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Beam σ-matrix under Target Scattering

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Abstract: In this note we summarize the results of calculations that we carried out with *G4Beamline* about the σ-matrix of 480 MeV proton beam under Coulomb scattering and energy loss straggling from the carbon foil/target traversal.

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In TRIUMF cyclotron we use stripping foil (carbon foil) to extract protons. The foil's typical thickness is ∼2 mg/cm^2. Through such a foil, the Coloumb scattering and energy loss straggling suffered by the protons of 480 MeV are not negligible to the primary beamline optics calculation. In addition, in BL1A we have T1 and T2 Beryllium targets of thicknesses being 12 mm and 50 mm (or even 100 mm) respectively. They cause large scattering, large energy loss and energy loss straggling to the proton beam. In order to characterize these effects, we carried out multi-particle (1 million particles) simulations with `G4Beamline` for a 480 MeV proton beam traversing a carbon foil/target of various thicknesses, followed by statistical calculations for the σ-matrix and correlation parameters. The results are compiled in the Table 1.

These σ-matrix elements were calculated using all the primary particles (protons, ∼1 million), without making any truncation to their positions, angles and momenta, which means that the small angle multiple scattering, large angle single/plural scattering and head-on collisons with electrons all are included. This is equivalent to covering the position, angle and momentum in the full space between $-\infty$ and $+\infty$, like in the analytical approach that integrals are taken between $-\infty$ and $+\infty$.

The first row of 0 target thickness actually gives the incident beam's property. It's seen that under the condition of thin foil where the average energy loss is less than 0.5 MeV (i.e. ≤0.1%), only the elements σ_{22}, σ_{44} and σ_{66} are changed, while σ_{12}, σ_{34}, σ_{16} and σ_{26} remain unchanged. But the correlation parameters r_{12}, r_{34}, r_{16} and r_{26} decrease in their absolute values simply because σ_{22}, σ_{44} and σ_{66} have increased. In other words, the correlation parameters satisfy the following relations:

$$r_{12}^{out} = r_{12}^{in}\sqrt{\sigma_{22}^{in}/\sigma_{22}^{out}}\,, \qquad r_{34}^{out} = r_{34}^{in}\sqrt{\sigma_{44}^{in}/\sigma_{44}^{out}}\,,$$
$$r_{16}^{out} = r_{16}^{in}\sqrt{\sigma_{66}^{in}/\sigma_{66}^{out}}\,, \qquad r_{26}^{out} = r_{26}^{in}\sqrt{\sigma_{22}^{in}\sigma_{66}^{in}/(\sigma_{22}^{out}\sigma_{66}^{out})}\,,$$

where the supersripts 'in' and 'out' denote at the entry and exit of the foil/target respectively.

Whereas under thick target through which the average energy loss exceeds 4 MeV (i.e. ≥0.8%), the elements σ_{12}, σ_{34}, σ_{16} and σ_{26} change significantly. In this case, the particle distributions in the phase spaces (x, x'), (y, y') & $\Delta p/p$ become so far from normal that the envelope calculation result from `TRANSOPTR` become unreliable. We have to use `G4Beamline`. Nevertheless, under such condition the correlation parameters r_{16} and r_{26} become close to zero.

As an example, Fig. 1 and 2 show the distributions of particles in (x, x'), (y, y'), $(\Delta p/p, x)$ and $(\Delta p/p, x')$ at the exit of foil/target of 0 and 2 mg/cm^2 as well as 10 mm thicknesses separately.

In the real life, the situation becomes somewhat complicated. Particles of angles above 5 mrad will likely get lost during propagation because the R-matrix element R_{34} reaches ∼1 cm/mrad at maximum while the pipe aperture is limited to 4 inch (10 cm, full). If we truncate the angles of ≥ 5 mrad prior to the statistics calculation, we will get results that appear to be very much different from the Table 1 especially on the elements σ_{22} and σ_{44}, even if for the 2 mg/cm^2 thick foil. This implies that the envelope calculation throughout the beamline with `TRANSOPTR` is just approximate rather than accurate. In particular, one would never take into consideration all the tails/halos, as measured with profile monitors, to calculate the rms beam sizes, because one can barely distinguish the halo particles from the background noise arisen from e.g. the electronics. Instead, one usually focuses on the core of the beam and makes a cut of the profile tails. This is already making an approximation.

Table 1: Statistically calculated σ-matrix (1rms) of $\sim$ 1 million protons at the exit of carbon foil/target.

Carbon thickness (mg/cm^2)	$< E_k >$ (MeV)	σ_{11} (mm^2)	σ_{22} ($mrad^2$)	σ_{12} (mm-mrad)	r_{12}	σ_{33} (mm^2)	σ_{44} ($mrad^2$)	σ_{34} (mm-mrad)	r_{34}	σ_{66}	σ_{16} (mm)	σ_{26} (mrad)	r_{16}	r_{26}
0	479.422	1.558	0.106	0.248	0.612	7.199	0.064	-0.628	-0.926	1.462e-07	-3.438e-04	-3.155e-05	-0.720	-0.254
1	479.414	1.558	0.170	0.249	0.483	7.199	0.137	-0.629	-0.634	1.465e-07	-3.438e-04	-3.110e-05	-0.720	-0.197
2	479.414	1.558	0.224	0.249	0.421	7.199	0.194	-0.629	-0.532	1.467e-07	-3.438e-04	-3.074e-05	-0.719	-0.169
5	479.407	1.558	0.375	0.249	0.326	7.199	0.323	-0.630	-0.413	1.474e-07	-3.438e-04	-3.062e-05	-0.717	-0.130
10	479.392	1.558	0.657	0.249	0.246	7.199	0.577	-0.630	-0.309	1.486e-07	-3.438e-04	-3.088e-05	-0.715	-0.099
20	479.369	1.558	1.243	0.249	0.179	7.199	1.162	-0.631	-0.218	1.509e-07	-3.438e-04	-3.186e-05	-0.709	-0.074
50	479.294	1.558	2.882	0.249	0.118	7.199	2.714	-0.632	-0.143	1.579e-07	-3.438e-04	-3.492e-05	-0.693	-0.052
100	479.167	1.558	5.103	0.250	0.089	7.199	4.794	-0.632	-0.108	1.692e-07	-3.439e-04	-3.858e-05	-0.670	-0.042
10 mm	475.206	1.564	55.767	0.501	0.054	7.188	56.338	-0.396	-0.020	5.178e-07	-3.488e-04	-3.650e-05	-0.388	-0.007
20 mm	470.978	1.582	113.720	1.335	0.100	7.187	112.127	0.430	0.015	9.009e-07	-3.524e-04	-6.660e-05	-0.295	-0.007
50 mm	458.228	1.817	286.759	7.158	0.314	7.365	280.198	6.102	0.134	2.130e-06	-3.681e-04	-5.224e-05	-0.187	-0.002
100 mm	436.573	3.495	580.213	28.106	0.624	8.936	582.217	27.014	0.375	4.530e-06	-3.930e-04	-3.399e-06	-0.099	-0.000

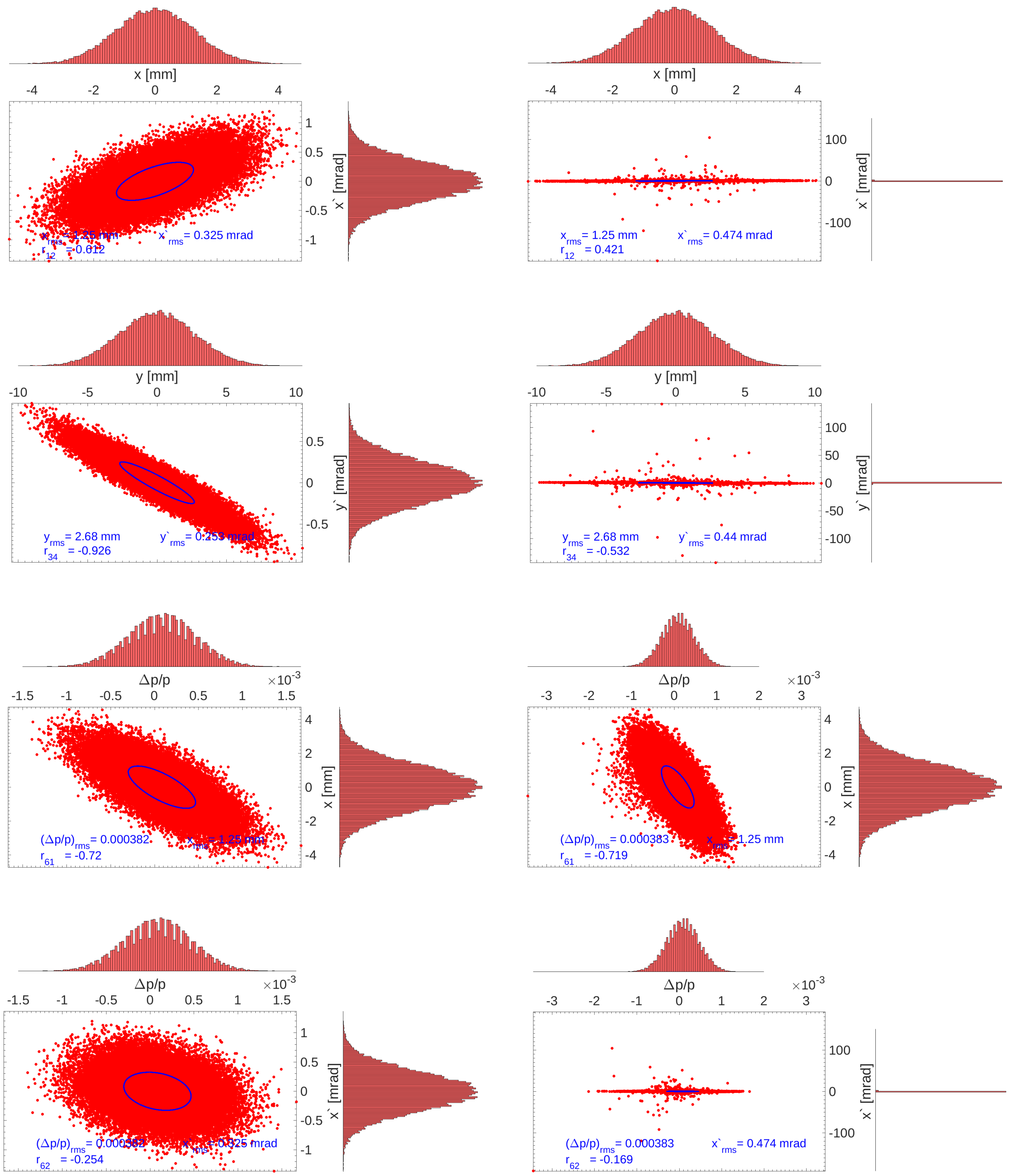

Figure 1: The distribution of particles in the phase spaces (x, x') , (y, y'), $(\Delta p/p, x)$ and $(\Delta p/p, x')$ (from top to bottom) at the exit of carbon foil of 0 (left) and $2\,\mathrm{mg/cm^2}$ (right) thicknesses respectively. Notice the large differences in scales of axes for visualizing the large angle scattering.

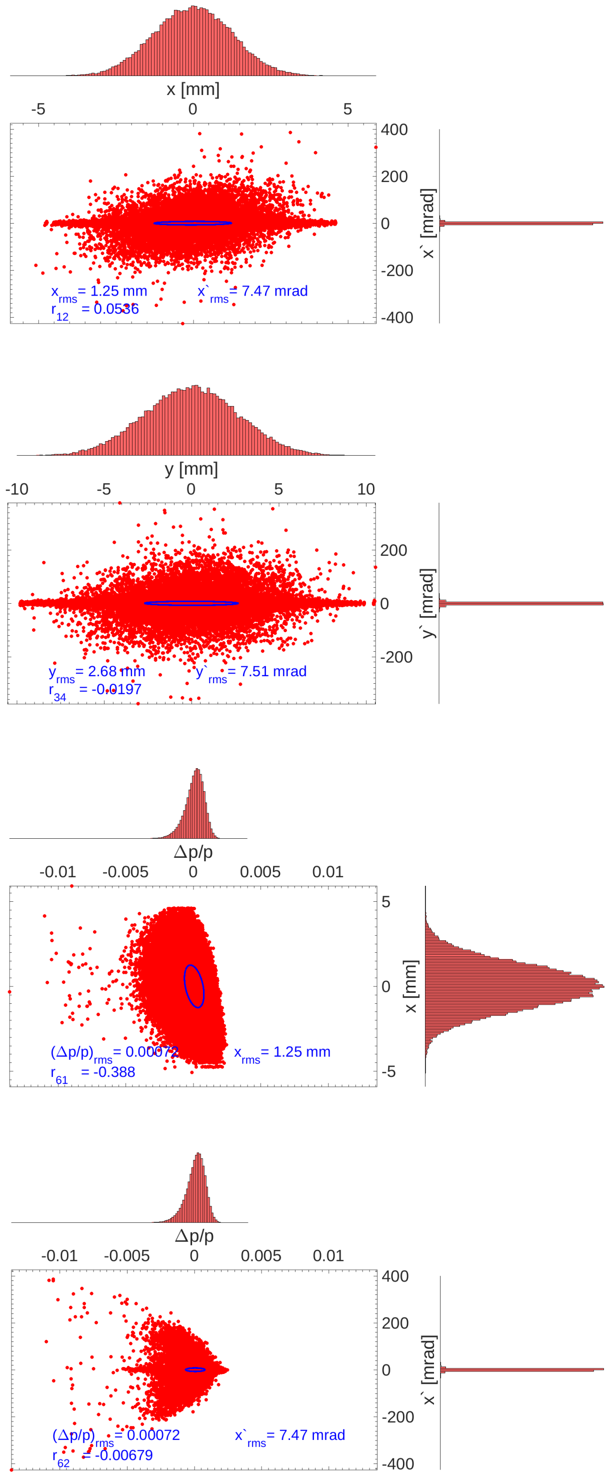

Figure 2: The distribution of particles in the phase spaces (x, x'), (y, y'), $(\Delta p/p, x)$ and $(\Delta p/p, x')$ (from top to bottom) at the exit of carbon target of 10 mm thicknesses. Notice the large differences in scales of axes for visulizing the large angle scattering.