frac{N}{mm^2}$ Ultimate Tensile Strength of bolt
Fabory [5, p.15-40-3]. BoltS_{ult} = 101526·psi
BoltS_{ult} = 700·MPa

$BoltSy := 0.9 \cdot BoltS_{ult} = 630 \cdot MPa$ Bolt Yield Strength

$NutS_{ult} := 35000 \cdot psi$ Ultimate Tensile Strength of nut UTS_1006 35KSI; Yield_1006 =25KSI
[2]

$NutSy := 25000 \cdot psi$

$D_{nom} := 12 \cdot mm$ Nominal Bolt diameter

$P := 1.75 \cdot mm$ Nominal Pitch

$E_{IforD2} := 0 \cdot mm$ Fundamental Deviation on D2
the pitch diameter Internal Thrd [3] MH29ed p.1886 Table 6

$esFORd2 := -0.034 \cdot mm$ Fundamental Deviation on d2
the pitch diameter Ext Thrd [3] MH29ed p.1886 Table 6

$TD1 := 0.335 \cdot mm$ Tol on D1 Internal Thrd [3] MH29ed p.1889 Table 8

$Td := 0.265 \cdot mm$ Tol on d the Major Dia Ext Thrd [3] MH29ed p.1890 Table 9

$TD2 := 0.200 \cdot mm$ Tol on D2 the Pitch Dia Internal Thrd [3] MH29ed p.1890 Table 10

$Td2 := 0.150 \cdot mm$ Tol on d2 the Pitch Dia Ext. Thrd [3] MH29ed p.1891 Table 11

$ALe := 18 \cdot mm$ ACTUAL Thread Engagement

Calculated values:

$E_p := D_{nom} - 0.649515 \cdot P = 10.863 \cdot mm$ Nominal Bolt Pitch diameter [3] MH29ed p.1529

$BoltMaxPitchDia := D_{nom} - 0.6495191 \cdot P - |esFORd2| = 10.829 \cdot mm$ [3] MH29ed p.1889

$E_{Smin} := BoltMaxPitchDia - Td2 = 10.679 \cdot mm$ Min. Bolt Pitch diameter [3] MH29ed p.1889

$es := |esFORd2| = 0.034 \cdot mm$

$$\text{BoltMaxMajorDia} := D_{\text{nom}} - e_s = 11.966\text{-mm} \quad [3] \text{ MH29ed p.1889}$$

$$D_{\text{smmin}} := \text{BoltMaxMajorDia} - T_d = 11.701\text{-mm} \quad \text{Bolt Min Major Dia = Min. Bolt Threads O.Dia} \\ [3] \text{ MH29ed p.1889.}$$

$$\text{NutMinPitchDia} := D_{\text{nom}} - 0.6495191 \cdot P + E_{\text{iforD2}} = 10.863\text{-mm} \quad [3] \text{ MH29ed p.1889}$$

$$E_{\text{nmax}} := \text{NutMinPitchDia} + TD2 = 11.063\text{-mm} \quad \text{Max Nut Pitch diameter [3] MH29ed p.1889}$$

$$\text{NutMinMajorDia} := D_{\text{nom}} + E_{\text{iforD2}} = 12\text{-mm}$$

$$\text{NutMinMinorDia} := \text{NutMinMajorDia} - 1.0825318 \cdot P = 10.106\text{-mm}$$

$$K_{\text{nmax}} := \text{NutMinMinorDia} + TD1 = 10.441\text{-mm} \quad \text{Nut Max Minor dia = Max. Nut I.Dia.}$$

$$TPI := \frac{1}{P} = 0.571\text{-mm}^{-1} \quad \text{Threads per inch}$$

$$\text{BSS}_{\text{ult}} := 0.577 \text{BoltSy} \quad \text{Bolt Ultimate Shear Stress using Maximum Shear Stress Theory. Shigley [6] p.169} \quad \text{BSS}_{\text{ult}} = 52723\text{-psi}$$

$$\text{BSS}_{\text{ult}} = 364\text{-MPa}$$

$$\text{NSS}_{\text{ult}} := 0.577 \text{NutSy} \quad \text{Nut Ultimate Shear Stress using Distortion-energy Stress Theory. Shigley [6] p.171} \quad \text{NSS}_{\text{ult}} = 14425\text{-psi}$$

$$\text{NSS}_{\text{ult}} = 99\text{-MPa}$$

$$A_S := \begin{cases} 0.785 \cdot \left(D_{\text{nom}} - \frac{0.9743}{TPI} \right)^2 & \text{if BoltS}_{\text{ult}} < 100000\text{-psi} \\ \left[\pi \cdot \left(\frac{E_{\text{Smin}}}{2} - \frac{0.16238}{TPI} \right)^2 \right] & \text{otherwise} \end{cases} \quad \text{Bolt [Tensile] Stress Area} \\ [3] \text{ MH29ed page 1529} \\ [1] \text{ Bickford p.23}$$

$$A_S = 80.293\text{-mm}^2$$

$$A_S = 0.124\text{-in}^2$$

Shear Area of bolt threads: See Bickford [1] eqns 2.9 and 2.13

$$\text{BoltATS} := \begin{cases} \pi \cdot TPI \cdot ALe \cdot K_{\text{nmax}} \left[\frac{1}{2 \cdot TPI} + 0.57735 \cdot (E_{\text{Smin}} - K_{\text{nmax}}) \right] & \text{if BSS}_{\text{ult}} \neq \text{NSS}_{\text{ult}} \\ \pi \cdot E_p \cdot \frac{ALe}{2} & \text{otherwise} \end{cases}$$

$$\text{BoltATS} = 341.7\text{-mm}^2$$

Shear Area of nut threads: See Bickford [1] eqns 2.11 and 2.13

$$\text{NutATS} := \begin{cases} \pi \cdot \text{TPI} \cdot A_{Le} \cdot D_{smin} \cdot \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right] & \text{if } BSS_{ult} \neq NSS_{ult} \\ \pi \cdot E_p \cdot \frac{A_{Le}}{2} & \text{otherwise} \end{cases}$$

$$\text{NutATS} = 470 \cdot \text{mm}^2$$

Length of thread engagement required to develop full strength of the threads:

$$L_e := \begin{cases} \frac{2 \cdot A_S}{\pi \cdot \text{TPI} \cdot K_{nmax} \cdot \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (E_{Smin} - K_{nmax}) \right]} & \text{if } NSS_{ult} > BSS_{ult} \\ \frac{4 \cdot A_S}{\pi \cdot E_p} & \text{if } NSS_{ult} = BSS_{ult} \\ \frac{BoltS_{ult} \cdot 2 \cdot A_S}{\text{NutS}_{ult} \cdot \pi \cdot \text{TPI} \cdot D_{smin} \cdot \left[\frac{1}{2 \cdot \text{TPI}} + 0.57735 \cdot (D_{smin} - E_{nmax}) \right]} & \text{otherwise} \end{cases}$$

$$L_e = 17.839 \cdot \text{mm}$$

$$F_{BoltBody} := BoltS_{ult} \cdot A_S \quad \text{Static Strength of Bolt Body} \quad F_{BoltBody} = 12635 \cdot \text{lbf}$$

$$F_{BoltThreads} := BSS_{ult} \cdot BoltATS \quad \text{Static Strength-Bolt Threads} \quad F_{BoltThreads} = 27925 \cdot \text{lbf}$$

Bickford [1] eqn 2.8

$$F_{NutThreads} := NSS_{ult} \cdot NutATS \quad \text{Static Strength-Nut Threads} \quad F_{NutThreads} = 10509 \cdot \text{lbf}$$

Bickford [1] eqn 2.8

$$F_0 := F_{BoltBody} \quad F_1 := F_{BoltThreads} \quad F_2 := F_{NutThreads}$$

The lowest static strength is $\min(F) = 10509.4 \cdot \text{lbf}$

$$F_{NutThreads} = 47 \cdot \text{kN} \quad \frac{F_{NutThreads}}{9.8 \cdot \frac{\text{m}}{\text{sec}^2}} = 4770 \cdot \text{kg} \quad \frac{F_{NutThreads}}{5 \cdot 9.8 \cdot \frac{\text{m}}{\text{sec}^2}} = 954 \cdot \text{kg}$$

$$\text{TorquePerPreload} := \frac{34.9 \cdot \text{N} \cdot \text{m}}{14533 \cdot \text{N}} = 2.401 \times 10^{-3} \cdot \text{m} \quad \text{Bickford [1] p.673}$$

$$\text{Preload} := \frac{24 \cdot \text{N} \cdot \text{m}}{\text{TorquePerPreload}} = 2246.7 \cdot \text{lbf}$$

References:

1. John Bickford, An Introduction to the design and behavior of bolted joints, 2nd Ed., Marcel Dekker, Inc. New York, 1990.
2. ArcelorMital USA HP Magnet Plate Steel Brochure, http://usa.arcelormittal.com/globalassets/arcelormittal-usa/what-we-do/steel/plate/plate-product-brochures/201309_magnet.pdf , accessed: May 29, 2015.
3. Oberg, Jones, Horton, and Ryffel, Machinery's Handbook 29th Ed., Industrial Press, New York, 2012.
4. Mathcad 15.0, Parametric Technology Corporation, Needham, MA, USA.
5. Fabory Metrican Masters in Fasteners Book 3, Metrican Fasteners Ltd., Mississauga, Ontario.
6. Joseph Shigley, Mechanical Engineering Design, 3rd Ed., McGraw-Hill, New York, 1977.

Appendix 6 Estimate HRS top/bottom yoke deflection
 based on Roark table 3 case 2d [1, page 100]
 Fixed end support beam with uniform loading on entire span
 uniform cross-section, a=0

HRS 23C62eng21 CONFIDENTIAL

$$E := 29 \cdot 10^6 \cdot \text{psi}$$

$$b := 1857 \cdot \text{mm} \quad \text{beam width see figure 11}$$

$$d := 205 \cdot \text{mm} \quad \text{Beam depth}$$

$$I := \frac{b \cdot d^3}{12}$$

$$L := 1004 \cdot \text{mm} \quad \text{Beam length}$$

For the beam weight use only the weight out between the supports. Not the weight directly over the supports.

$$\text{BeamWeight} := b \cdot d \cdot L \cdot 0.2833 \cdot \frac{\text{lbf}}{\text{in}^3} = 6608 \cdot \text{lbf}$$

$$\text{PoleVolume} := 252173130.37 \cdot \text{mm}^3 \quad \text{from SolidWorks 23C62eng21}$$

$$\text{PoleWeight} := \text{PoleVolume} \cdot 0.2833 \cdot \frac{\text{lbf}}{\text{in}^3} = 4360 \cdot \text{lbf}$$

$$\text{MagForce} := 64.22 \cdot \frac{\text{N}}{\text{mm}} \cdot \frac{\pi}{2} \cdot 1200 \cdot \text{mm} = 27214 \cdot \text{lbf}$$

64.22 from Opera2D
 Force to opposite pole
 Apr. 29, 2015

TOP YOKE

The top yoke has the three forces in the same direction.

$$w_a := \frac{(\text{BeamWeight} + \text{PoleWeight} + \text{MagForce})}{L} = 965.9 \cdot \frac{\text{lbf}}{\text{in}}$$

$$\text{MaxY} := \frac{-w_a \cdot L^4}{384 \cdot E \cdot I} = -1.68 \times 10^{-6} \cdot \text{m} \quad + \text{ is up}$$

$$R_a := w_a \cdot \frac{L}{2} = 19090 \cdot \text{lbf}$$

$$\text{MaxM} := \frac{wa \cdot L^2}{12} = 125765.9 \cdot \text{lbf} \cdot \text{in}$$

$$\sigma := \frac{\text{MaxM} \cdot d}{2 \cdot I} = 158.5 \cdot \text{psi}$$

Bottom Yoke

For the bottom yoke the magnetic force direction is opposite the weight.

$$wa := \frac{(\text{BeamWeight} + \text{PoleWeight} - \text{MagForce})}{L} = -411 \cdot \frac{\text{lbf}}{\text{in}}$$

$$\text{MaxY} := \frac{-wa \cdot L^4}{384 \cdot E \cdot I} = 7.15 \times 10^{-7} \cdot \text{m} \quad + \text{ is up} \quad \text{Note Sign!}$$

[1] Roark and Young, Formulas for Stress and Strain, Fifth Edition, McGraw-Hill, New York, 1975.

Appendix 7 Estimate HRS Pole deflection
 Long direction
 HRS 23C62eng21 CONFIDENTIAL

$$E := 29 \cdot 10^6 \cdot \text{psi}$$

$$b := 760 \cdot \text{mm}$$

beam/pole width

$$d := 183.3 \cdot \text{mm}$$

Beam/pole depth

$$I := \frac{b \cdot d^3}{12} = 937 \cdot \text{in}^4$$

$$L := 1885 \cdot \text{mm}$$

Beam/pole length

$$\text{Beam Weight} := b \cdot d \cdot L \cdot 0.2833 \cdot \frac{\text{lbf}}{\text{in}^3} = 4540 \cdot \text{lbf}$$

downward

$$\text{NetMagDistLoad} := (73.68 - 63.28) \cdot \frac{\text{N}}{\text{mm}}$$

towards Yoke
 from Opera2D April 16, 2015

$$\text{MagForce} := \text{NetMagDistLoad} \cdot \frac{\pi}{2} \cdot 1200 \cdot \text{mm} = 4407 \cdot \text{lbf}$$

Deflection due to the distributed load
 Machinery's Handbook (21st Ed., Oberg, Jones, & Horton, Industrial Press New York, 1979) p.404 case 1
 Simply supported both ends W is total load

$$y_{\text{dist}}(W, x) := -W \cdot \frac{x \cdot (L - x)}{24 \cdot E \cdot I \cdot L} \cdot [L^2 + x \cdot (L - x)]$$

Deflection due to a point load at any point
 Machinery's Handbook (21st Ed., Oberg, Jones, & Horton, Industrial Press New York, 1979) p.404
 case 3 Simply supported both ends. W is the point load force. a is where the force is.

$$y_{\text{point}}(x, W, a) := \begin{cases} b \leftarrow L - a \\ v \leftarrow L - x \\ \frac{W \cdot b \cdot x}{6 \cdot E \cdot I \cdot L} \cdot (L^2 - x^2 - b^2) & \text{if } x \leq a \\ \frac{W \cdot a \cdot v}{6 \cdot E \cdot I \cdot L} \cdot (L^2 - v^2 - a^2) & \text{otherwise} \end{cases}$$

TOP Pole (weight down and magnet force up). 4 EQUAL spans

$$\begin{aligned} W &:= (\text{Beam Weight} - \text{MagForce}) = 132.7 \cdot \text{lbf} && \text{power on} \\ W_{\text{PowerOff}} &:= (\text{Beam Weight}) = 4539.8 \cdot \text{lbf} \\ FB &:= 37.8 \cdot \text{lbf} && FC := 82.9 \cdot \text{lbf} - 1.3744 \cdot FB = 30.9 \cdot \text{lbf} && W = 133 \cdot \text{lbf} \\ XB &:= 471.25 \cdot \text{mm} && XC := 942.5 \cdot \text{mm} && \frac{XB}{L} = 0.25 \\ FD &:= FB && FA := \frac{W - FB - FC}{2} = 32 \cdot \text{lbf} && FE := FA \\ XD &:= L - XB = 1413.75 \cdot \text{mm} && XA := 0 \cdot \text{mm} && XE := L \end{aligned}$$

$$y1(x) := y_point(x, FA, XA) + y_point(x, FB, XB) + y_point(x, FC, XC) + y_point(x, FD, XD) + y_point(x, FE, XE)$$

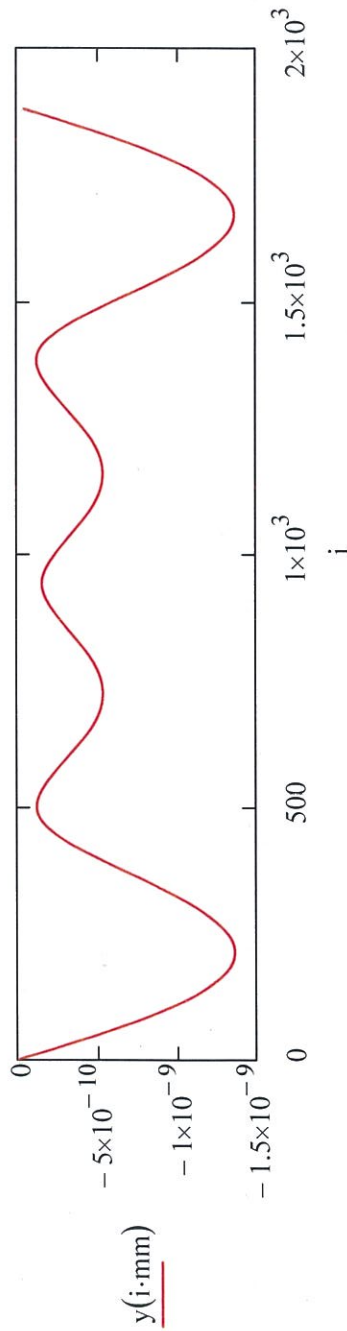
$$y(x) := y_dist(W, x) + y1(x)$$

$$y(XB) = -1.617 \times 10^{-10} \text{ m}$$

$$y(XC) = -1.612 \times 10^{-10} \text{ m} \quad y(220 \cdot \text{mm}) = -1.366 \times 10^{-9} \text{ m}$$

$$i := 1 .. 1880$$

TOP Pole 4 EQUAL spans



Bottom Pole (both weight and magnet force are downward). 4 Equal Spans

$$W := (\text{Beam Weight} + \text{MagForce}) = 8946.8 \cdot \text{lbf} \quad \text{power on}$$

$$W_{\text{PowerOff}} := (\text{Beam Weight}) = 4539.8 \cdot \text{lbf}$$

Bottom Pole Equal Spans

$$FB := 2550 \cdot \text{lbf} \quad FC := 5590 \cdot \text{lbf} - 1.3744 \cdot FB = 2085 \cdot \text{lbf} \quad W = 8947 \cdot \text{lbf}$$

$$XB := 471.25 \cdot \text{mm} \quad XC := 942.5 \cdot \text{mm}$$

$$FD := FB \quad FA := \frac{W - FB - FC}{2} = 2156 \cdot \text{lbf} \quad FE := FA$$

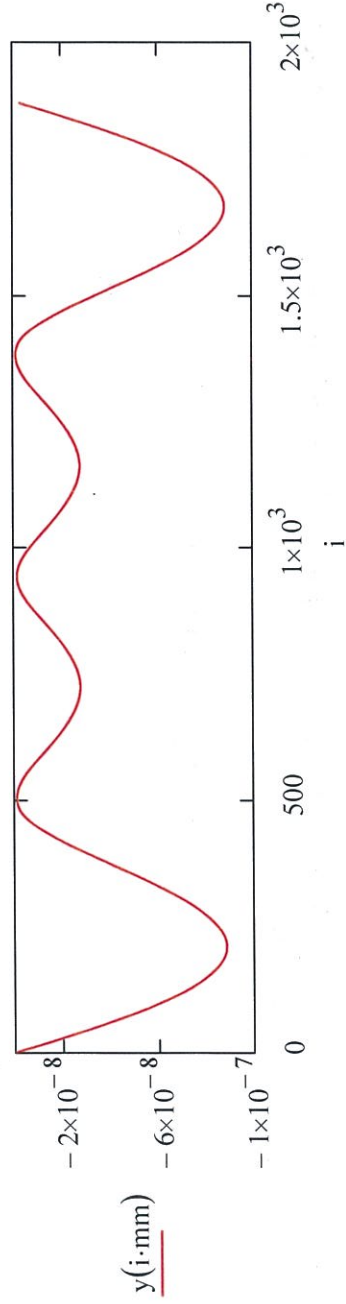
$$XD := 1413.75 \cdot \text{mm} \quad XA := 0 \cdot \text{mm} \quad XE := L$$

$$y1(x) := y_point(x, FA, XA) + y_point(x, FB, XB) + y_point(x, FC, XC) + y_point(x, FD, XD) + y_point(x, FE, XE)$$

$$y(x) := y_dist(W, x) + y1(x)$$

$$y(XB) = -3.95 \times 10^{-9} \text{ m}$$

$$i := 1 .. 1880 \quad y(XC) = -1.801 \times 10^{-9} \text{ m} \quad y(220 \cdot \text{mm}) = -8.85 \times 10^{-8} \text{ m}$$



Appendix 8. Estimate HRS Pole deflection
Short direction
Confidential

$$E := 29 \cdot 10^6 \cdot \text{psi}$$

$$b := 1885 \cdot \text{mm}$$

beam/pole width

$$d := 183.3 \cdot \text{mm}$$

Beam/pole depth

$$I := \frac{b \cdot d^3}{12} = 2324 \cdot \text{in}^4$$

$$L_{\text{beam}} := 760 \cdot \text{mm}$$

Beam/pole length

$$\text{Beam Weight} := b \cdot d \cdot L_{\text{beam}} \cdot 0.2833 \cdot \frac{\text{lbf}}{\text{in}^3} = 4540 \cdot \text{lbf}$$

downward

$$\text{NetMagDistLoad} := (73.68 - 63.28) \cdot \frac{\text{N}}{\text{mm}}$$

towards Yoke
from Opera2D April 16, 2015

$$\text{MagForce} := \text{NetMagDistLoad} \cdot \frac{\pi}{2} \cdot 1200 \cdot \text{mm} = 4407 \cdot \text{lbf}$$

TOP Pole 1 Span (weight down and magnet force up).

power on

$$W := (\text{BeamWeight} - \text{MagForce}) = 132.7 \cdot \text{lbf}$$

$$W_{\text{PowerOff}} := (\text{BeamWeight}) = 4539.8 \cdot \text{lbf}$$

Deflection due to the distributed load
 Machinery's Handbook (21st Ed., Oberg, Jones, & Horton, Industrial Press New York, 1979) p.404 case 1 Simply supported both ends, uniform load
 W is total load

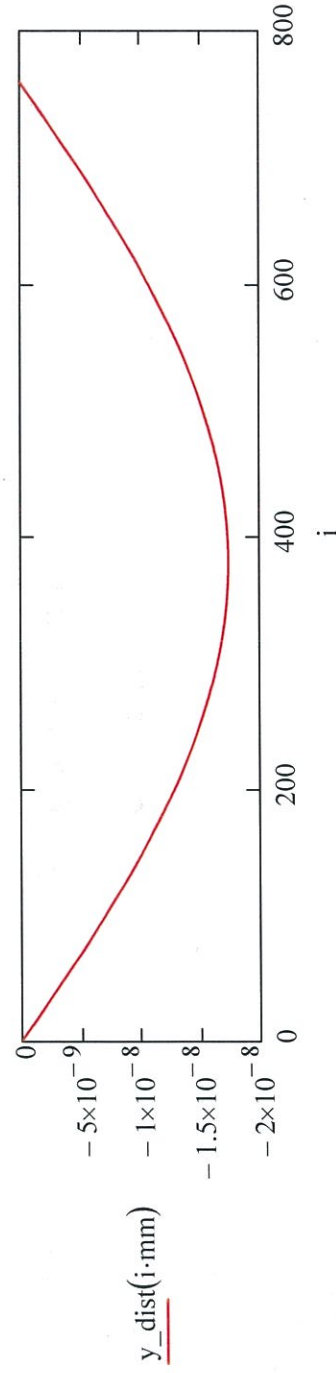
$$L := L_{\text{beam}} = 760 \cdot \text{mm}$$

L is span between supports

$$y_{\text{dist}}(x) := -W \cdot \frac{x \cdot (L - x)}{24 \cdot E \cdot I \cdot L} \cdot [L^2 + x \cdot (L - x)]$$

$$y_{\text{dist}}\left(\frac{L}{2}\right) = -1.74 \times 10^{-8} \text{ m}$$

i := 1 .. 760



Bottom Pole (both weight and magnet force are downward).
 1 Span version

$$W := (\text{BeamWeight} + \text{MagForce}) = 8946.8 \cdot \text{lbf} \quad W \text{ is total load on span, power on}$$

$$W_{\text{PowerOff}} := (\text{BeamWeight}) = 4539.8 \cdot \text{lbf}$$

Deflection due to the distributed load
 Machinery's Handbook (21st Ed., Oberg, Jones, & Horton, Industrial Press New York, 1979) p.404 case 1 Simply supported both ends, uniform load
 W is total load

$$L := L_{\text{beam}} = 760 \cdot \text{mm}$$

L is span between supports

$$y_{\text{dist}}(x) := -W \cdot \frac{x \cdot (L - x)}{24 \cdot E \cdot I \cdot L} \cdot [L^2 + x \cdot (L - x)] \quad y_{\text{dist}}\left(\frac{L}{2}\right) = -1.18 \times 10^{-6} \text{ m}$$

$$i := 1 .. 760$$

